Algorithm for the numerical assessment of the conjecture of a subglacial lake at Svalbard, Spitzbergen

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Abstract

The melting of glaciers coming with climate change threatens the heritage of the last glaciation of Europe likely contained in subglacial lakes in Greenland and Svalbard. This aspect urges specialists to focus their studies (theoretical, numerical and on-field) on such fascinating objects. Along this line we have built up a numerical procedure for validating the conjecture of the existence of a subglacial lake beneath the Amundsenisen Plateau at South-Spitzbergen, Svalbard. In this work we describe the algorithm and significant representative results of the related numerical test. The conjecture followed the Ground Penetrating Radar measurements of that area exhibiting several flat signal spots, sign of the presence of a body of water. Actually, numerical simulation results appear in support to the decision of drilling operations above the presumed ice/water front where subglacial lake water bio-chemicals might be traceable.

The time dependent mathematical model, structuring the numerical algorithm, includes the description of dynamics and thermodynamics of the icefield and of the subglacial lake, with heat exchange and liquid/solid phase change mechanisms at the interface. Critical modeling choices and confidence in the algorithm are granted by the numerical results of the sensitivity analysis versus the contribution of ice water content, of firn and snow layers at top of the icefield and versus the approximation of ice sliding on bedrock, that have been issued in previous recent works also including successful comparison with measured quantities.

Keywords – Temperate ice, Glen's law, Subglacial lake, Phase-change, Large Eddy Simulation, Svalbard, Finite volume.

1 Introduction

Subglacial lakes are a peculiar important component of the basal hydrological system of the Antarctic ice sheet, about 400 in number; in Arctic, just four subglacial lakes have been, so far, clearly identified by airborne radio echo sounding in Greenland (Palmer et al. 2013, Howat et al. 2015 and Willis 2015). Isolated (*not* interconnected) subglacial lakes result a possible source of information tracing back to past geological periods, being such their life length. In those cases, any local inspection by deep drilling operation must be conducted with extreme care in order to avoid water spoiling and preserve the physical, chemical and (presumably) biological equilibria. Studies based on mathematical modelling and numerical simulation are, clearly, encouraged. It is worth to remind that from the study of the subglacial hydrological network in the polar regions, lakes included, hints might be extrapolated also on the characteristics of the sub-surface ocean conjectured underneath the icy crust of several satellites of the solar system (Carr et al. 1998, Mitri & Showman 2005) (e.g. Europa and Enceladus, respectively Jupiter's and Saturn's satellites), possibly revealing extra-terrestrial environments potentially suitable for life.

This work deals with a numerical algorithm for the assessment of the conjecture of a subglacial lake underneath Amundsenisen Plateau at South-Spitzbergen, Svalbard, eventually the first one to be localized there. The interest of this investigation is increased by the fact that likely deposits from the period before glaciation of Europe and Scandinavia might have gathered in the basins.

The structure of the paper is the following: Section 2 deals with the geographical and physical description of the problem; in Section 3, adopted simplifying assumptions are presented and discussed; in Section 4, the numerical algorithm is described; in Section 5 conclusions are drawn on the response of the algorithm versus the conjecture assessment via representative numerical results.

2. Conjecture set-up

Ground-based radio-echo sounding records of the Amundsenisen Plateau have revealed several 'flat' bedrock sections, the largest one being 450 *m* long (Glowacki et al. 2007). Amundsenisen Plateau is an accumulation icefield, 80 km^2 in area, with the thickest firn-ice cover of the archipelago (*631 m* maximum thickness), occupying a large depression surrounded by mountain ridges, controlled by geological structure, with three outlet channels.

From thermal viewpoint this one is a temperate icefield. By reflecting properties and estimates of the hydraulic potential field, the 'flat' bed sections might be interpreted as from near-bottom water bodies, indeed very similar to radar reflection from subglacial lakes.

In the upper left part of Fig. 1, the top surface of Amundsenisen Plateau is plotted with the record of the ice velocity measured at upper reaches of Högsterbreen, Böygisen and Paierlbreen outlet glaciers: the values are 7.22, 10.65 and 9.30 cm d^{-1} respectively (Glowacki et al. 2008). The lower right part of the same figure shows the top surface ice velocity field as it results (Mansutti et al. 2015a) by ignoring the lake and running the 3D ELMER/ICE code up to regime on the geometrical domain occupied by Amundsenisen Plateau (open source http://www.csc.fi/elmer/, only ice dynamics and thermodynamics, refer to Gagliardini et al.

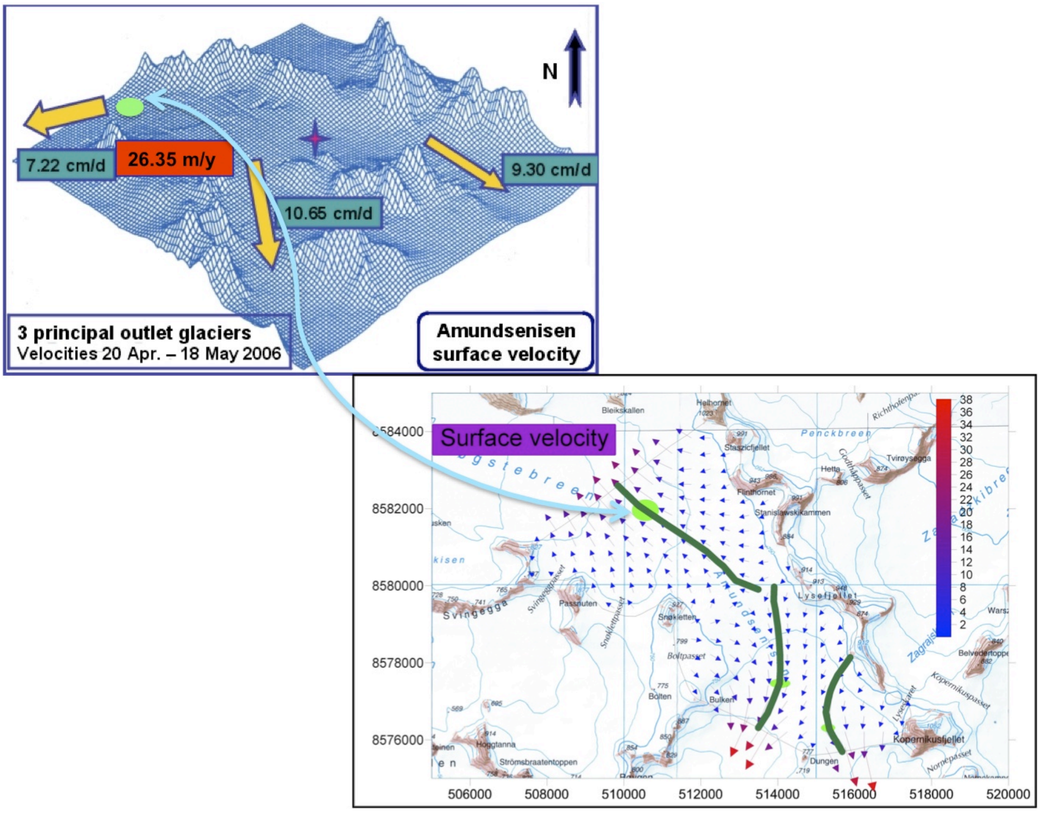

Fig. 1 Amundsenisen Plateau (Svalbard): external surface with measured top surface ice velocity values at outlets (from Glowacki et al. 2008) *(above)* and simulated top surface ice flow field with light-green ovals in correspondence of ground-based radio-echo sounding flat signal spots (from Mansutti et al. 2015a) *(below).*

2013). The dark-green ice flow lines end respectively to the three outlet glaciers. The ground-based radio-echo sounding flat signal spots are highlighted in light-green; we have focussed on the largest one overtopped by the ice flowing to Högsterbreen. It is worth to stress that an ice divide is visible between the considered flow and the central branch. The fact that the flat signal spots are placed beneath a flowing ice stream and in the proximity of an ice divide, where water drainage occurs, is in support to the conjecture of the existence of corresponding subglacial lakes (Ridley et al. 1993). The locations of the focussed lake in the two drawings are connected with a light-blue arrowed line.

3. Simplifying assumptions

The proposed algorithm is linked to some environmental aspects that have inspired important simplifications of the modelling. Here we summarize these items.

i) The spatial structure of the problem can be assumed two-dimensional - This aspect is derived from the fact that, in the preliminary numerical simulation above mentioned that was developed on the real bedrock, ice flow above the considered conjectured lake is parallel rectilinear. In present case, such a structure should persist also when the presence of the lake is included, because of its length (the measure of the lake surface diameter parallel to the ice flow) - about *460 m* – being of the same order of Amundsenisen icefield thickness (Glowacki et al. 2007) - from *420 m* to *631 m*. Mansutti et al. (2015a) have already discussed this point by contrast to arguments that Kwok et al. (2000) introduced for the case of Vostok lake (Antarctica) and inferred that, for its relatively *small* extension, conjectured lake cannot influence the overall sliding behaviour of the overlying icefield and, much less, impress a three-dimensional deviation to a rectilinear parallel ice flow. On the contrary, ERS1-SAR images of the area around Vostok lake exhibit curvy ice flow-lines, effect of the sliding velocity increase upon the subglacial lake surface followed by abrupt containing effect of the downhill rocky bottom. But Vostok lake maximum diameter measures *45 km,* that is ten times the ice cover thickness in Antarctica.

We have developed the numerical simulation on the two-dimensional vertical section parallel to the ice flow-lines where the driving force of the icefield on the lake water is maximum. Supposing that, for the sake of simplicity, the conjectured lake has the shape of a spherical cap, in any other vertical two-dimensional section of the lake, as the buoyancy force supported by the geothermal heat amounts essentially the same, the composition of both forces – ice driven and buoyancy - ends in a system that is weaker than the corresponding one in the simulated section.

In any case we can expect that a more realistic three-dimensional numerical simulation of the conjectured subglacial lake might end in a slower flow field, with the extention of the evolution time of the overall process but, presumably, without changes in the conclusions of the investigation.

ii) Time change of the icefield top surface elevation can be neglected - Several factors contribute to the change of a grounded icefield top surface elevation, most important are the accumulation/ablation of snow on top (with the formation of a firn layer for successive melting and regelation local events), melting at the bottom of the icefield with water drainage through the soil and ice surge occurrences. The mathematical modelling of these aspects is closely linked to environmental data either only episodically available, as it is for the snow fall time series, or technically difficult to collect, as it is for the porosity of the soil, the local value of the geothermal heat flux and the periodicity of ice surges. However, in the case of Amundsenisen Plateau, the laser altimetry data collected by J. Jania, L. Kolondra and P. Glowacki show top surface elevation profiles essentially stationary during the time span

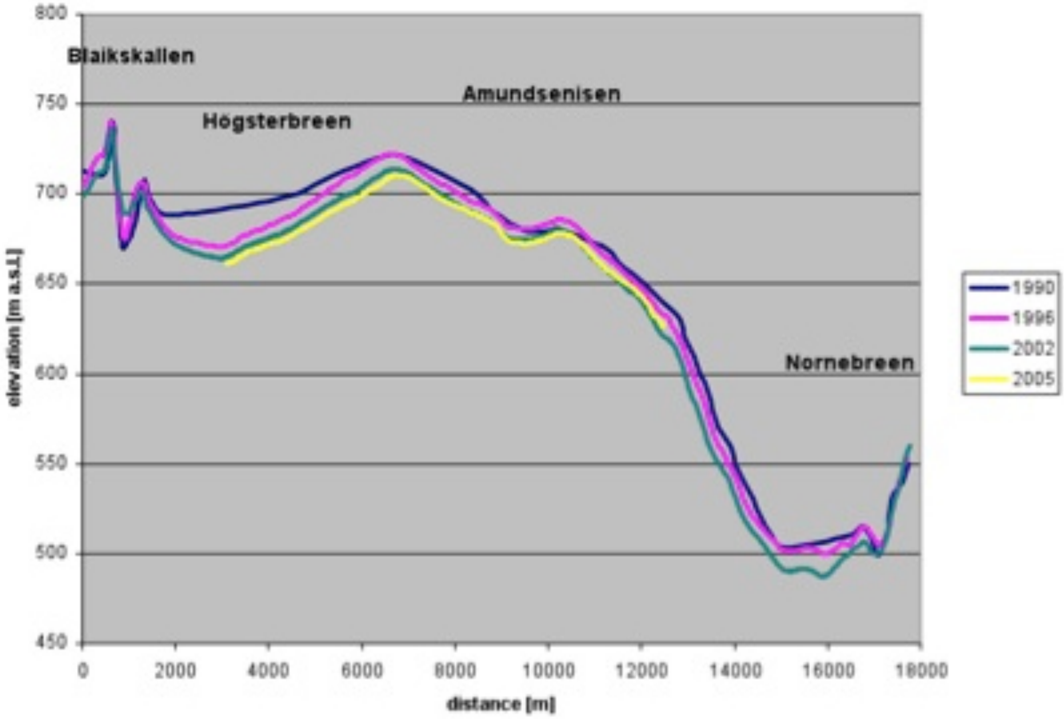

Fig. 2 Top surface elevation along the icefield profile Hogsterbreen-Amundsenisen-Nornebreen registered at several years (unpublished laser altimetry data and picture by J. Jania and L. Kolondra, University of Silesia, and P. Glowacki, Polish Academy of Science).

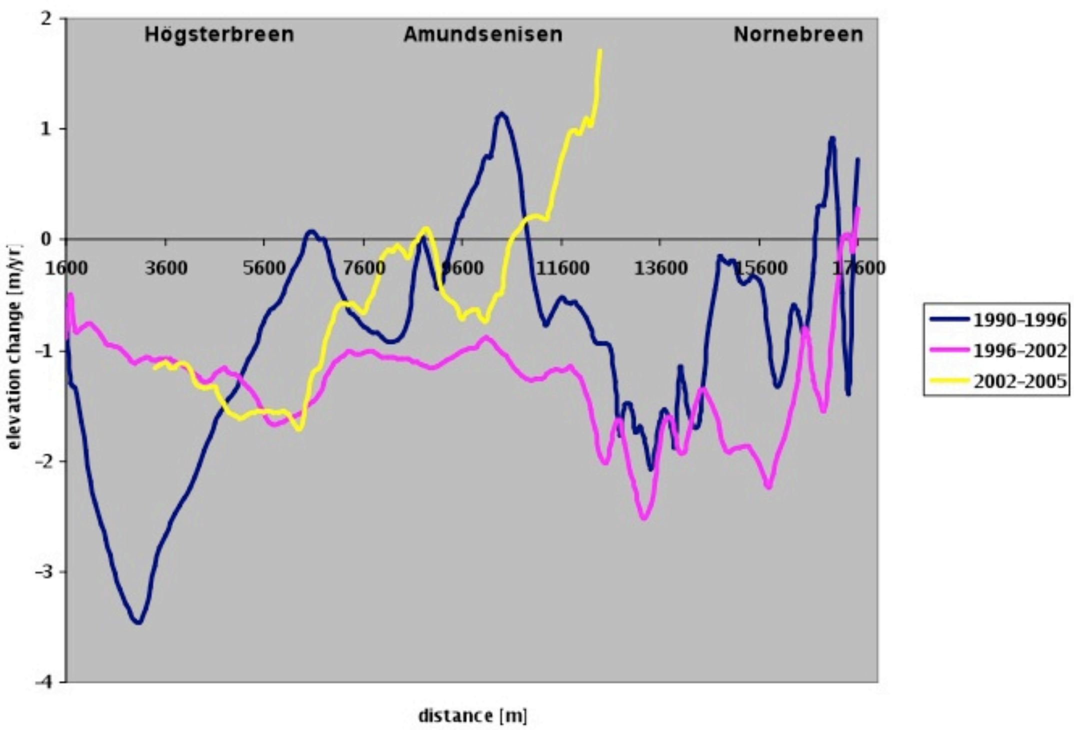

Fig. 3 Annual average elevation change along the icefield profile Hogsterbreen-Amundsenisen-Nornebreen at several successive year intervals (unpublished laser altimetry data and picture by J. Jania and L. Kolondra (University of Silesia) and P. Glowacki (Polish Academy of Science).

1990 – 2005: Fig. 2 and Fig. 3 display the top surface elevation along the profile of interest, Hogsterbreen-Amundsenisen-Nornebreen, and the related annual average elevation change respectively. It is noticeable that when at north-side Amundsenisen (Hogsterbreen) a decrease was captured, an increase of comparable entity corresponded at south-side of it (Nornebreen) so that the spatial average of the elevation change at Amundsenisen is indeed quite small and may be considered negligible. In addition, Nuth et al. (2010) pointed out that observation data on the long-term (over the past 40 years) confirm this trend. This means that the overall effect of snow accumulation/ablation on top surface, water drainage through the bottom rocks and surge occurrences, in terms of ice mass balance, is negligible during this time span. We have assumed this peculiarity as permanent and, then, ignored such processes within mathematical modeling; moreover we have fixed the icefield top surface at the observed position (equilibrium of the air/ice front) and imposed a Weertman-type sliding condition to the ice facing the rocky bottom (effect of the residual water film).

iii) The impact of the temporal changes of air temperature and snow/firn layer temperature on the system can be neglected – For the check of the likelihood of the existence of a subglacial lake, direct impact aspects within the system air/icefield/bedrock, either for thermo-dynamical effects and mechanical ones, are the peculiar temperate state of the ice above the conjectured lake and the geothermal heat flux from the rocky cavity where it would be hosted, that are experimental data. In such a context the snow and firn layer, typically *25 m* thick on top of the temperate ice core (Paterson 2006), must have an insulating effect on the ice underneath so to guarantee that its temperature keeps at melting point: alike a sort of thick interface between air and the icefield core, it undergoes the effect of the external harsh temperature and its seasonal changes, damping the impact on the ice. Conversely no drawback should come in maintaining, as we have chosen to do, the external temperature and the snow/firn temperature profile constant in time, fixed at known values, collected from the literature and from a meteorological database as it follow. From field data on Amundsenisen icefield reported by Zagorodnov et al. (1985), we have acquired the *17 m* in-depth temperature vertical profile along the firn layer. Then we have estimated the temperature vertical profile of the *8 m* thick snow layer covering firn by interpolating the top firn value and the annual average temperature at Amundsenisen Plateau, estimated from the temperature of the Hornsund meteorological station (a.s.l.), http://www.yr.no/sted/Norge/Svalbard/Svalbard_lufthavn_m%C3%A5lestasjon/varsel.rss – the closest one to the tested place available - with altitude correction (in that area amounting to *-1 °C* per *100 m* added altitude). In Fig.4 the resulting plot is sketched.

4. Numerical algorithm

The leading idea, here, has been to check if the inclusion of a subglacial lake underneath the icefield within our numerical simulation yield results compatible with the available measured data. At this purpose, with the assumptions above, we have studied the time evolution of the dynamics and thermodynamics of the icefield and of the subglacial lake within a fixed two-dimensional vertical rectangular section along the upper dark green flow-line in Fig. 1, with a

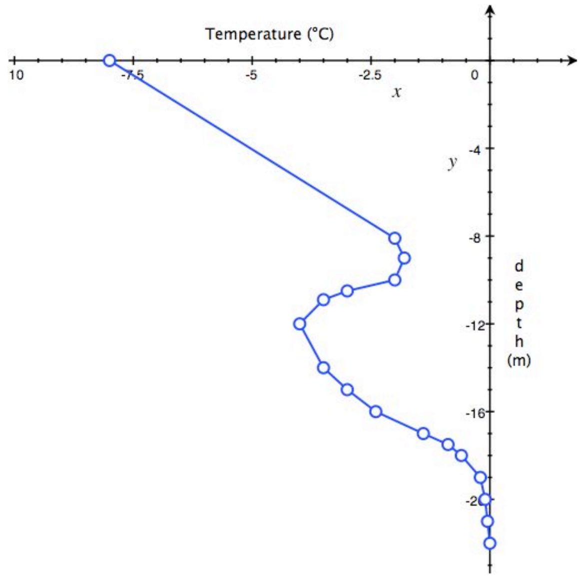

Fig. 4 In-depth temperature profile of the snow/firn layer at Amundsenisen Plateau reconstructed by combining the field data by Zagorodnov et al. (1985) and the annual average temperature at Hornsund as from records of the meteorological station (with correction by altitude).

vertical side in correspondence of an ice-divide (see description in Section 2.) and the opposite one at the beginning of Högsterbreen, outlet glacier. Bedrock topography and top surface of the glacier respond to real data appeared in Glowacki et al. 2007 and Glowacki et al. 2008, whereas the shape of the conjectured subglacial lake resulted from a numerical sensitivity study in Mansutti et al. 2015a.

The numerical algorithm, that we have built up, is based on a mathematical numerical model whose details have been presented and justified in Mansutti et al. 2015a and Mansutti et al. 2015b. Here, we do not provide neither the equations nor their discretized form but we only recall, in the following points, the main elements and, then, focus on the structure of the algorithm.

a) Dynamics and thermodynamics of ice:

- the icefield is at melting temperature that is assumed as function of the pressure field, $T_m(p)$, according to Clausius-Clapeyron law;
- for rheology, Glen's law for temperate ice with n=3, accounting for tertiary creep (Paterson 2006), is adopted. Except in snow and firn layer, ice water content is computed as solution of a transport equation with source term for water released via general strain heating (Hutter 1982 and 1993, Greve 1995). In the upper layer, where only sliding motion occurs, correlation

functions obtained for a similar glacier (Breuer et al. 2006) provide estimates of ice water content in terms of measured temperature values. Intergranular drainage is neglected as permeability of temperate ice is indeed extremely low (Paterson 2006). We solve the unsteady Stokes formulation of the dynamics equations with incompressibility constrain;

- a Weertman-type sliding condition is imposed for the motion of the icefield on the bedrock where a water film forms. As justified in Section 3., point i), same sliding condition is imposed also above counjectured lake surface. Parameter calibration has been developed in order to match with the measured top surface ice velocity at the outlet.

b) Dynamics and thermodynamics of the subglacial lake:

- Large Eddy Simulation (LES) formulation of the unsteady non-isothermal hydrodynamics equations with constant eddy viscosity and eddy diffusion coefficients is adopted. As water is quasi-thermal-incompressible (Gouin and Ruggeri 2012), buoyancy force is included with Chen and Millero's density function (equation of state) (Chen and Millero 1986). After choosing a spatial mesh, the calibration of the eddy coefficients has been accomplished by meeting the order of magnitude of the velocity field of other subglacial lakes (i.g. Vostok lake);

- the estimate of the geothermal heat flux, $q_{geo}=37.5\ mW/m^2$, is taken from literature referring to similar environment.

c) Ice/water phase change process:

- at the phase front, in line with the related conservation laws, the continuity of momentum, mass and energy is imposed via the so-called jump conditions. The phase-change mechanism, as release/absorption of latent heat, contributes to the energy balance and provides the local value of the progression velocity of the icefield/subglacial lake front.

d) The equations resulting from a), b) and c) are coupled; they are completed with boundary conditions corresponding to the natural set-up so to form a closed partial differential system. This one is solved via a second order space and time finite volume discretization method and with a linearization technique, which preserves good coupling among the equations. At each time step and for each phase, a linear system is solved and the updated value of processed variables is promptly introduced in next step. At each time iteration phase front is re-drawn, according to the output from energy jump condition, space mesh in both phase domains is adjusted and related discretized variable values are re-computed via linear interpolation on the new points (extrapolation might be required only at phase front).

The computational algorithm can be summarized with the pattern illustrated in Fig. 5: the sketched time cycle includes the computational procedure for one time step numerical integration of the equations for the subglacial water (package I), for the icefield (package II) and for the ice/water interface (package III); package IV) re-computes the space mesh of the updated icefield/lake domain. As water time scale is much smaller than ice time scale, we have obtained a significant cut-off of the computing time by advancing in time solely water solution – with a proper fractional time step - up to reach a cumulative time step suitable for ice, when package II and package III are run to update ice variables and ice/water front position.

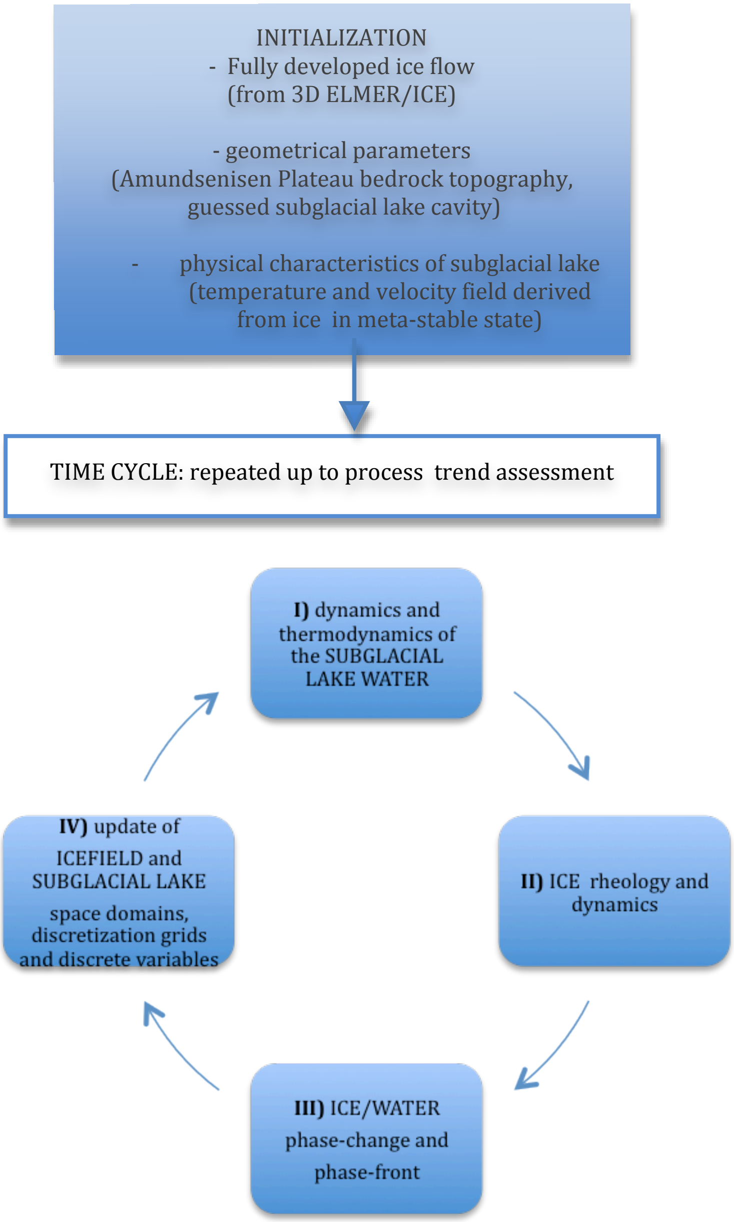

Fig. 5 Plan of the computational algorithm

In this context, we show that a lower limit of time step for the integration of ice and subglacial water differential equations, called $\Delta t_i^{\min}$ and $\Delta t_w^{\min}$, differ approximately of three orders of magnitude. This result is obtained considering the order of magnitude of the estimate of the maximum value of ice and subglacial water velocity fields, respectively $O(u_i^{\max})$ and $O(u_w^{\max})$, and the minimum step of the adopted finite volume discretization mesh of the two phases equations, respectively $\Delta s_i^{\min}$ and $\Delta s_w^{\min}$:

$$\Delta t_i^{\min} = \frac{\Delta s_i^{\min}}{O(u_i^{\max})} = \frac{2m}{8.23\cdot 10^{-7}\, m/s} = 2.4\cdot 10^6\, s,$$

$$\Delta t_w^{\min} = \frac{\Delta s_w^{\min}}{O(u_w^{\max})} = \frac{1\, m}{10^{-3}\, m/s} = 10^3\, s.$$

The above implies that, within the chosen space resolution for the numerical solution of the two phases equations, in order to capture a change in time of the magnitude of the ice velocity field, it is required a time step amounting at least to $\Delta t_i^{\min} \approx 10^3 \cdot \Delta t_w^{\min}$, that is 10^3 times the time step suitable to capture a change in time of the magnitude of the water velocity and temperature fields.

With the above considerations we obtain a safe of computing time equivalent to 10^3 times the computing time of ice phase one time step.

Another important aspect for the choice of the time step is numerical stability imposing upper limitations that we have managed by trial-and-error.

5. Numerical results. Conclusions

As benchmark quantities for the numerical simulation we have adopted the measured value of the top surface ice velocity at the outlet glacier (Högsterbreen), that amounts to *26.35 m/y* (see Fig. 1), and the interface profile between the supposed subglacial lake and the icefield which is known by Ground Penetrating Radar (GPR) measurements. Prior to comparison, it is worth to remind that the measured surface ice velocity value is subject to an intrinsic error as it results from a local (in space and time) measurement operation. About the second target, it is known that GPR measurements undergo a systematic error of *+/- 3.5 m.*

From initial time *t = 0 d,* we have pushed forward the numerical simulation up to time *t = 20000 d* , when the ongoing process has clearly revealed its trend.

In Fig. 6 we plot the computed ice horizontal velocity iso-surfaces and related vertical profile at the outlet at time *t = 20000 d*; a subglacial lake is conjectured in the abscissa subinterval [640, 1060] (it is not drawn here). It is apparent that the numerical top surface velocity at the outlet reaches the value *30 m/y,* that is affected by 14% relative error vs. measured value. Ice initial flow was assigned equal to the preliminary simulation result computed with 3D ELMER/ICE, partially shown in Fig. 1 (bottom).

In Fig. 7 a time sequence of numerical icefield/subglacial lake interfaces is shown. The initial profile was assigned equal to that one acquired from the GPR survey; it can be seen that, as the simulated time goes on, the lake front converges within a range certainly covered by the typical GPR measurement error. This implies that, within set conditions and time *t = 20000 d*, the inclusion of a subglacial lake in the numerical simulation leads to results in agreement with measured datum. Finally, let us examine how flow and temperature fields of ice evolve in the cavity of the conjectured lake. Fig. 8 (top) shows the temperature iso-surfaces and the vector plot of the horizontal velocity along the vertical mid-line at initial time when it is supposed that

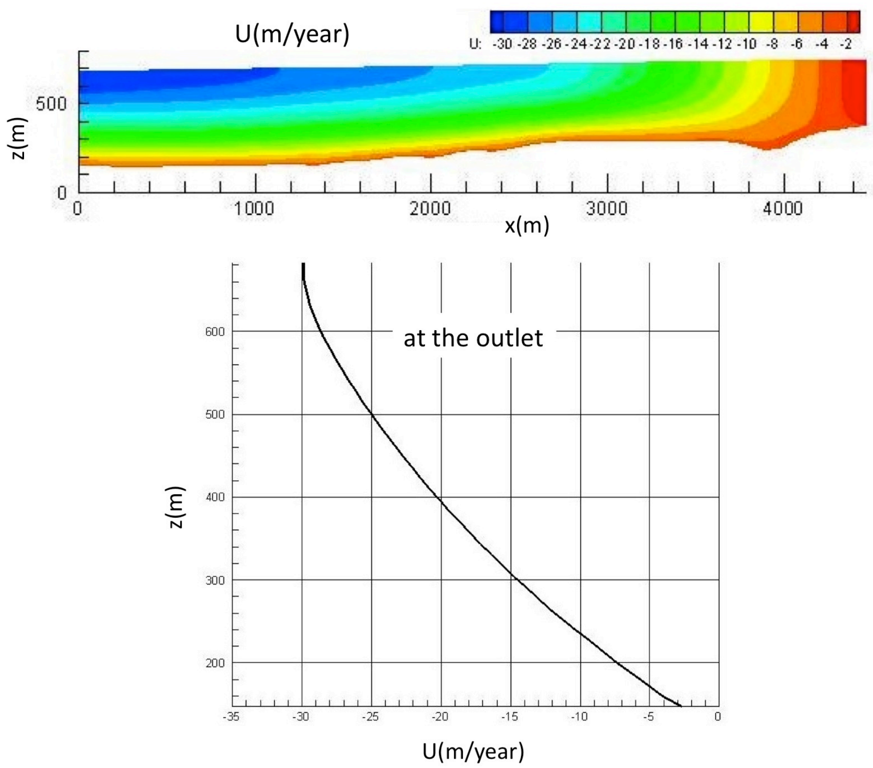

Fig. 6 Ice horizontal velocity at steady regime, *t =20000 d*: iso-surfaces (top) and vertical profile at the outlet.

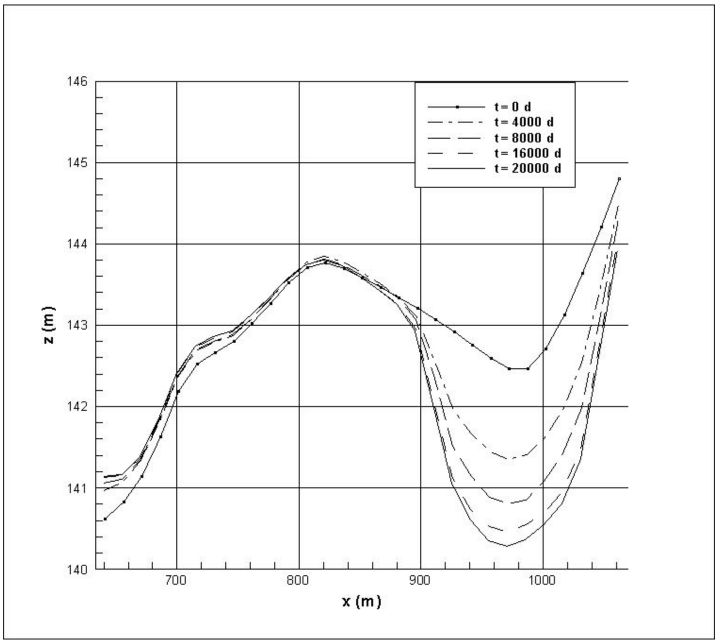

Fig. 7 Time evolution of the icefield/subglacial lake interface.

the ice there has overcome metastability with a temperature just slightly over melting point and, being still similar for rheology and dynamical behaviour, flows together with the ice of the overlain icefield. Fig. 8 (bottom) shows the temperature iso-surfaces and the particle tracks in the basin cavity at final simulated time: it is clearly visible a definite increase of temperature that confirms the progress of the ice/water phase change, and also the onset of a weakly asymmetric recirculation driven by the icefield motion and enforced by the geothermal heat flux. The comprehension of the trend of the overall process is provided by Fig. 9 where we have reported a time sequence of metastability lines, separating the newly formed water from

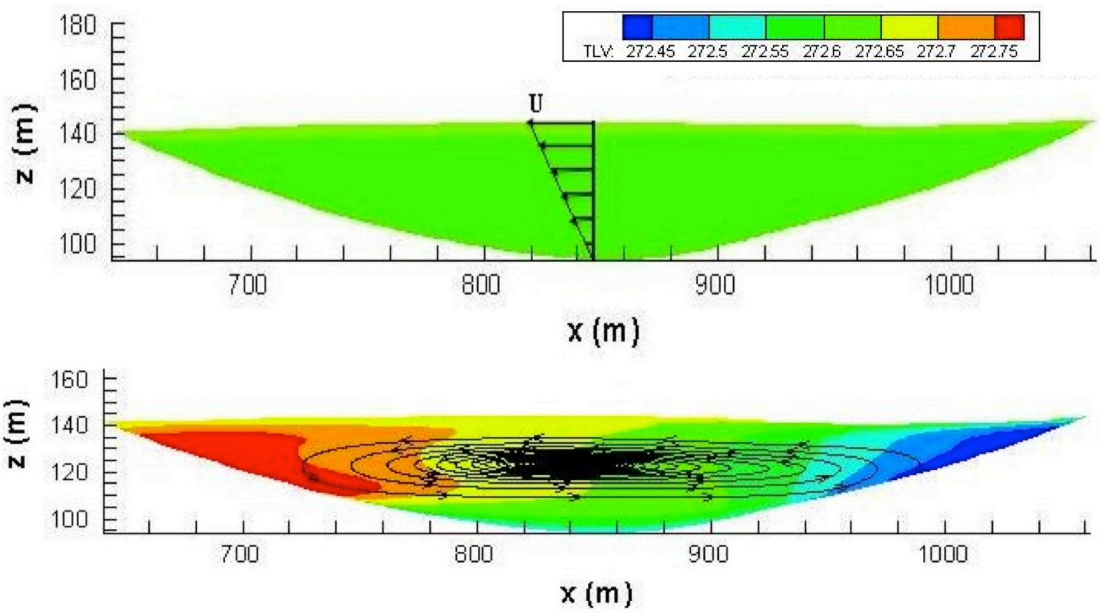

Fig. 8 Subglacial lake: temperature distribution and horizontal velocity along the vertical midline at *t=0 d* (top) and temperature iso-surfaces with particle tracks at *t = 20000 d* (bottom).

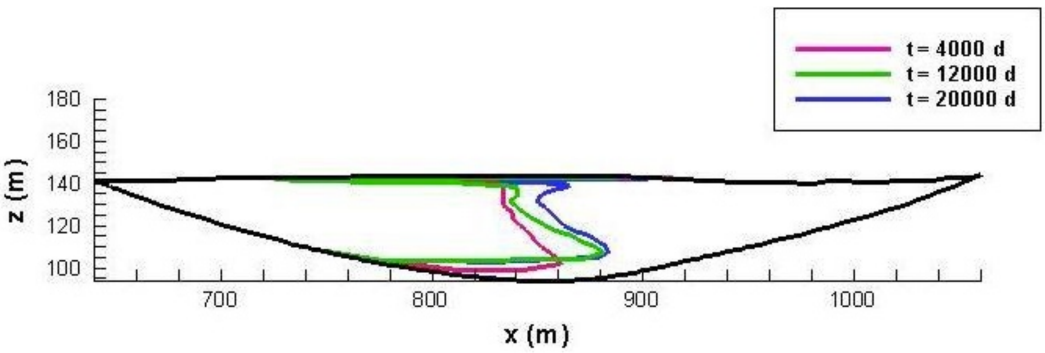

Fig. 9 Subglacial lake formation: time progression of the metastability line attesting the increase of the molten ice mass.

the metastable ice: we notice that such curve moves forward that is the size of the cavity occupied by water grows in time due to ongoing melting.

Within set conditions, we can, ultimately, conclude that numerical simulation results are compatible with the formation of a subglacial lake and support the decision to undertake on-field investigations aimed at further characterization of the basin.

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