Muon fluxes derived from the directly measured primary cosmic-ray elemental spectra using hadron-proton machine interaction results

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Summary. — The directly measured elemental spectra of primary cosmic rays obtained from JACEE, MSU, SOKOL and CRN data on p, He, CNO, Ne-S and Fe fluxes have been considered for the calculation of elemental nucleon spectra at the top of the atmosphere. Treating these spectra as parents of the secondary mesons, the sea-level muon fluxes at zenith angles 0° and 89° from each species have been derived using *Z*-factors based on the FNAL cross-section data for meson production and duly corrected for A-A collisions. The conventional muon spectra at 0° and 89° estimated from the individual elemental spectra and the prompt muon contribution arising from the charmed meson decays have been found comparable with the directly measured muon fluxes determined by Thompson *et al.*, Allkofer *et al.*, Matsuno *et al.*, Khalchukov *et al.*, Ivanova *et al.*, Afanesieva *et al.*, Bakatanov *et al.*, Gettert *et al.* and Zatsepin *et al.* The muon spectra derived from the primary nucleon spectrum with a constant spectral index of value -2.73 are found in approximate agreement with the present results.

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1. – Introduction

The secondary cosmic-ray spectra in the atmosphere are strongly dependent on the elemental abundance of primary cosmic rays. The recent investigation of the Fuji Kanbala experiment [1] shows the absence of knee of the primary cosmic-ray proton spectrum beyond 1000 TeV energy. Moreover, the fits to the directly measured primary elemental fluxes at TeV energies obtained from the balloon-borne results of JACEE [2], MSU [3] along with the satellite-borne SOKOL [4] and CRN [5] experiments exhibit different primary spectral indices. It is found from the survey of Swordy [6] that below 1 PeV energy these spectral shapes differ from one another with the EAS predictions of Maryland and MIT experiments [7]. Till now due to the limitation of accelerator experiments in the search for the high-energy A-A collisions and also for the lack of proper cascade distribution formulation, the precise estimation of the secondary cosmic-ray spectra in the atmosphere is difficult.

In the present investigation we have taken the elemental fluxes obtained from the direct measurements [2-5] to estimate the spectra of p, He, CNO, Ne-Si and Fe at the top of the atmosphere for the derivation of muon spectra at zenith angles 0° and 89° from each individual elemental primary spectra using standard formulations. The sum of total muon fluxes from non-prompt and prompt meson decay has been compared with the observed results of Thompson *et al.* [8], Allkofer *et al.* [9], Matsuno *et al.* [10], Khalchukov *et al.* [11], Ivanova *et al.* [12], Afanasieva *et al.* [13], Bakatanov *et al.* [14], Gettert *et al.* [15] and Zatsepin *et al.* [16].

2. – Nuclear physics and kinematics

The differential primary cosmic-ray elemental spectra for the i-th species usually follow the power law

$$n_i(E)\,\mathrm{d}E = K_i E^{-\gamma_i}\,\mathrm{d}E, \tag{1}$$

where K_i and γ_i are the elemental spectral amplitudes and indices of the i-th species. Using the conventional superposition model, one can convert the primary nuclei fluxes to nucleon fluxes by the relation

$$N_i(E)\,\mathrm{d}E = A_i K_i E^{-\gamma_i}\,\mathrm{d}E, \tag{2}$$

where A_i represents the mass number of the i-th elemental species.

We have used the Fermilab Single Arm Spectrometer data on the Lorentz invariant differential cross-sections as functions of x_{F} Feynman variable for $\pi^{\pm}$ and $\mathrm{K}^{\pm}$ productions at 175 GeV/n after Brenner *et al.* [17] to estimate the fractional energy moments for meson production. The usual p_{T}-integrated invariant cross-sections follow the form

$$x(\mathrm{d}\sigma/\mathrm{d}x) = C(1-x)^n, \tag{3}$$

and the Z-factor for the $\mathrm{a}+\mathrm{b}\to\mathrm{c}+\mathrm{X}$ inclusive reactions can be estimated using the form

$$Z_{\mathrm{ac}} = \frac{C\Gamma(\gamma)\Gamma(n+1)}{\sigma_{\mathrm{in}}\Gamma(\gamma+n+1)}. \tag{4}$$

The constants C and n are fitting parameters of the p_{T}-integrated invariant cross-sections obtained from the fits to inclusive reactions for $\pi^{\pm}$ and $\mathrm{K}^{\pm}$ productions [17]. The Z-factors for pp, πp and Kp collisions can be corrected for pA and AA collisions using the procedure of Minorikawa and Mitsui [18]. The production spectra of pions and kaons from each elemental species can be estimated using the form

$$i(E)\,\mathrm{d}E = (Z_{\mathrm{pi}^+} + Z_{\mathrm{pi}^-}) N_{\mathrm{i}}(E)\,\mathrm{d}E. \tag{5}$$

The kinetic equation for the passage of charged i ($\pi^{\pm}$ or $\mathrm{K}^{\pm}$) mesons in the atmosphere is:

$$\frac{\delta i}{\delta E} = \frac{i(E)\exp[-y/\Lambda_{\mathrm{p}}]}{\lambda_{\mathrm{p}}} - (1/\lambda_{\mathrm{i}} + H_{\mathrm{i}}/[yE])\,i(E,y), \tag{6}$$

where λ_{p} and λ_{i} are the interaction mean free paths of nucleons and i-mesons in the atmosphere, respectively; H_{i} is the critical energy of i-meson decay; Λ_{p} is the absorption mean free path of nucleons in air. The solution of the above equation for i-mesons is generated through $pp \to i^{\pm} X$ inclusive reaction channels considered for muon flux calculation from N-generations of the parent mesons by using the procedure of Bhattacharyya and Pal [19]

$$M_{pi}(E) = \alpha^{(N-1)\gamma} \cdot D_{pi}^{N}(E), \tag{7}$$

where α is the elasticity, and

$$D_{pi}^{N}(E) = N_i(E) \langle Z'_{pi} \rangle^{N} A_i \langle \lambda_i / \lambda_p \rangle^{N} \frac{1}{(N-1)!} \cdot \sum_{m=1}^{10} \frac{(1-\lambda_i/\Lambda_p)^{m-1} m(m+1)\dots(m+N-1)}{1+(m+N-1) R_i E/H_i(\theta)}. \tag{8}$$

$N_i(E)$ is the primary elemental flux of species i;

$$A_i = \frac{1 - r_i^{2(\gamma+1)}}{(\gamma+1)(1-r_i^2)}; \qquad R_i = \frac{(\gamma+2)}{(\gamma+1)} \left[\frac{1-r_i^{2(\gamma+1)}}{1-r_i^{2(\gamma+2)}} \right], \tag{9}$$

$r_i = (m_i^2 + m_\mu^2)/(2m_i^2)$; $H_i = m_i c^2 H_i/(c\tau_i)$; m_i and τ_i are the mass and lifetime of i-mesons (*viz.* $i = \pi^{\pm}$ or $K^{\pm}$), respectively.

The muon flux from the first generation ($N = 1$) by high-energy pions in collisions with the atmospheric nuclei through the inclusive reaction channel $\pi^{\pm} p \to \pi^{\pm} X$ can be estimated by following the procedure of Erlykin *et al.* [20]:

$$M_{\pi\pi}^{1}(E, y) = \frac{\lambda_\pi A_\pi Z'^{A}_{\pi\pi} N(E)}{\lambda_p [1 + 2R_\pi E/H_\pi(\theta)]} [1 - b_{\pi\pi} (H_\pi(\theta)/r_\pi E)^{a_{\pi\pi}}]. \tag{10}$$

The total differential spectrum of atmospheric muons at a depth y_0(g cm^{-2}) may be estimated from the decay of atmospheric mesons using the relation:

$$M_\mu^{atm}(E, y_0) = \left[\sum_{N=1}^{10} M_{p\pi}^{N}(E, y_0) + \sum_{N=1}^{10} b_{K\mu} M_{pK}^{N}(E, y_0) + M_{\pi\pi}^{1}(E, y_0) \right] W(E, y, y_0), \tag{11}$$

where $b_{K\mu}$ is the branching ratio for $K \to \mu_2$ decay. $W(E, y, y_0)$, representing the survival probability of muons produced at a mean atmospheric depth $y = Y \sec\theta^*$ reaching a depth $y_0 = Y_0 \sec\theta^*$, can be calculated for the respective vertical depths Y and Y_0 for muon production usign the form

$$W(E, y, y_0) = \left[\frac{y}{y_0} \frac{E}{E + (\alpha + \beta E)(y_0 - y)} \right]^{H_\mu \sec\theta^* / [E + (\alpha+\beta E) y_0]}, \tag{12}$$

where θ^* is the effective angle of incidence with the vertical direction, α and β are the energy loss parameters; the first one accounts for the loss due to the ionisation and the other due to pair production, bremsstrahlung and nuclear interaction at the time of propagation through the atmosphere. The usual energy loss formulation for muons is

$$-\frac{dE}{dx} = \alpha + \beta E. \tag{13}$$

The contributions of direct muons from D-mesons and Λ_c-hyperons have been accounted for from the primary nucleon spectrum with spectral amplitude A and index γ following the recent procedure of Pal *et al.* [21] which is based on the conventional standard formulation:

$$G^{\mu}(E_{\eta}, y, \theta) = A \sum_{n=0}^{\alpha} (I_{n\eta}/\lambda_{N})(-1/\Lambda_{p} + 1/\lambda_{\eta})^{n-1}(y^{n}/n!)\cdot \cdot E_{\eta}^{-(\gamma+1)} \exp[-y/\lambda_{\eta}][1 + H_{\eta}^{cr}(\theta)/(nE_{\eta})]^{-1}, \tag{14}$$

where the term $I_{n\eta}$ represents the number of D-mesons or Λ_c-hyperons at an energy E_{η} produced in the atmosphere due to cosmic nucleon-air collisions and can be calculated using the relation

$$I_{n\eta} = \int_{0}^{1} dx (A^{\alpha}/\sigma_{in}^{pp}) \left(x \frac{d\sigma}{dx} \right)_{pp \rightarrow iX}. \tag{15}$$

The integrated Lorentz-invariant cross-section $x(d\sigma/dx)$ for the $pp \rightarrow D^{\pm} X$, $pp \rightarrow \Lambda_c^{+} X$, $pp \rightarrow D^{0} X$, $pp \rightarrow \overline{D}^{0} X$ inclusive reactions can be considered from conventional QCD calculations of Bugaev *et al.* [22]. The term $dF^{\mu}(E_{\eta}, E_{\mu})/dE_{\mu}$ is the probability of generating muons in the interval E_{μ} and $E_{\mu} + dE_{\mu}$ due to the decay of η-particle and taken from Pal *et al.* [21]. The prompt muon spectrum from the decay of η-charmed meson from $D \rightarrow K + \mu + \nu$ and $\Lambda_c \rightarrow \Lambda_0 + \mu + \nu$ decay for $E_{\eta} \ll H_{\eta}^{cr}(\theta)$ may have been estimated by neglecting the energy loss and decay of muons in the atmosphere by using the relation

$$P_{\eta}^{\mu}(y, E_{\mu}, O^{0}) = \frac{\Lambda}{\lambda_{N}} B_{N} W_{sl}^{\eta} \int_{E_{\eta}^{min}}^{E_{\eta}^{max}} E_{\eta}^{-(\gamma+1)} I_{N\eta} \frac{dF_{\mu}(E_{\eta}, E_{\mu})}{dE_{\mu}} dE_{\eta}, \tag{16}$$

which incorporates the three-body decay of charged particles $D \rightarrow K\mu\nu$ and $\Lambda_c \rightarrow \Lambda_0 \mu\nu$. The other terms are $E_{\eta}^{max} = E_{\mu}(m_{\eta}/m_{\mu})^2$ and $E_{\eta}^{min} = E_{\mu}/[1 - (m_K/m_{\eta})^2]$ for $E_{\mu} \ll H_{\eta}^{cr}(\theta)$; $I_{n\eta}$ may be evaluated from expression (15); the term $dF^{\mu}(E_{\eta}, E_{\mu})/dE_{\mu}$ is a three-body phase space integral [23] with the usual form

$$\frac{dF^{\mu}(E_{\eta}, E_{\mu})}{dE_{\mu}} = = \frac{1}{R_3 E_{\eta}} \left| \left[1 - \frac{E_{\mu}}{E_{\eta}} \right] (0.96 m_{\eta}^2) - m_K^2 \left(1 + \ln \frac{(1 - E_{\mu}/E_{\eta})(0.96 m_{\eta}^2)}{m_K^2} \right) \right|. \tag{17}$$

The corrected kinematical form of R_3 due to three-body decay after Mitsui *et al.* [24] is

$$R_3 = \frac{\pi^2}{4 m_{\eta}^2} \int_{m_K^2}^{(m_{\eta} - m_{\mu})^2} dM^2 \frac{(M^2 - m_K^2)}{M^2} \left| [M_{\eta}^2 - (M + m_{\mu})^2][m_{\eta}^2 - (M - m_{\mu})^2] \right|^{0.5}; \tag{18}$$

using the relations (15) to (18) and the usual symbolic conventional notations, one can evaluate the contribution of the direct muon fluxes from the charmed-particle decays which can be added to the spectrum of muons from the atmospheric π and K meson decays to obtain the total high-energy muon spectrum.

3. – Results and discussion

The fits to elemental spectra of primary cosmic nuclei obtained from the balloon-borne direct measurements of JACEE [2] and MSU [3] and the satellite-borne experimental results of SOKOL [4] and CRN [5] surveyed by Swordy [6] are found to obey a power law behaviour. Figures 1 and 2 show the p, He, CNO, Ne-S and Fe spectra obtained from the directly measured data [2-5] along with the best-fit lines using relation (1) whose parametric values K_i and γ_i are presented in table I. The constancy of the elemental spectral indices is almost absent. With the application of the superposition model we have estimated the elemental spectra of p, He, CNO, Ne-S and Fe using relation (2) and the spectral parameters of $N_i(E)$, *i.e.* A_i, K_i and γ_i are displayed in table I.

We have found that the FNAL data [17] for p_T-integrated invariant cross-section follow relation (2), and the fitted parameters C and n are shown in table II for $\pi^\pm$ and $K^\pm$ production. The estimated fractional energy moments, *viz.* Z-factors for $\pi^\pm$ and $K^\pm$ productions estimated by using the relation (3), are corrected for nucleus-nucleus collisions using the straightforward formulation of Minorikawa and Mitsui [18] taking the constant values of $\sigma_{\text{p-air}}$ and $\sigma_{\pi\text{-air}}$ cross-sections as 273 mb and 213 mb from the observations of Boziev *et al.* [25]. The Z_{pi}^{A} is assumed to be energy independent and has an error of $\sim 7\%$ and the energy dependence of p-air collision is neglected since the meson flux is not sensitive to the growth of the cross-section with energy. The derived Z-factors for different inclusive reactions have been displayed in fig. 3. The Z-factors for A-A collision have an average error of about 7‰. We have considered the relations (5)-(10) for the evaluation of muon spectra derivation on Earth at zenith angles 0° and

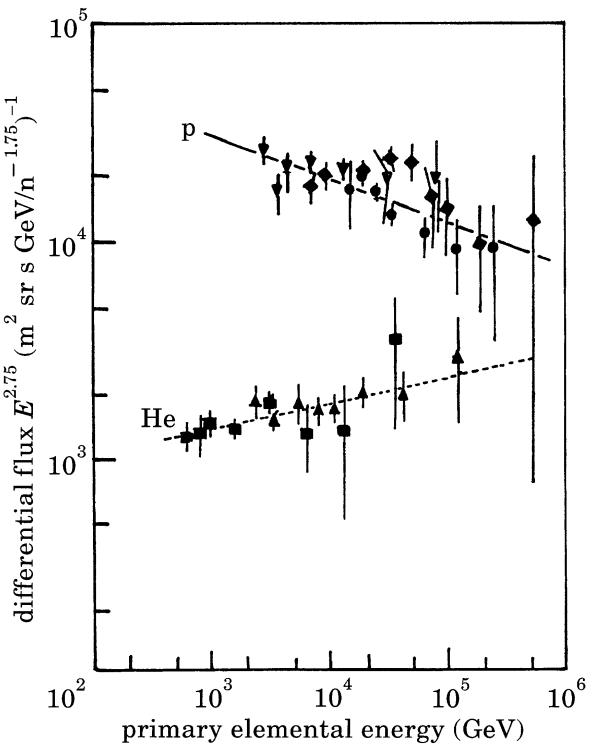

Fig. 1. – Energy spectra of p and He nuclei obtained from the direct measurements using balloons and satellites surveyed by Swordy [6]: Experimental results: p: ◆ JACEE [2]; ● MSU [3], ▼ SOKOL [4]. He: ▲ JACEE [2]; ■ SOKOL [4]. Full and broken curves are the power law fits to the p, He fluxes whose spectral amplitudes and indices are shown in table I.

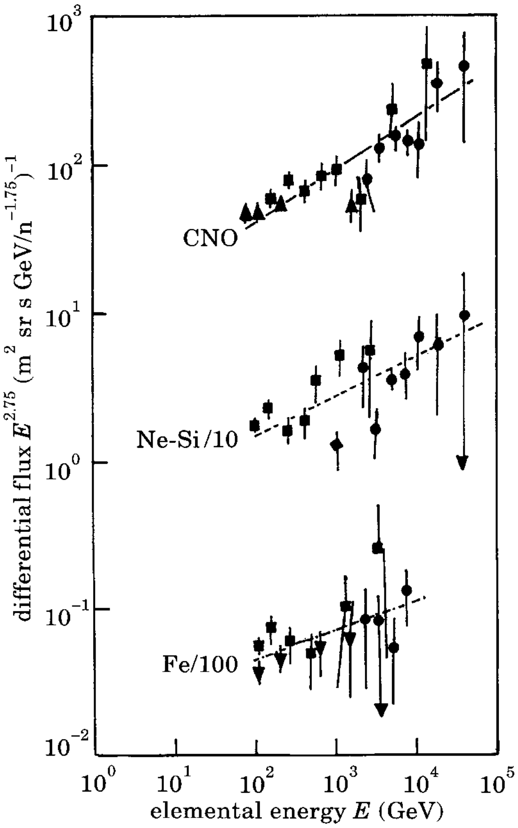

Fig. 2. – Energy spectra of primary cosmic-ray CNO, Ne-Si and Fe nuclei fluxes directly measured using balloons and satellites surveyed by Swordy [6]; CNO fluxes: ● JACEE [2]; ■ SOKOL [4]; ▲ CRN [5]. Ne-Si fluxes: ● JACEE [2], ■ SOKOL [4]; ◆ CRN [5]. Fe fluxes: ● JACEE [2], ■ SOKOL [4]; ▼ CRN [5]. Full, broken and chain lines are the fits to CNO, Ne-Si and Fe nuclei fluxes data whose parametric values are displayed in table I.

89° using the parameters cited in table III along with the following interaction parameters: $\lambda_p = 85$ g cm^{-2}, $\lambda_\pi = \Lambda_p = 110$ g cm^{-2}, $\lambda_K = 150$ g cm^{-2}, $r_\pi = 0.78$, $r_K = 0.52$, $a_{\pi\pi} = 0.579$, $b_{\pi\pi} = 0.271$, $\alpha = [1 - (1 - \lambda_p/\Lambda_p)^{1/\gamma}]$, $b_{K\mu} = 0.635$; $W(E, y, y_0)$ is

TABLE I. – *Amplitudes and indices of the elemental primary fluxes obtained from the superposition model.*

Nuclei	A_i	K_i (m^2 s sr GeV)$^{-1}$	$K = A_i \cdot K_i$ (m^2 s sr GeV)$^{-1}$	γ_i
p	1	$12 \cdot 10^4$	$12 \cdot 10^4$	2.95
He	4	520.17	2080.68	2.62
CNO	14	7.62	106.68	2.39
Ne-Si	24	4.49	107.76	2.49
Fe	56	1.78287	99.84	2.55

TABLE II. – *The table shows the fitted parametric values like c and n of the invariant cross-sections after relations* (3) *of different charged particles produced in the inclusive reactions* $a + b \to c + X$.

Inclusive reactions	c (mb)	n
$pp \to \pi^+ X$	24.7	3.50
$pp \to \pi^- X$	16.77	4.36
$pp \to K^+ X$	2.38	2.77
$pp \to K^- X$	2.21	5.48
$\pi^+ p \to \pi^+ X$	15.54	1.495
$\pi^+ p \to \pi^- X$	13.88	3.56
$\pi^- p \to \pi^+ X$	13.25	3.75
$\pi^- p \to \pi^- X$	14.46	1.515

TABLE III. – *Effective zenith angle, atmospheric depths, critical energies for meson decay and other cosmic propagation parameters.*

Effective zenith angle θ^*	Vertical 0°	Large zenith angle 89° 84.4°
Production depth y	100 g/cm^2	1025 g/cm^2
Final depth y_0	1033 g/cm^2	27600 g/cm^2
Critical energies:		
for pion decay H_π	121 GeV	840 GeV
for kaon decay H_K	897 GeV	6450 GeV
for muon decay H_μ	1.055 GeV	10.81 GeV

estimated using relation (10), where $H_\mu = 1.055\,\text{GeV} \sec\theta^*$, θ^* is the effective zenith angle for 0° and 84.8°. The muon energy loss parameters α and β in the atmosphere have been taken from Zas *et al.* [26] and are found to be of values 0.002 GeV cm^2 g^{-1} and $6.25 \cdot 10^{-7}$ cm^2 g^{-1}, respectively; the values of the production depth are found to be $y = 100$ g cm^{-2}, $y_0 = 1033$ g cm^{-2} for muon traversals at 0° and $y = 1025$ g cm^{-2}, $y_0 = 27600$ g cm^{-2} for muon traversals at 89° zenith angles, respectively. Using the above-mentioned interaction parameters and other results from table I-II, the spectra of sea level muons from the decay of pions and kaons have been estimated from different primary cosmic-ray elemental spectra and are displayed in fig. 4 and 5 along with the total muon spectra from π and K decay at zenith angles 0° and 89° calculated from the single-spectral–indexed primary nucleon spectrum of the form $\sim 2.75\,E^{-2.73}$ obtained in a recent investigation [27].

The inclusive spectra of $pp \to \eta^{\pm} X$ for $\eta = D$ or Λ_c have been found to follow the form (3) whose parametric values from the QCD calculations after Bugaev *et al.* [22] are displayed in table IV and the estimated $I_{n\eta}$ from the relation [15] are displayed in table V for different primary spectral indices γ_i. The values of $H_\eta^{cr}(\theta)$ are shown in table VI. The charmed particles are the main sources of prompt leptons. The inclusive production cross-sections obtained by Bugaev *et al.* [22] are proportional to the product of two- or three-quark distributions in the hadron and quark recombination function integrated on momentum fractions of quark partons. Bugaev *et al.* have also assumed

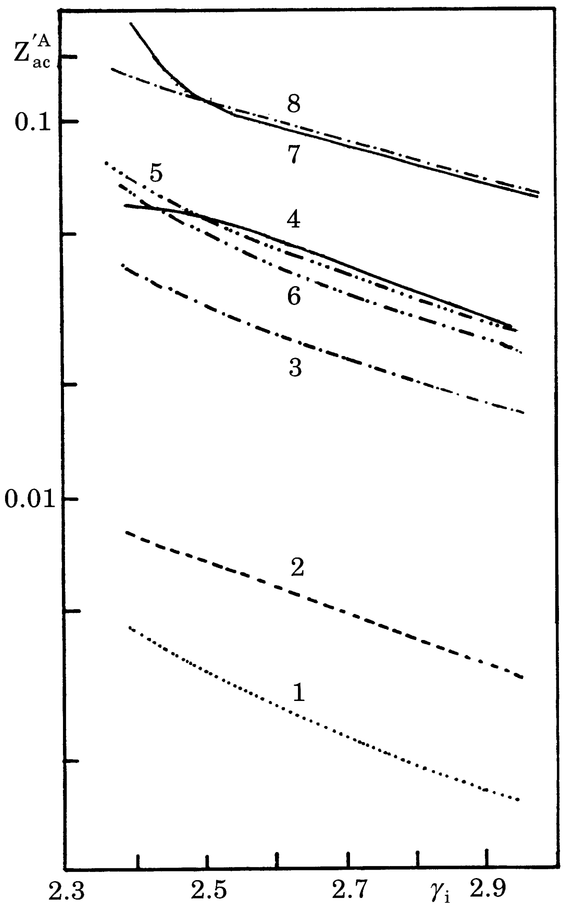

Fig. 3. – The Z_{ac} factors corrected for h-A, A-A and QG obtained from FNAL data [17] are displayed as functions of primary spectral indices γ_i. Curves 1-8 represent the distribution of Z_{ac}^{A} for pK, pK^+, $p\pi^-$, $p\pi^+$, $\pi^+\pi^-$, $\pi^-\pi^+$, $\pi^-\pi^-$ and $\pi^+\pi^+$, respectively, obtained from different inclusive reactions like $a + b \rightarrow C + X$.

that momentum distributions of c-quarks can be obtained by a statistical approach. The contribution of direct muon spectrum from the decay of charmed mesons has been estimated by using the relation (16) after Pal *et al.* [21], adopting the charmed-particle cross-sections obtained from QCD calculations by Bugaev *et al.* [22] along with the parametric values of R_3 displayed in table VII, shown in fig. 6 and also accounted with the muon spectrum expected from the conventional meson decays and displayed in figs. 4 and 5. The prompt muon flux from charmed decay is independent of the zenith angle upto 10^8 GeV, since the charmed particles are more likely to decay than interact and the flux is independent of target. The experimental results of different authors [8-15] are shown in fig. 4 and 5.

The present investigation allows the evaluation of cosmic-ray muon spectra at zenith angles 0° and 89° in the TeV energy range from individual primary elemental spectra during the collisions of primary cosmic nuclei with the atmosphere. The

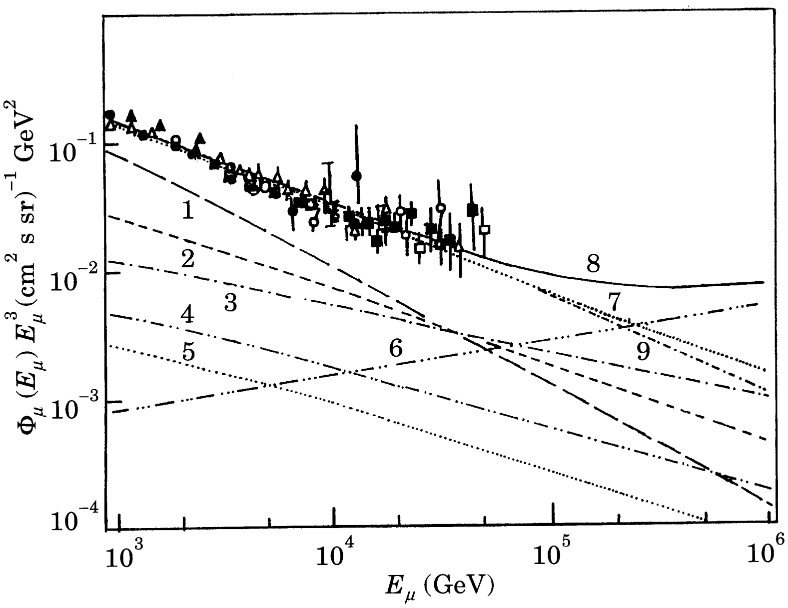

Fig. 4. – Differential muon flux at the zenith angle 0° obtained from the decay of atmospheric non-prompt and prompt mesons: Curves 1-5 represent the separate contribution to the muon flux generated from different groups of elements like p, He, CNO, Ne-Si and Fe in the primary cosmic rays. Curve 6 shows the total prompt muon spectrum obtained from the charmed meson decays. Curve 7 represents the total muon spectrum obtained from the non-prompt meson decay (π and K). Curve 8 shows the total muon spectrum obtained from the non-prompt and prompt mesons in the atmosphere. Curve 9 shows the muon spectrum obtained from the single-slope primary cosmic-ray nucleon spectrum [26]: Experimental data: ▲ Thompson *et al.* [8]; △ Matsuno *et al.* [10]; ● Khalchukov *et al.* [11]; □ Ivanova *et al.* [12]; ○ Bakatanov *et al.* [14]; ■ Zatsepin *et al.* [16].

estimated muon flux is mainly dependent on $Z^{A}_{N\pi}$ which has an error of $\pm 7\%$ and the energy dependence of p-air collision is neglected since the meson flux is not sensitive to the growth of cross-section with energy.

The spectral indices of the elemental abundances of primary cosmic rays are different and we assume that the cosmic-ray nuclei of atomic mass A_i are constituted by A independent nucleons when the superposition model is applied, which in turn generates the spectra of secondary mesons due to the high-energy interactions of primary nuclei with the atmosphere. The mesons are generated by the high-energy interactions of primary nuclei with the atmosphere and the muons are their final decay products reaching the Earth. The current information on the primary elemental spectra yields discrete spectral-shaped muon spectra. In a similar manner [7] we have neglected the slow increase of hadronic cross-sections and used the Feynman [28] scaling hypothesis for inclusive cross-sections in the fragmentation region. More recently Parente *et al.* [7] have pointed out that the cross-section expected from Quark Gluon String Model [29] shows a very small energy dependence on the cross-section, $< 5\%$ in the lab energy range 1–100 TeV, which gives a strong support for the selection of the scaling hypothesis for the present investigation. Recently Ranft [30] has found

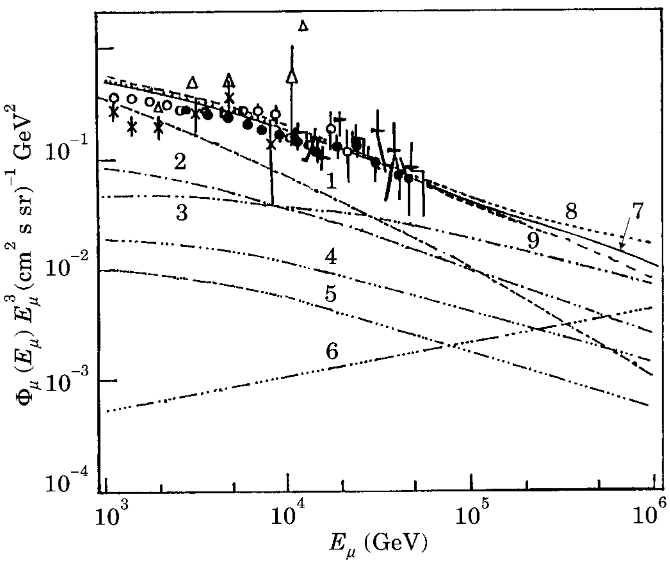

Fig. 5. – Differential muon flux at the zenith angle 89° obtained from the decay of atmospheric non-prompt and prompt mesons: Curves 1–5 represent the separate contribution to the muon flux generated from different groups of elements like p, He, CNO, Ne-Si and Fe in primary cosmic rays. Curve 6 shows the total prompt muon spectrum obtained from the charmed particle decay. Curve 7 represents the total muon spectrum obtained from the non-prompt meson decay (π and K). Curve 8 shows the total muon spectrum obtained from the non-prompt and prompt mesons in the atmosphere. Curve 9 shows the muon spectrum obtained from the single slope primary cosmic-ray nucleon spectrum: Experimental data: × Allkofer *et al.* [9]; ○ Matsuno *et al.* [10]; + Afanasieva *et al.* [13]; △ Gettert *et al.* [15]; ● Zatsepin *et al.* [16].

an excellent agreement with the expected results from the scaling hypothesis of Feynman [28] with the DMPJET model in the large part of the x_F region in h-h and h-A collisions, which favours the Feynman x_F distribution in the projectile fragmentation region, *viz.* for $0 < x_F < 1$. This observation reveals the fact that the accelerator data on Feynman x_F distributions in the fragmentation region are relevant for the analysis of secondary cosmic rays on Earth's atmosphere. A recent study supports the earlier observation of Gaisser and Yodh [31], who have shown that the superposition model is approximately applicable to the investigation of A-A collisions in high-energy cosmic-ray propagation. It may be pointed out that the muon fluxes decrease by 20% when corrected for pp collisions due to A-A collisions.

The separate differential muon fluxes from atmospheric π and K decays at zenith angles 0° and 89° originated from different primary cosmic-ray elemental groups displayed in fig. 4 and 5 show that the muon spectrum obtained from the primary CNO group increases adequately and exceeds the muon spectra obtained from the parents created by p, He beyond 30 TeV energy but below 10^6 GeV. Hence the muon spectra are mostly generated by A-air nucleus collisions, where the majority of A are CNO, as found from the muon spectrum at 89° above ~ 30 TeV energy. In figs. 5 and 6 we have also plotted the derived muon spectra of muons produced from the

TABLE IV. – *The fitting parameters of the invariant cross-sections of the charm particle production cross-sections produced in the* $pp \to \eta^{\pm} X$ *during primary nuclei-air collisions estimated from Bugaev et al.* [22].

η-particle	B	n
D^-	0.179	1.50
D^+	0.161	2.20
$\overline{D}^0$	0.200	1.50
Λ_c	0.111	1.00

TABLE V. – $I_{N\eta}$ *estimated from relation* (15) *using the fitting parameters of table* IV *for* $\gamma = 1.73$.

Particle (η)	$I_{n\eta}$	Spectral range (GeV)
$D^{\pm}$	0.00633	$10^4 < E < 10^6$
$D^0 \overline{D}^0$	0.00530	$10^4 < E < 10^6$
Λ_c	0.00264	$10^4 < E < 10^6$

TABLE VI. – *The branching ratios, mass, lifetime of the charmed particles* η *and the critical energies of the charmed particle decay for* $pp \to \eta^{\pm} X$ *inclusive reactions producing* η *charmed mesons through the decay* $D \to K + \mu + \nu$ *and* $\Lambda_c \to \Lambda + \mu + \nu$ [31].

	Particle		
	$D^{\pm}$	$D^0 \overline{D}^0$	Λ_c
W_{sl}	$(17.2 \pm 1.9)\%$	$(7.7 \pm 1.2)\%$	$(3.2 \pm 0.7)\%$
m_i(Mev)	(1869.3 ± 0.5)	(1864.5 ± 0.5)	(2284.0 ± 0.6)
τ_i(s)	$(10.66 \pm 0.23) \cdot 10^{-13}$	$(4.20 \pm 0.08) \cdot 10^{-13}$	$(1.91 \pm 0.5) \cdot 10^{-13}$
$H_{\eta}^{cr}(\theta = 0°)$GeV	$3.97 \cdot 10^7$	$1.05 \cdot 10^8$	$2.70 \cdot 10^8$

TABLE VII. – *The calculated kinematical constant* R_3 *for the two-body decay obtained from the formulation of Hagedorn* [23] *corrected by Mitsui et al.* [24].

Particle	$D^{\pm}$	$D^0 \overline{D}^0$	Λ_c
R_3	$5.5240 \cdot 10^6$	$5.4865 \cdot 10^6$	$15.5212 \cdot 10^6$

single-slope total primary nucleon spectrum [26] of the form

$$N(E)\,dE = 2.75E^{-2.73}\,dE[\text{cm}^2\,\text{s}\,\text{sr}\,\text{GeV}]^{-1}. \tag{19}$$

The comparison of the derived total single muon spectra in figs. 4 and 5 indicates that the single-slope primary nucleon spectrum can give an estimate of sea-level muon spectra in the vertical direction in good agreement with the present macroscopic muon flux estimate for energies below 20 TeV. In fig. 7 the integral TeV muon spectra from the primary nucleon spectrum with constant spectral index and the total sum of muon

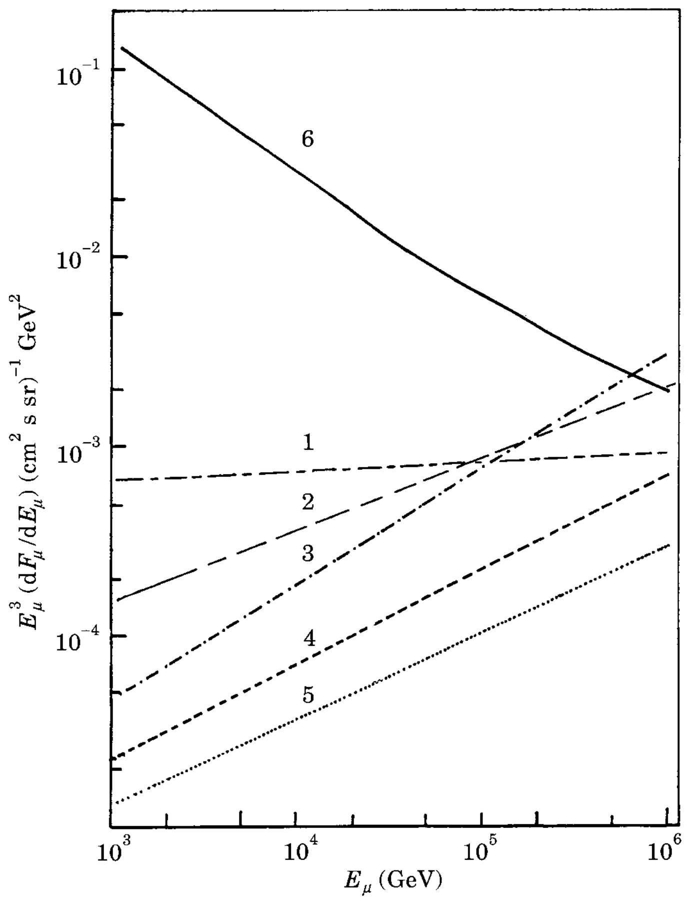

Fig. 6. – The plot shows the prompt muon spectra obtained from different elemental fluxes like p, He, CNO, Ne-Si and Fe primary cosmic-ray nuclei along with the total muon spectrum obtained from the conventional atmospheric meson decay: Curves 1-5 represent the derived prompt muon spectra obtained from the elemental spectra of primary cosmic rays like p, He, CNO, Ne-Si and Fe, respectively. Curve 6 shows the total sum of the muon fluxes obtained from the individual primary elemental spectra through the decay of atmospheric mesons like pions and kaons.

spectra, obtained from different primary elemental spectra produced through the decay of non-prompt and prompt mesons have been compared with the experimental results obtained from the underground KGF [32], ERPM [33] and BAKSAN [34] measurements. Both of the muon spectra resulted to be in accord with KGF [32] and BAKSAN [34] up to 20 TeV but are in approximate good agreement with ERPM data [33] in the energy range (5–100) TeV. The derived prompt muon spectra of the earlier authors [26, 27, 35-38] have also been compared with our result. It is found that the present muon spectrum expected from the individual primary nuclei spectra is in approximate agreement with our earlier expected muon spectra [27] from the total primary nucleon spectrum with constant energy spectral index.

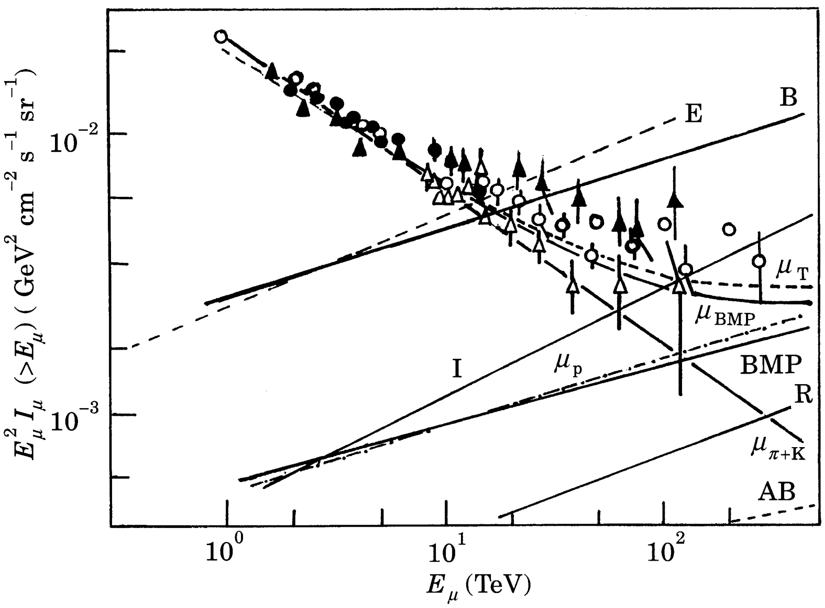

Fig. 7. – The plot shows the comparison of the derived integral energy spectra of muons expected from the non-prompt and prompt meson decay in the atmosphere with the experimental results obtained from the underground experiments: Experimental data: ●, ○ KGF [32] data at a rock depth of 3375 hg/cm^2 and 7000 hg/cm^2, respectively; △ ERPM data [33]; ▲ BAKSAN [34]. Theoretical results: derived spectra of atmospheric muons generated by the decay of non-prompt mesons obtained from the present analysis: μ_T represents the total muon spectrum expected from the non-prompt and prompt meson decay; μ_{BPM} represents the total muon spectrum expected from the primary nucleon spectrum with single energy spectral index -2.73. $\mu_{\pi+K}$ represents the muon spectrum from the decay of non-prompt mesons. Derived spectra of prompt muons derived by different authors: B: Bugaev *et al.* [27]; E: Elbert *et al.* [35]; I: Inazawa *et al.* [36]; BPM: Bhattacharyya *et al.* [26]; AB: Allkofer and Bhattacharyya [37]; R: Ramanamurthy [38]; μ_p: present work.

4. – Conclusion

The muon spectra have been derived from the individual elemental primary spectra obtained from direct measurements with active and passive detectors borne by balloons and satellites by adopting the *Z*-factors based on the machine interaction results. The total muon spectra at zenith angles 0° and 89° are found in accord with the muon spectra obtained from the single-slope primary nucleon spectrum of the form $\sim 2.75\, E^{-2.73}$. The contribution of prompt muon spectrum was calculated and found to be negligible for muon energies below 50 TeV. The derived results are comparable with the experimental results of Matsuno *et al.*, Khalchukov *et al.*, Ivanova *et al.*, Bakatanov *et al.*, Gettert *et al.* and Zatsepin *et al.* The derived integral TeV muon spectrum displayed in fig. 7 is in approximate agreement with the muon intensity data of KGF, BAKSAN and ERPM experiments. No significant difference is observed between the expected muon spectrum and the derived results obtained from the single-slope primary nucleon spectrum. The contribution of total prompt muon flux is adequate especially beyond 100 TeV muon energy.

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