Weak-scattering theory for ionospheric scintillation

A. K. SINGH, D. NARAYAN and R. P. SINGH
Atmospheric Research Laboratory, Physics Department, Banaras Hindu University
Varanasi 221 005, India

(ricevuto il 18 Marzo 1996; revisionato il 17 Gennaio 1997; approvato il 27 Febbraio 1997)

Summary. — The VHF scintillations recorded at Varanasi (geomag. lat. = 14°55′N; geomag. long. = 154 °E) are mostly weak scintillations. Using a thin phase screen model for ionospheric irregularities, under the approximation of weak-scattering theory, attempts have been made to evaluate theoretically the time evolution of the scintillation index, S_4, for different models of electron densities. The present formulation is used to study the impact of turbulence strength, C_s, and its outer scale length L_0 on the variation of the scintillation index S_4. The S_4 index increases with the increase in spectral index ν but decreases with increasing outer scale length L_0. Diurnal, seasonal and solar-cycle variations of the S_4 index have also been studied and it is found that S_4 lies between 0.2 and 0.9 as the Sun passes through solar maxima to solar minima. The results are found to be in good agreement with those reported by other workers.

PACS 92.60 – Meteorology.
PACS 94.20 – Physics of the ionosphere.

1. – Introduction

A radio signal propagation through the ionosphere and the magnetosphre undergoes amplitude and phase variations which are known as amplitude and phase scintillations, respectively (Aarons, 1982). Satellite communication system designers require detailed knowledge of ionospheric scintillation index statistics for specific system designs. For this purpose good propagation models are required which must predict frequency and propagation geometry dependence.

The electron density irregularities responsible for ionospheric scintillation appear as random refractive index irregularities along the propagation path. A complete theory for wave propagation through a medium with a random refractive index or dielectric-constant fluctuations is not available, and hence approximate solutions for models have been proposed from time to time (Booker *et al.*, 1950; Booker, 1957; Fante, 1975; Prokherov *et al.*, 1975); these all have restrictions on their application.

Analytical expressions for field quantities for the general problem of wave propagation through random media are not available. In the absence of analytical solution, a perturbation method is used to derive the field quantities. First-order perturbation solutions for the field quantities provide the basis for the weak-scintillation theory or single-scattering theory. Two weak-scattering models are available: a) the single scattering or Born approximation to the scalar wave equation (Budden, 1965) and b) the single scattering or Born approximation to the equation for the logarithm of the amplitude of the wave (the so-called Rytov method or the method of smooth perturbation) (Tatarski, 1967). Crane (1977) has investigated the range of validity of these approximations and found that Rytov approximation is valid over a range of scintillations (up to $S_4 = 0.9$) and larger than that for the Born approximation (up to $S_4 = 0.3$).

Booker *et al.* (1950) proposed that diffraction by fluctuation of the electron density in the ionosphere could cause the observed scintillation and that the effect of the electron density fluctuations could be modeled by a thin phase-changing diffraction screen. An amalgamation of geometrical optics and diffraction theory has also been used extensively for an approximate description of scintillation effects (Rufenach, 1975).

The development of theoretical models is based on the assumed shape of the power spectrum of electron density fluctuations along the path. Early models assumed a Gaussian shape (Briggs and Perkins, 1963; Budden, 1965; Fremouw and Rino, 1973) whereas recent models have used a power law form (Rumsey, 1975; Rufenach, 1975; Singleton, 1974; Rino, 1979; Uscinski *et al.*, 1981; Booker and Majidi Ahi, 1981).

Apart from weak-scintillation theory, theories of strong scintillation have also been attempted. In the analysis of strong scintillation, the concept of multiple thin screens is used to avoid geometrical optics approximation to thin-screen model. The fourth-moment equation is used in the estimation of higher moments of the electron density fluctuations within each layer (Yeh *et al.*, 1975; Uscinski, 1977; Ishimaru, 1978).

The size distribution of the irregularities is characterized by a power law spectral density function (SDF) (Rufenach, 1975; McClure *et al.*, 1977). Booker and Ferguson (1978) have inferred an inner-scale cut-off near the ion gyroradius (5 m). The low-frequency cut-off or outer scale is at present not well understood. The available evidence shows that the ionospheric outer scale is at least several tens of kilometers.

In this paper the weak-scattering theory (Rino, 1979) is discussed to show explicitly the implications for data interpretation of an arbitrarily large outer scale. The generally accepted thin phase screen model for ionospheric scintillation has been used. The validity of this approach has been discussed by Bramley (1977). The phase screen model provides a means of obtaining unique, equivalent lumped parameters that characterize the average irregularity structure along the propagation path. We have considered only intensity scintillation because we have been recording only amplitude fluctuations. The observed scintillations at Varanasi are mostly weak and moderate (Singh *et al.*, 1993), hence these scintillations are modelled by weak-scattering theory. A closed-form expression for the S_4 intensity scintillation index has been derived (sect. **2**) and computed for some representative values. An intermediate parameter C_s called the strength of turbulence (Rumsey, 1975), generally used in classical turbulence studies, has also been derived and computed for different electron density distribution models. The ionospheric models used in the present numerical computations are described in sect. **3** and the results and discussions are given in sect. **4**. Finally, sect. **5** summarises the result reported in the present paper.

2. – Theoretical formulation

The ionospheric irregularities causing scintillations in the propagating signal move in the ionosphere and hence are the functions of time and space coordinates. This implies a space-time–dependent refractive index for the ionosphere, which for $\omega \gg \omega_p$, is written as

$$\mu = \left(1 - \frac{\omega_p^2}{\omega^2}\right)^{1/2}, \qquad \omega_p^2 = \frac{N_e e^2}{m\varepsilon_0}, \tag{1}$$

where $N_e(r, t)$ is the electron density at time t and space position r, e is the charge of the electron, m is the mass of the electron, ε_0 is the dielectric permittivity of the free space and ω is the angular wave frequency of the wave. ω_p is the angular plasma oscillation frequency. The phase variation $d\phi$ imposed on the wave propagating through the ionosphere is given by

$$d\phi = \frac{2\pi}{\lambda} \int \Delta\mu(r, t)\, dl, \tag{2}$$

where λ is the signal wavelength and the integral is evaluated along the straight line path from the signal source to the receiver and $\Delta\mu$ is the change in the refractive index. In writing eq. (2) it is considered that there is no change in the refractive index during the time interval required for the signal to travel through the irregularity layer. Substituting for $\Delta\mu$ from eq. (1), eq. (2) is rewritten as

$$d\phi = -r_e \lambda \int \Delta N_e\, dl (1 - f_p^2/f^2)^{-1/2}, \tag{3}$$

where $r_e = e^2/4\pi m \varepsilon_0 c^2$ is the classical electron radius and ΔN_e is the local electron density perturbation. The three-dimensional spectral density function (SDF) of ΔN_e for $f \gg f_p$ is given by (Rino and Fremouw, 1977)

$$\phi_{\Delta N_e}(\mathbf{k}, -\tan\phi\, \mathbf{a}_{k_r} \cdot \mathbf{k}) = ab \langle \Delta N_e^2 \rangle Q(q), \tag{4}$$

where ϕ is the zenith angle, and $\mathbf{a}_{k_r}$ is a unit vector along the xy-plane (projection of the propagation vector $\mathbf{k}$). In this case the z-axis is normal to the scattering layer (fig. 1). The parameters a and b are axial ratios along and transverse to the principal axis of the irregularity, respectively. The function $Q(q)$ normalized to unit area gives the shape of the spectral density function. For a power law SDF with no inner-scale cut-off, $Q(q)$ is given by (Rino, 1979)

$$Q(q) = \frac{8\pi^{1/2}\Gamma(\nu + 1/2)/\Gamma(\nu - 1/2)\, q_0^{2\nu - 2}}{(q_0^2 + q^2)^{\nu + 1/2}}, \tag{5}$$

where q_0 is the outer-scale cut-off wave number. For all wave numbers $\mathbf{k}$ such that $q \gg q_0$, the spectral density function is written as

$$\phi_{\Delta N_e}(q) = ab\, C_s\, q^{-(2\nu + 1)}, \tag{6}$$

where $2\nu + 1$ is the three-dimensional spectral index of the irregularities and C_s, the strength of turbulence (irregularity), is closely related to the structure constant used

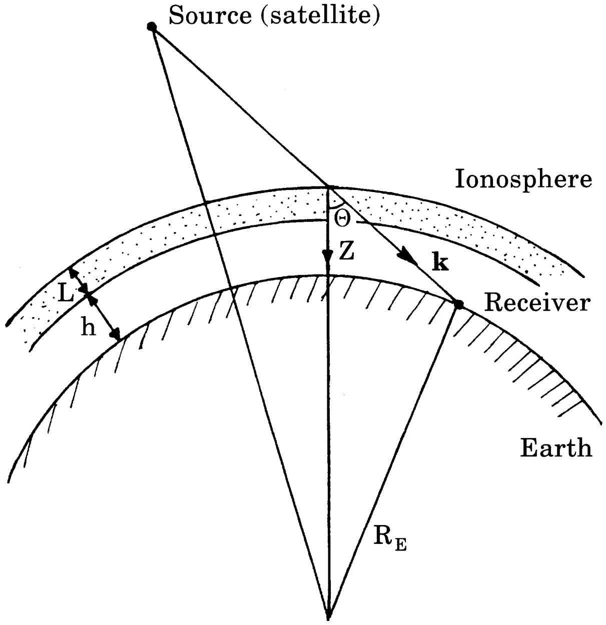

Fig. 1. – Typical ionospheric scintillation observing geometry.

in classical turbulence studies and is given by (Rumsey, 1975)

$$C_s = 8\pi^{3/2}\langle \Delta N_e^2\rangle\, q_0^{2\nu-2}\,\Gamma(\nu+1/2)/\Gamma(\nu-1/2)\,. \tag{7}$$

The intensity auto-correlation function (ACF) characterizes the scintillation index S_4, which measures the level of scintillations. The auto-correlation function and spectral density functions are related with each other (Rino and Fremouw, 1977). For weak-scattering theory, the intensity ACF is written as (Rino, 1979)

$$R_{\delta I}(\Delta\varrho,\, \Delta z) = 4 r_e^2 \lambda^2 L \sec^2\phi \int\!\!\int \phi_{\Delta N_e}(\mathbf{k},\, -\tan\phi\, \mathbf{a}_{k_r}\cdot\mathbf{k}) \times \times \sin^2(h\mathbf{k}Z)\cos(\mathbf{k}\cdot\Delta\varrho_S)\,\frac{\mathrm{d}k}{(2\pi)^2}\,, \tag{8}$$

where $h(\mathbf{k}) = k^2 - \tan^2\phi(\mathbf{a}_{k_r}\cdot\mathbf{k})$ and $Z = \lambda z_R \sec\phi/4\pi$, $z_R = z z_S/(z+z_S)$. z_S is the distance to the source.

The scintillation index S_4 is defined as (Rino, 1979)

$$S_4^2 = R_{\delta I}(0,\, 0)\,. \tag{9}$$

Substituting eqs. (4), (5), (7) and (8) into eq. (9) and performing a series of manipulations (Rino, 1979), we can write

$$S_4^2 = r_e^2\lambda^2 (L\sec\phi)\, C_s Z^{\nu-1/2}\left[\frac{\Gamma((2.5-\nu)/2)}{2\pi^{1/2}\,\Gamma((\nu+0.5)/2)(\nu-0.5)}\right] I\,, \tag{10}$$

where I is a combined geometry and propagation factor, defined for a two-dimensional irregularity model ($a \gg 1$, $b = 1$) as

$$I \cong \frac{\Gamma(\nu)}{\pi^{1/2}\Gamma(\nu + 1/2)} . \tag{11}$$

Using the appropriate values for the parameters in eq. (7), (10) and (11), the strength of the turbulence C_s, the scintillation index S_4 and their temporal evolution are evaluated in the next section.

3. – Ionospheric models used in numerical computations

For the present theoretical studies, we have considered the electron density variation with time as reported by Mahajan and Saxena (1976). For ready reference these are shown in fig. 2a) and b). From fig. 2a), it is evident that the electron density peak position can increase with the progress of time, and the location of the peak position decreases in height with the increase in the time during the night. Another measurement of electron density is considered in which the position of the peak electron density decreases in altitude with the increase in time but the peak value of the electron density remains the same (fig. 2b)). These two models are used for computations in order to see how S_4 and C_s change with the change in the variation of the electron density fluctuation with time. Further, it is known that the electron density varies with solar activity. To account for the possible variation in S_4 and C_s, we have also considered the electron density representative of solar maxima and solar minima (Rajaram, 1977). For ready reference they are shown in fig. 3. In the numerical evaluation of S_4 and C_s, the irregularity patch is assumed to be field aligned and hence highly anisotropic, so that $a \gg 1$ and $b = 1$. The expression for S_4 and C_s becomes much simpler when $a \to \infty$.

The variation in the power law spectral index governs the shape and size of the electron density irregularities. Vats and Shah (1989), computing the auto-correlation

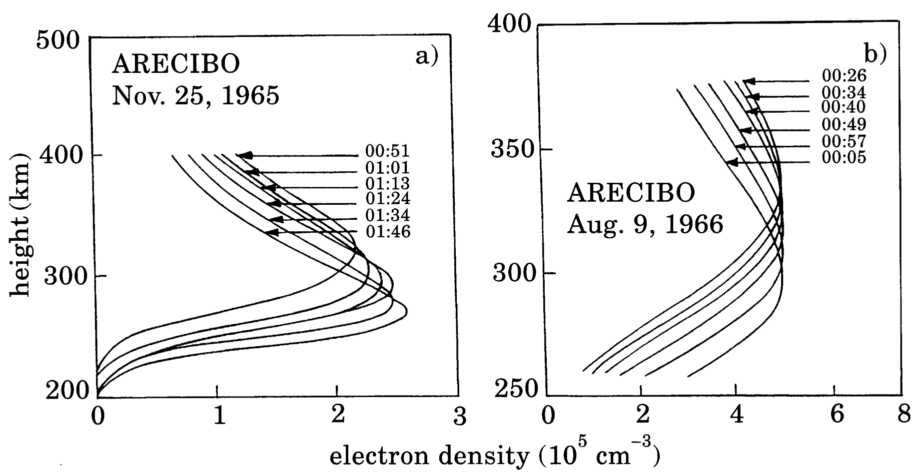

Fig. 2. – Electron density profiles during a sharp decrease in the height of the F-layer peak: a) NmF increases. b) No change in NmF (Mahajan and Saxena, 1976).

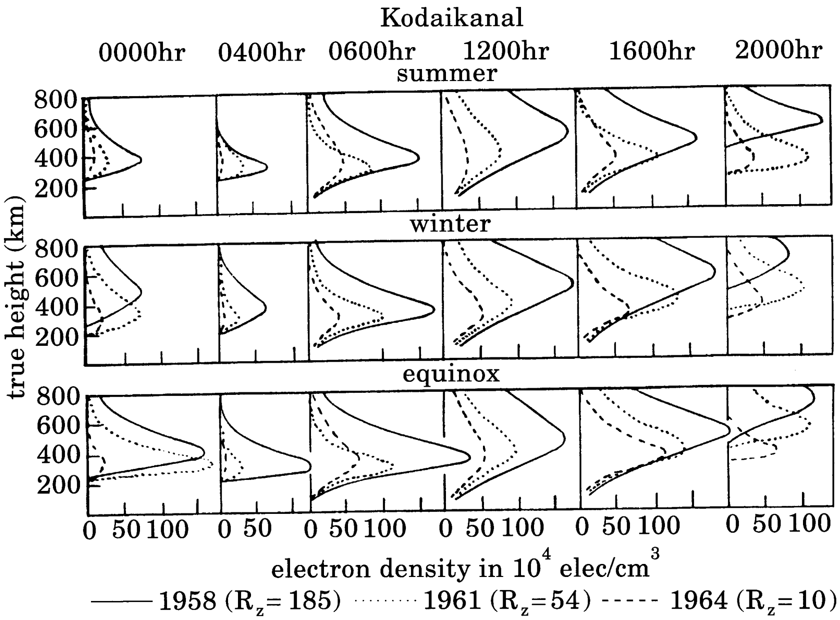

Fig. 3. – Response of the altitude profile of the equatorial electron density to seasonal and solar cycle (Rajaram, 1977).

functions for power law spectral index 2, 3, 4 and 5, have shown that the half correlation distance (irregularity scale) increases almost linearly with the power law index. In the present paper, the spectral index $p = 2\nu$ has been used. The value of ν for the present computation is taken as 1.25, 1.3 and 1.33 which means that the spectral index is 2.5, 2.6 and 2.66 which is in close agreement with those taken by Rino (1979). Crane (1977) has reported $p = 3$, which seems to be slightly higher than the measured values.

The outer-scale cut-off wave number is given by $q_0 = 2\pi/L_0$, where L_0 is the outer scale of the irregularity spectrum. Vats *et al.* (1981) have considered L_0 to be 50 km for F-region irregularities whereas Rino (1979) has taken the value of L_0 to be 200 km. The *in situ* measurements of electron density aboard low-orbiting satellites (Dyson *et al.*, 1974) and scintillation spectra (Singleton, 1974) suggest a power law spectrum in the F-region ionosphere with outer scale dimension (L_0) varying between 10 and 100 km. In the present model calculations, the outer scale is taken to be 50, 100, 200 and 250 km.

Another parameter which is important in the evaluation of S_4 is the layer thickness L of the scattering layer. This is the equivalent thickness of the horizontally stratified irregularity layer. Rino (1979) has considered $L = 200$ km whereas Rufenach (1975) has taken $L = 100$ km. In the present computation the equivalent layer thickness is taken to be 200 km. At Varanasi we are recording amplitude scintillations for the signal frequency (244.168 MHz) from FLEETSAT satellite parked at 73 °E. For this source, the distance between the source and the observation plane is 6.6 R_e and the zenith angle is $(\phi) \simeq 2.5°$. For F-region irregularities, the mean height is considered as 350 km. Similar values for mean height have been considered by Vats *et al.* (1981).

Using these model parameters for ionospheric irregularities, the value of the turbulence strength C_s and scintillation index S_4 and their dependence on various parameters are studied in the next section. For the numerical evaluation, eq. (7) is used for C_s and eqs. (10) and (11) have been used for S_4.

4. – Results and discussions

Figure 4 shows the variation of turbulence strength C_s with electron density for different values of outer scale length (L_0) and spectral index p $(=2\nu)$. The turbulence strength increases with the increase in electron density fluctuations and decreases with the increase of L_0 and ν or p. The figure clearly shows that the turbulence strength decreases as the size of the irregularity (outer scale) increases, for the same ΔN_e, and also as ν increases. As ΔN_e changes from 10^9 m^{-3} to 1.4×10^{10} m^{-3}, C_s changes from 9×10^{16} to 1.6×10^{19} for $L_0 = 100$ km and $\nu = 1.25$, whereas for the same change in density it changes from 4.5×10^{16} to 8×10^{19} for $L_0 = 100$ km and $\nu = 1.3$. Keeping $\nu = 1.25$ and varying L_0 from 100 km to 250 km, C_s changes from 5.75×10^{16} to 1.2×10^{19} for the variation in ΔN_e from 10^9 m^{-3} to 1.4×10^{10} m^{-3}. This shows that the thin phase screen model is very sensitive to small changes in the spectral index ν.

The variation of the scintillation index S_4 with turbulence strength C_s for different values of the spectral index ν is shown in fig. 5. The figure shows that S_4 increases with C_s. The rate of increase is low at lower values of $C_s(<10^{18})$ and it increases considerably at higher C_s values ($C_s > 10^{18}$). It is further seen that with the increase in the spectral index, S_4 increases. The change in S_4 is predominant at higher C_s values

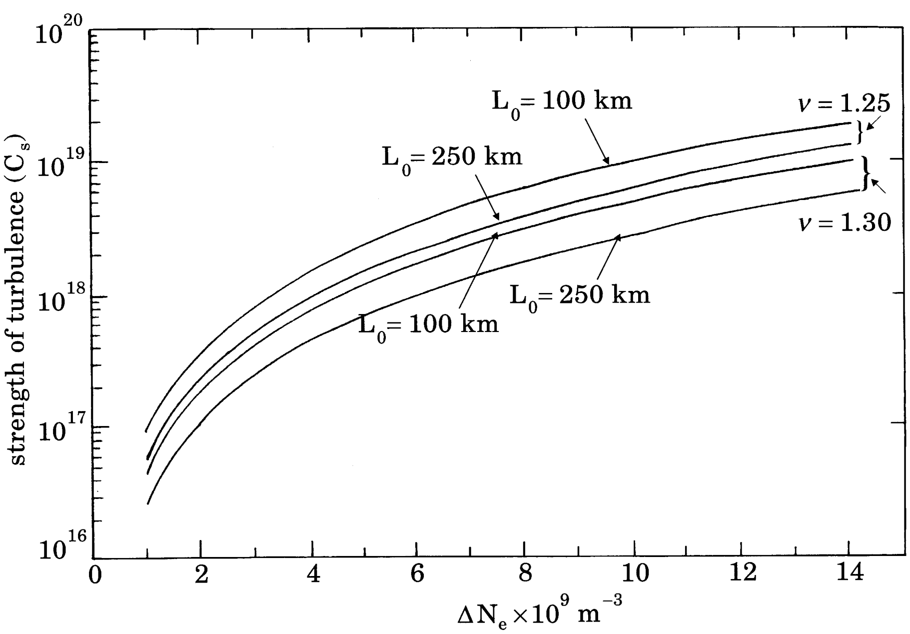

Fig. 4. – Variation of strength of turbulence, C_s, with electron density fluctuations for different values of the spectral index, ν, and the outer scale length, L_0.

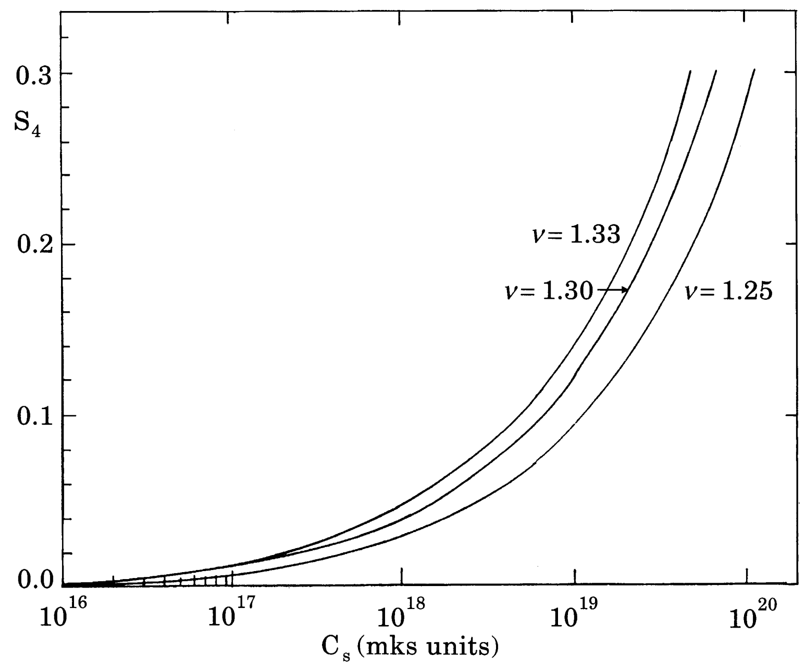

Fig. 5. – Variation of the scintillation index, S_4, with the strength of turbulence, C_s (in $m^{-6.6}$) for different values of the spectral index ν.

than at lower values. A similar nature of the variation of S_4 in magnitude and shape has been reported by Basu *et al.* (1988) using observed phase spectral index.

Figure 6a) and b) shows the time development of the scintillation index S_4 for the electron density model shown in fig. 2a), b) for outer scale length $L_0 = 100$, 200 and 250 km and $\nu = 1.3$. The S_4 varies with time irregularly and the values of S_4 lie in the range 0.03 to 0.22. The variations of S_4 with time have been obtained for an electron density deviation ΔN_e measured at the height of the maximum ionization density. In both the models the scintillation index S_4 decreases with an increase in the outer scale length L_0.

The response of the equatorial *F*-region changes with season and solar-cycle changes. Using the time development of the *F*-region peak electron density (Rajaram, 1977), the turbulence strength C_s and the scintillation index S_4 have been computed. The diurnal variation of S_4 for summer, winter and equinox seasons are given in fig. 7. The S_4 indices are computed for solar-cycle maxima and solar-cycle minima. It is found that the value of the scintillation index is normally minimum for solar-cycle minima and maximum for solar-cycle maxima. Of course, the absolute value changes from season to season. For example, the S_4 index for the solar-minima year lies between 0.2 and 0.4 in the summer season, between 0.3 and 0.45 in the winter season and between 0.35 and 0.7 at the equinox. The diurnal variation of S_4 shows more scattered variations in equinoctal months as compared with summer and winter months. The S_4 index for solar-maxima year shows a variation between 0.45 and 0.75 in summer months, between 0.4 and 0.72 in winter months and between 0.4 and 0.9 for equinoctal months. Thus, it is found that the scintillation index S_4 varies between 0.2 and 0.9 as the Sun passes

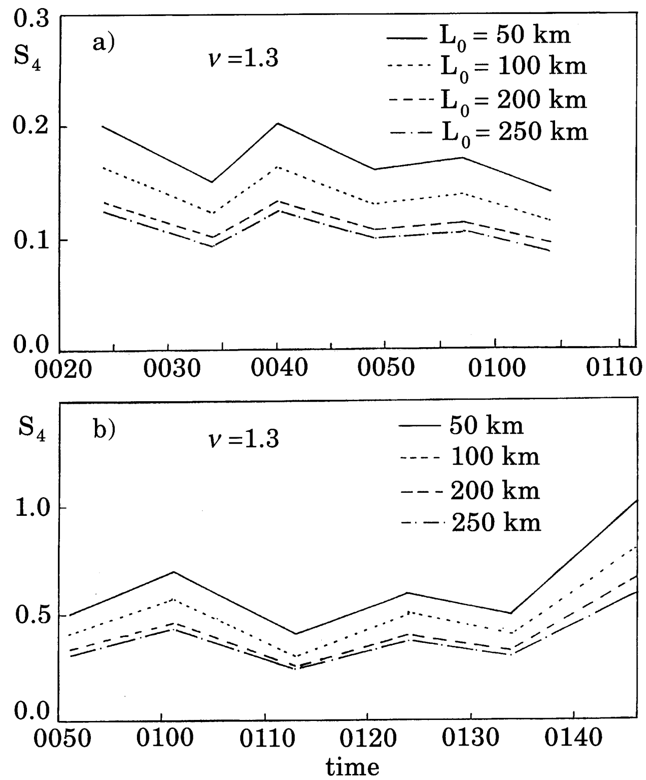

Fig. 6. – Time development of the scintillation index, S_4, for the electron density model: a) shown in fig. 2a) and b) shown in fig. 2b) for different values of the outer scale lengths, L_0.

through solar maxima to solar minima. Further, the trend of diurnal variation of the S_4 index varies from season to season and it also varies as the sunspot number varies.

5. – Conclusions

The F-region at equatorial and low latitudes is a fascinating part of the upper atmosphere which, apart from its academic interest, has an applied value in radio communications. The physics of this region is greatly influenced by the presence and movement of ionospheric irregularities. The movement of these irregularities produces amplitude and phase scintillations. In this paper, we have briefly discussed the theory of the time evolution of the scintillation index S_4 for different models of electron densities. We have also studied the diurnal, seasonal and solar-cycle variation of the S_4 index. Based on the study presented in this paper we conclude that:

i) The F-region ionospheric irregularities are described by a power law spectrum.

ii) The turbulence strength C_s is found to decrease with increasing spectral index ν and outer scale length L_0.

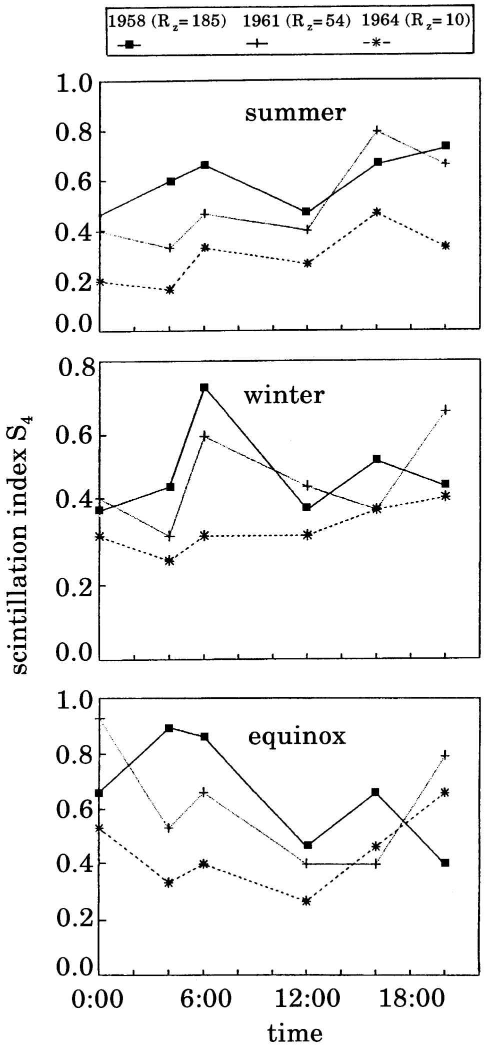

Fig. 7. – The diurnal variation of the scintillation index, S_4, for summer, winter and equinox seasons for solar-cycle maxima and minima.

iii) The scintillation index S_4 is found to decrease with increasing outer scale length L_0 and with increasing spectral index ν.

iv) Further, it is observed that the thin phase screen model is very sensitive to small changes in the spectral index ν.

v) The diurnal, seasonal, and solar-cycle variation of the scintillation index S_4 does not follow a systematic trend. It is also found that the scintillation index S_4 is almost minimum for solar minima and maximum for solar maxima except near sunset. Further, S_4 varies between 0.2 and 0.9 as the Sun passes through solar maxima to minima.

* * *

One of us (AKS) thanks CSIR for providing financial assistance as Research Associateship. This work is partially supported by Department of Science and Technology under AICPITS second phase program.

REFERENCES

AARONS J., *Global morphology of ionospheric scintillation*, *Proc. IEEE*, **70** (1982) 360-378.

BASU SU., BASU S., WEBER E. J. and COLEY W. R., *Case study of polar cap scintillation modeling using DE2 irregularity measurements at 800 km*, *Rad. Sci.*, **23** (1988) 545-553.

BOOKER H. G., RATCLIFF J. A. and SHINN D. H., *Diffraction from an irregular screen with applications to ionospheric problems*, *Philos. Trans. R. Soc. Ser. A*, **242** (1950) 579-607.

BOOKER H. G., *The use of radiostars to study irregular refraction of radio waves in the ionosphere*, *Proc. IRE*, **46** (1957) 298-314.

BOOKER H. G. and FERGUSON J. A., *A theoretical model for equatorial ionospheric spread-F echoes in the H.F. and VHF bands*, *J. Atmos. Terr. Phys.*, **40** (1978) 803-829.

BOOKER H. G. and MAJIDI AHI G., *Theory of refractive scattering in scintillation phenomena*, *J. Atmos. Terr. Phys.*, **43** (1981) 1199-1214.

BRAMLEY E. N., *The accuracy of computing ionospheric radiowave scintillation by the thin phase screen approximation*, *J. Atmos. Terr. Phys.*, **39** (1977) 367.

BRIGGS B. H. and PERKINS I. A., *On the variation of radio star and satellite scintillations with zenith angle*, *J. Atmos. Terr. Phys.*, **25** (1963) 239-265.

BUDDEN K. G., *The amplitude fluctuations of the radiowave scattered from a thin ionospheric layer with weak irregularities*, *J. Atmos. Terr. Phys.*, **27** (1965) 155-172.

CRANE R., *Ionospheric scintillation*, *Proc. IEEE*, **65** (1977) 180-199.

DYSON P. L., MCCLURE J. P. and HANSON W. B., *In situ measurements of the spectral characteristics of F-region ionospheric irregularities*, *J. Geophys. Res.*, **79** (1974) 1495-1502.

FANTE R. L., *Electromagnetic beam propagation in turbulent media*, *Proc. IEEE*, **63** (1975) 1669-1692.

FREMOUW E. J. and RINO C. L., *An empirical model for average F-layer scintillation at VHF/UHF*, *Rad. Sci.*, **8** (1973) 213-222.

ISHIMARU A., *Wave Propagation and Scattering in Random Media*, Vol. **2** (Academic Press, New York) 1978.

MAHAJAN K. K. and SAXENA O. P., *Continuous measurements of nighttime electron concentration profiles and estimation of neutral wind velocities at Arecibo*, *J. Geophys. Res.*, **81** (1976) 3165-3170.

MCCLURE J. P., HANSON W. B. and HOFFMAN J. H., *Plasma bubbles and irregularities in the equatorial ionosphere*, *J. Geophys. Res.*, **82** (1977) 2650.

PROKHEROV A. M., BUNKIN F. V., GOCHELASHVILY K. S. and SHISKOV V. I., *Laser irradiance propagation in turbulent media*, *Proc. IEEE*, **63** (1975) 790-811.

RAJARAM G., *Structure of the equatorial F-region topside and bottomside—a review*, *J. Atmos. Terr. Phys.*, **39** (1977) 1125-1144.

RINO C. L. and FREMOUW E. J., *The angle dependence of singly scattered wavefields*, *J. Atmos. Terr. Phys.*, **39** (1977) 859-868.

RINO C. L., *A power law phase screen model for ionospheric scintillation* (1). *Weak Scatter*, *Rad. Sci.*, **14** (1979) 1135-1145.

RUFENACH C. L., *Ionospheric scintillation by a random phase screen: spectral approach*, *Rad. Sci.*, **10** (1975) 155-165.

RUMSEY V. H., *Scintillations due to a concentrated layer with a power law turbulence spectrum*, *Rad. Sci.*, **10** (1975) 107-114.

SINGH R. P., SINGH U. P., SINGH A. K. and SINGH R. N., *Scintillation study at Varanasi*, *Ind. J. Rad. Space Phys.*, **22** (1993) 17-21.

SINGLETON D. G., *Power spectra of ionospheric scintillations*, *J. Atmos. Terr. Phys.*, **36** (1974) 113-133.

TATARSKI V. I., *The Effects of the Turbulent Atmosphere on Wave Propagation* (Nauka, Moscow) 1967.

USCINSKI B. J., *The Elements of Wave Propagation in Random Media* (McGraw Hill, New York) 1977.

USCINSKI B. J., BOOKER H. G. and MARIANS M., *Intensity fluctuations due to a deep phase screen with a power-law spectrum*, *Proc. R. Soc. London, Ser. A*, **374** (1981) 503-530.

VATS H. O., BOOKER H. G. and MAJIDI AHI G., *Quantitative explanation of strong multifrequency intensity scintillation spectra using refractive scattering*, *J. Atmos. Terr. Phys.*, **43** (1981) 1235-1241.

VATS H. O. and SHAH K. J., *Power law irregularities and their diffraction pattern*, *Ind. J. Rad. Space Phys.*, **18** (1989) 85-89.

YEH K. C., LIU C. H. and YOUAKIM M. V., *A theoretical study of the ionospheric scintillation behaviour caused by multiple scattering*, *Rad. Sci.*, **10** (1975) 97-106.