IL NUOVO CIMENTO VOL. 20 C, N. 6 Novembre-Dicembre 1997

Theoretical models for MHD turbulence in the solar wind (*)

P. VELTRI and F. MALARA

Dipartimento di Fisica, Università della Calabria - I-87030, Arcavacata di Rende (CS), Italy

(ricevuto il 5 Dicembre 1996; approvato il 18 Marzo 1997)

Summary. — The *in situ* measurements of velocity, magnetic field, density and temperature fluctuations performed in the solar wind have greatly improved our knowledge of MHD turbulence not only from the point of view of space physics but also from the more general point of view of plasma physics. These fluctuations which extend over a wide range of frequencies (about 5 decades), a fact which seems to be the signature of turbulent nonlinear energy cascade, display, mainly in the trailing edge of high-speed streams, a number of features characteristic of a self-organized situation: i) a high degree of correlation between magnetic and velocity field fluctuations, ii) a very low level of fluctuations in mass density and magnetic-field intensity, iii) a considerable anisotropy revealed by minimum variance analysis of the magnetic-field correlation tensor. Many fundamental processes in plasma physics, which were largely unknown or not understood before their observations in the solar wind, have been explained, by building up analytical models or performing numerical simulations. We discuss the most recent analytical theories and numerical simulations and outline the limits implicit in any analysis which consider the low-frequency solar-wind fluctuations as a superposition of linear modes. The characterization of low-frequency fluctuations during Alfvénic periods, which results from the models discussed, is finally presented.

PACS 96.60 – Solar physics.
PACS 52.35.Bj – Magnetohydrodynamic waves.
PACS 01.30.Cc – Conference proceedings.

1. – Introduction

The study of low-frequency fluctuations in solar wind not only is important in itself, since it represents a way to understand the behavior of the magnetohydrodynamic (MHD) turbulence in a parameter region which is not accessible in terrestrial laboratories, but it also furnishes information that is useful in many different astrophysical problems. In the last years a considerable amount of work has been done concerning both the analysis of

(*) Paper presented at the VII Cosmic Physics National Conference, Rimini, October 26-28, 1994.

the solar-wind data and the theoretical understanding of the physical mechanisms which determine the features observed in the solar-wind turbulence.

Low-frequency fluctuations, *i.e.* fluctuations with frequencies lower than the ion-cyclotron frequency, extend over a very wide range

$$10^{-6}\ \mathrm{Hz} < f < 1\ \mathrm{Hz}. \tag{1}$$

All over this frequency range these fluctuations display a power law spectrum

$$E(k) \propto k^{-\alpha}, \tag{2}$$

with spectral indices α comprised between 1 and 2 [1,2]. This fact seems to be the signature of a fully developed MHD turbulence resulting from a nonlinear energy cascade.

In spite of this, the fluctuations display, mainly in the trailing edges of high-speed streams and at small scales ($1\ \mathrm{min} < T < 1\ \mathrm{day}$) some striking features, which seem to show that these fluctuations are in some sense *organized*:

i) *a high degree of correlation* between velocity and magnetic-field fluctuations [2]

$$\delta \mathbf{v} \simeq \frac{\sigma\, \delta \mathbf{B}}{\sqrt{4\pi\rho}}, \qquad \text{with } \sigma = \pm 1 \tag{3}$$

($\delta\mathbf{v}$ and $\delta\mathbf{B}$ are, respectively, the velocity and magnetic-field fluctuations, ρ represents the mass density and the sign of the correlation ($\sigma = \pm 1$) turns out to be that corresponding to nonlinear ($|\delta\mathbf{B}|/|\mathbf{B}_0| \simeq 1$) Alfvén waves propagating away from the Sun).

ii) *a low level of fluctuations* in mass density and magnetic-field intensity [2]

$$\frac{\delta\rho}{\rho} \simeq \frac{\delta|\mathbf{B}|}{|\mathbf{B}|} \simeq \text{few percents.} \tag{4}$$

The apparent contradiction [3] between turbulent spectrum and organization of fluctuations has been the origin of a lot of fruitful theoretical work.

2. – Incompressible and statistically homogeneous MHD turbulence models

The first attempts to solve the apparent contradiction between the high degree of correlation and the presence of a turbulent spectrum have been performed in the framework of *incompressible* ($\delta\rho \simeq 0$) MHD.

In terms of the Elsasser's [4] variables $\mathbf{z}^\sigma$, defined by

$$\mathbf{z}^\sigma = \mathbf{v} + \sigma \frac{\mathbf{B}}{\sqrt{4\pi\rho}}, \text{ with } \sigma = \pm 1, \tag{5}$$

the equations governing incompressible MHD are written

$$\frac{\partial \mathbf{z}^\sigma}{\partial t} + (\mathbf{z}^{-\sigma}\cdot\nabla)\mathbf{z}^\sigma = -\frac{1}{\rho}\nabla\left(p + \frac{B^2}{8\pi}\right) + \text{ dissipation terms.} \tag{6}$$

It is easily seen that in these variables the conditions (4) found in the solar wind are written as

$$\delta\mathbf{z}^- = 0 \quad \text{and} \quad |\delta\mathbf{z}^+| \simeq v_{\mathrm{A}}\,. \tag{7}$$

v_{A} represents the Alfvén velocity associated with the mean magnetic field.

Ideal incompressible MHD conserves both energy per mass unit and cross-helicity [5]. In terms of Elsasser's variables this corresponds to the conservation of two pseudo-energies ϵ^σ

$$\epsilon^\sigma = \int \frac{|\mathbf{z}^\sigma|^2}{2}\,\mathrm{d}^3\mathbf{r}\,. \tag{8}$$

Dobrowolny *et al.* [6], building up a model of nonlinear cascade for *incompressible* and *statistically homogeneous* MHD, which takes into account the conservation of both pseudo-energies, suggested that an initial unbalance between the pseudo-energies (*e.g.* $|\delta\mathbf{z}^+| > |\delta\mathbf{z}^-|$) is progressively enhanced by nonlinear interaction: the final state of the turbulence being that where only one propagating mode, the one initially dominating, survives (*i.e.* $|\delta\mathbf{z}^+| \gg |\delta\mathbf{z}^-|$). Coming back to velocity and magnetic-field variables, this result means that MHD turbulence displays a strong tendency to develop *self-organized* states where the correlation between $\delta\mathbf{v}$ and $\delta\mathbf{B}$ is maximum and the energy is distributed on the different wavevectors according a power law spectrum of Kraichnan's [7] type ($\alpha = 3/2$).

This conjecture has successively been confirmed by rather different mathematical techniques: numerical integration of statistical equations, obtained via closure hypothesis [8-10], simplified models for the nonlinear energy cascade [11, 12], direct numerical simulations of MHD equations in both 2D [13-15] and 3D [16, 17]. In fig. 1 we show a simulation of the evolution of the Orszag-Tang vortex [18]: it is clearly seen that starting with a situation where the structure of the velocity field lines is completely different from that of the magnetic-field lines, at the end of the simulation the velocity and magnetic fields are almost parallel to each other.

3. – Compressible and inhomogeous MHD turbulence models

The picture of the evolution of *incompressible* MHD turbulence which comes out from these theoretical models is rather nice, but the solar-wind turbulence, which stimulated all this work, displays a more complicated behavior.

Data analysis by Roberts *et al.* [19, 20], Bavassano and Bruno [21], Grappin *et al.* [22] show that solar-wind turbulence evolves in the reverse way: the correlation is high near the Sun. At larger radial distances from 1 AU to 10 AU the correlation is progressively lower, while the level of fluctuations in mass density and magnetic-field intensity increases. The spectra, initially flatter than a Kolmogorov's ($\alpha = 5/3$) or Kraichnan's ($\alpha = 3/2$) spectrum, increase their indices up to $\alpha = 5/3$ at 10 AU.

Alfvén waves propagating in opposite directions are both convected by solar wind only beyond the Alfvénic point, where the flow speed becomes greater than the Alfvén speed. One should then expect that only those Alfvén waves, which propagate away from the Sun can leave the Sun and arrive at the solar wind [2]. By supposing that the Sun is the main source of low-frequency fluctuations, the high level of correlation near the Sun can be understood. Even if the situation is somewhat more complicated [23], this explanation remains the most widely accepted.

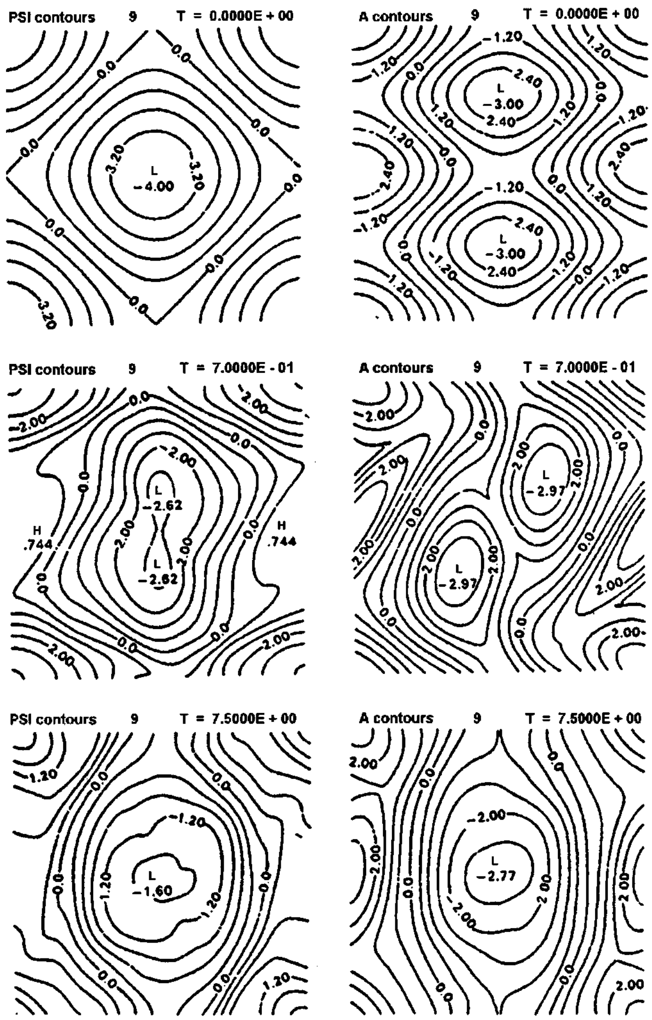

Fig. 1. – Time evolution of velocity (on the left) and magnetic (on the right) field lines for the Orszag-Tang vortex. The Reynolds number is 400. Time values are, respectively, 0, 0.7, 7.5 from top to bottom.

What is more difficult to understand is the reason why correlation is progressively destroyed in the solar wind if the natural evolution of MHD turbulence is towards a state of maximal normalized cross-helicity. A possible solution to such paradox can be found in the fact that the solar wind is neither *incompressible* nor *statistically homogeneous*. Some attempts to take into account compressibility and/or inhomogeneity of the solar wind have thereinafter been performed.

Roberts *et al.* [20] suggested that stream shear velocity gradients should be directly responsible for the decrease of correlation. Goldstein *et al.* [24] carried out incompressible 2D simulations of a slow flow within two fast streams and studied the decrease of the correlation coefficient near the shear layer. More recently, Roberts *et al.* [25,26] simulated the evolution of Alfvénicity (the correlation mentioned in (3)) near a magnetic neutral sheet, showing that in this case the decay of the correlation is accelerated.

Veltri *et al.* [27] performed numerical simulations which show that, in a compressible medium, the interaction between small-scale waves and large-scale magnetic-field gradients on the one hand, and the parametric instability on the other hand, reduce the correlation between the velocity and magnetic-field fluctuations and let develop a compressive component of the turbulence characterized by $\delta\rho \neq 0$ and $\delta|\mathbf{B}| \neq 0$.

Grappin *et al.* [28] observed that the overall solar-wind expansion increases the lengths normal to the radial direction, thus producing a sort of inverse energy cascade which competes with the direct nonlinear energy cascade. As a result, nonlinear interactions are slowed down, at least at large scales. To describe the effect of the solar-wind expansion they have built up a numerical simulation where MHD equations are solved in an expanding box comoving with the solar wind (Expanding Box Model). The results of their simulations show that, after a first stage of evolution, nonlinear interactions are effectively stopped.

In conclusion, it is now clear that in a *compressible* and *inhomogeneous* medium there are a lot of processes which may be responsible for the decorrelation of the turbulence and for the development of a compressive component of the fluctuations. To explain the fact that, in fast streams, the correlation lives longer (up to 1 AU), Veltri *et al.* [27] proposed that Landau damping could play a role in keeping the density and magnetic-field intensity fluctuations at their observed low level.

4. – A model for the 3D magnetic-field correlation spectra of solar-wind MHD turbulence

For Alfvénic periods Bavassano *et al.* [29] found for the eigenvalues $\lambda_1,\ \lambda_2,\ \lambda_3$ of the magnetic-field correlation matrix

$$\lambda_1 : \lambda_2 : \lambda_3 = 10 : 3 : 1. \tag{9}$$

The minimum variance direction turns out to be almost parallel to the average magnetic field $\mathbf{B}_0$, while the maximum variance direction is perpendicular to the $\mathbf{v}_{\mathrm{SW}}$-$\mathbf{B}_0$ plane, which is almost coincident with the ecliptic plane.

Bavassano *et al.* [29] also performed minimum variance analysis over five different time basis (168 s, 8 min, 22.5 min, 1 h, 3 h) and for observations at three different heliocentric distances (0. 29 AU, 0. 65 AU, 0. 87 AU). Starting from these data, Carbone *et al.* [30] tried to obtain information on the 3D spectra of the magnetic-field fluctuations. They assumed for these spectra the following phenomenological expressions:

$$I^{[s]}(\mathbf{k}) = \frac{C^{[s]}}{[(\ell_x^{[s]}k_x)^2 + (\ell_y^{[s]}k_y)^2 + (\ell_z^{[s]}k_z)^2]^{1+\alpha^{[s]}/2}}, \qquad \text{with } s = 1, 2. \tag{10}$$

The indices $s = 1, 2$ correspond to magnetic energy density polarized, respectively, along

$$\mathbf{e}^{[1]}(\mathbf{k}) = \frac{i\mathbf{k} \times \mathbf{B}_0}{|\mathbf{k} \times \mathbf{B}_0|} \quad \text{and} \quad \mathbf{e}^{[2]}(\mathbf{k}) = \frac{i\mathbf{k}}{|\mathbf{k}|} \times \mathbf{e}^{[1]}(\mathbf{k}). \tag{11}$$

In the limit of small amplitude fluctuations (linear approximation) the first polarization turns out to be that of Alfvén waves, while the second is that of both fast and slow magnetosonic waves. Also if solar-wind fluctuations are by no means linear, for the sake of

TABLE I. – *Parameters of the spectra* $I^{[s]}(\mathbf{k})$.

R (AU)	Alfvénic Polarization						Magnetosonic Polarization					
	$C^{[1]}$	$\alpha^{[1]}$	$\ell_x^{[1]}$	$\ell_y^{[1]}$	$\ell_z^{[1]}$	$E^{[1]}$	$C^{[2]}$	$\alpha^{[2]}$	$\ell_x^{[2]}$	$\ell_y^{[2]}$	$\ell_z^{[2]}$	$E^{[1]}$
0.29	7.2	1.10	100	30	1.0	0.25	1.1	1.46	110	1.3	1.0	0.13
0.65	7.8	1.23	70	20	1.0	0.26	1.7	1.73	100	1.2	1.0	0.12
0.87	5.3	1.31	50	10	1.0	0.30	1.9	1.81	90	0.7	1.0	0.10

simplicity in the following we will refer to polarizations 1 and 2, respectively, as Alfvénic and magnetosonic polarization.

By performing a best fit of the spectra (10) with the Bavassano *et al.* [29] data, Carbone *et al.* [30] have been able to determine the parameters of the spectra.

Looking at table I, where we have reported these parameters, it can be seen that the spectral index $\alpha^{[1]}$ of the Alfvénic polarization ranges between 1.1 and 1.31, corresponding to rather flat spectra. The spectral index $\alpha^{[2]}$ ranges between 1.46 and 1.81 (table I); *i.e.* it is larger by a factor of 1.3–1.4 than the corresponding value of $\alpha^{[1]}$. The spectral indices of both polarizations increase with increasing distance from the Sun, showing a tendency of both spectra to become steeper at larger distances.

In table I we have also reported the energy contents $E^{[1]}$ and $E^{[2]}$ of the two polarizations, both normalized to B_0^2. Their square root ratio represents a measure of the relative fluctuation level for the two polarizations. This ratio $E^{[1]}/E^{[2]}$ is comprised between 2 and 3 and tends to increase with the distance from the Sun.

In Table I $\ell_x^{[s]}$ and $\ell_y^{[s]}$ (we set $\ell_z^{[s]} = 1$) determine the shape of the energy distributions of the two polarizations in the $\mathbf{k}$ space. Looking at these parameters we can note that:

i) $\ell_x^{[1]} > \ell_y^{[1]} \gg \ell_z^{[1]}$ which indicates that, in the energy distribution of Alfvénic polarization fluctuations, wave vectors quasi-parallel to $\mathbf{B}_0$ (z-direction) largely dominate. The corresponding contour surfaces in the $\mathbf{k}$ space are sort of "cigars" (fig. 2) aligned along the $\mathbf{B}_0$-direction; in particular, the first inequality shows that these surfaces are rather flat in the $\mathbf{B}_0$-$\mathbf{v}_0$ plane.

ii) $\ell_x^{[2]} \gg \ell_y^{[2]}, \ell_z^{[2]}$, indicating that the spectrum $I^{[2]}(\mathbf{k})$ is strongly flat on the $\mathbf{B}_0$-$\mathbf{v}_0$ plane. Within this plane (fig. 2) the energy distribution does not present any relevant anisotropy.

The very large values of $\ell_x^{[1]}$ and $\ell_x^{[2]}$ (x is the direction perpendicular to the $\mathbf{v}_{\mathrm{SW}}$-$\mathbf{B}_0$ plane) indicate that both spectra are very flat in the $\mathbf{v}_{\mathrm{SW}}$-$\mathbf{B}_0$ plane. This feature has been found also by Dobrowolny *et al.* [31] and by Carbone *et al.* [32] using more simplified models.

5. – An insight into the nature of solar-wind MHD fluctuations

The model by Carbone *et al.* [30], which directly concerns only magnetic-field fluctuations, is usefully compared with observations of the energy level and the spectra of compressive quantities, *i.e.*, density and magnetic-field intensity. Simple correlations between the magnetic-field fluctuations and the fluctuations of these compressive quantities exist only in the weak turbulence framework, *i.e.* in the small-amplitude limit. Due to the amplitude of the solar-wind fluctuations ($(E^{[1]} + E^{[2]})^{1/2} \simeq 0.6$), some caution should be

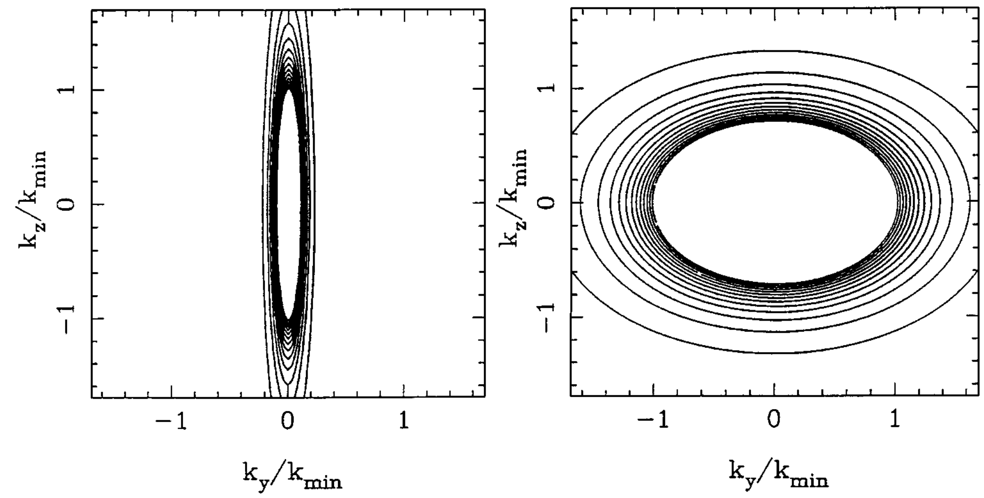

Fig. 2. – Level curves of the power spectra in the plane $\mathbf{v}_{SW}$-$\mathbf{B}_0$: Alfvénic polarization $I^{[1]}(0, k_y, k_z)$ on the left, magnetosonic polarization $I^{[2]}(0, k_y, k_z)$ on the right. $\mathbf{B}_0$ is parallel to the z-axis. The heliocentric distance is $R = 0.87$.

taken when considering the turbulence as a superposition of linear modes, in particular, none of the correlations predicted by the linear theory can expected to be satisfied *a priori*. The comparison with the observations of compressive fluctuations must be used to verify to what extent the correlations predicted by the linear theory actually survive in the solar-wind MHD turbulence.

In this respect, it is worth to note that the nonlinear Alfvénic solution in a compressible medium is characterized by $\delta\mathbf{v} = \pm\delta\mathbf{B}/(4\pi\rho)^{1/2}$ and B^2 and ρ both uniform, regardless of the polarization. This means that in the presence of such type of solution, we can have any value of $E^{[2]}$ and, at the same time, $\delta\rho = \delta|\mathbf{B}| = 0$.

Let us consider now the relation satisfied by magnetic-field intensity in the solar wind $\mathbf{B}^2 \simeq \mathrm{const}$. By developing this relation with respect to the fluctuation amplitude, the following relation is obtained

$$B^2 = \mathbf{B}_0^2 + \delta\mathbf{B}^{[1]^2} + \mathbf{B}_0 \cdot \delta\mathbf{B}^{[2]} + \delta\mathbf{B}^{[2]^2} + 2\delta\mathbf{B}^{[1]} \cdot \delta\mathbf{B}^{[2]} \simeq \mathbf{B}_0^2 + \mathbf{B}_0 \cdot \delta\mathbf{B}^{[2]} \simeq \mathrm{const}, \tag{12}$$

where $\delta\mathbf{B}^{[1]}$ and $\delta\mathbf{B}^{[2]}$ represent the magnetic-field fluctuations associated, respectively, with Alfvénic and magnetosonic polarizations, and we have used the fact that $\mathbf{B}_0 \cdot \delta\mathbf{B}^{[1]} = 0$.

Relation (12) implies that the amplitude of magnetic-field intensity fluctuations is related to the amplitude of magnetosonic polarization fluctuations by

$$\frac{\delta|\mathbf{B}|}{B_0} \simeq \frac{\mathbf{B}_0 \cdot \delta\mathbf{B}^{[2]}}{B_0^2}. \tag{13}$$

This gives a level of magnetic-field intensity fluctuation of the order of $\delta|\mathbf{B}|/B_0 \simeq (E^{[2]}/2)^{1/2} \simeq 0.22$. The corresponding values calculated by Bavassano *et al.* [29] for the same data sets range from 0.05 to 0.07. However, to obtain expression (13), we have neglected in (12) nonlinear terms like $\{\delta\mathbf{B}^{[1]}\}^2$, $\{\delta\mathbf{B}^{[2]}\}^2$, $2\delta\mathbf{B}^{[1]}\cdot\delta\mathbf{B}^{[2]}$, which can be estimated to be of the same order as the linear term we have retained. This indicates that the nonlinear terms tend to counterbalance the linear term, to keep the magnetic-intensity fluctuations to the very low observed level.

Similar conclusions can be drawn also for the density fluctuations. For the magnetosonic modes the linear relation between density and magnetic-field fluctuations is given by [33]

$$\frac{\delta B^{[2]}(\mathbf{k})}{B_0} = -\frac{\delta\rho(\mathbf{k})}{\rho_0}\sin\theta\frac{\left\{1+\beta\pm\left[(1+\beta)^2-4\beta\cos^2\theta\right]^{1/2}\right\}}{1+\beta-2\cos^2\theta\pm\left[(1+\beta)^2-4\beta\cos^2\theta\right]^{1/2}}, \tag{14}$$

where $\beta = c_{\mathrm{S}}^2/c_{\mathrm{A}}^2$, c_{S} and c_{A} are the sound and the Alfvén velocity, respectively, θ is the angle between $\mathbf{k}$ and $\mathbf{B}_0$, and the upper (lower) sign refers to the fast (slow) magnetosonic mode. Evaluating expression (14) for $\theta \neq 0$ and slow mode fluctuations, it is found that

$$\frac{\Delta\rho}{\rho} \simeq \frac{1}{\beta}\frac{\Delta B^{[2]}}{B_0} \simeq \frac{\sqrt{E^{[2]}}}{\beta}. \tag{15}$$

The typical measured level $\Delta\rho/\rho \simeq 0.1$ [34] is obtained from equation (15) for $\beta \simeq 3$. Lower values of β, which are likely to be found in the solar wind, would yield an estimation of $\Delta\rho/\rho$ which is higher than the observed value. Then also in the case of the density fluctuations, nonlinear terms neglected in the above approach should play a role in keeping these fluctuations to a level lower than that predicted by the linear theory.

Marsch and Tu [35] have calculated the spectra of proton density and magnetic-field intensity, during the same periods as the analysis by Carbone *et al.* [30]. Looking at their Figures 1 and 3, panels e ($R = 0.29$ AU), it is seen that the slope of the magnetic-intensity spectrum is almost equal to the slope calculated by Carbone *et al.* [30] for the magnetosonic polarization ($\alpha^{[2]} = 1.46$). The density spectrum displays the same slope only in the lower-frequency range. In the high-frequency range the density spectrum is much flatter. This high-frequency component of the density fluctuation spectrum, which is decoupled from magnetic-intensity fluctuations, could be interpreted in terms of a population of high-frequency sound-like fluctuations propagating parallel to $\mathbf{B}_0$. This idea is supported by the fact that in this frequency range the magnetic-field magnitude spectrum is steeper than the density spectrum (Figures 1e and 3e in [35]). In fact, sound waves can affect neither the energy spectra of the magnetic fluctuations nor the magnetic-field magnitude fluctuation spectrum, while they would give a contribution to the density fluctuation spectrum.

A further insight into the nature of solar-wind fluctuations can be obtained by comparing with observations by Matthaeus *et al.* [36]. Looking at the obtained distribution of the wave vectors, these authors identified two distinct populations in the magnetic fluctuations: the first one with wave vectors nearly parallel to the average magnetic field ("slab-like" Alfvénic fluctuations), the second one with wave vectors nearly perpendicular to $\mathbf{B}_0$ (quasi–two-dimensional turbulence). In their analysis Matthaeus *et al.* [36] assumed that the correlation tensor is axisymmetric around the direction of $\mathbf{B}_0$. The analysis by Carbone *et al.* [30] shows that this assumption is not valid, since the wavevectors distributions in both polarizations are strongly flat in the $\mathbf{B}_0$-$\mathbf{v}_0$ plane. Nonetheless, when the results of the model by Carbone *et al.* [30] are restricted to the $\mathbf{B}_0$-$\mathbf{v}_0$ (yz) plane, a comparison is still significant.

The "slablike" population by Matthaeus *et al.* [36], is obtained by the superposition of both Alfvénic and magnetosonic polarizations with wave vectors parallel to $\mathbf{B}_0$, *i.e.* the corresponding fluctuations are mainly polarized in the plane perpendicular to $\mathbf{B}_0$; on the contrary, the second population, identified by Matthaeus *et al.* [36] as a quasi–two-dimensional turbulence, is only due to waves in the magnetosonic polarization, *i.e.* these waves are polarized nearly parallel to $\mathbf{B}_0$.

In the introduction of their paper, Matthaeus *et al.* [36] suggest that a nearly two-dimensional (2D) incompressible turbulence characterized by wave vectors and magnetic-field fluctuations both perpendicular to $\mathbf{B}_0$ is present in the solar wind. This interpretation, however, does not arise from the analysis they performed on the solar-wind magnetic-field fluctuations, but has been based on 2D simulations of the decay of anisotropic incompressible turbulence [37] and on analytical studies of quasi-2D turbulence in presence of strong dc magnetic field [38]. It is worth to note, however, that in the former approach, which is strictly 2D, when $\mathbf{k}$ is perpendicular to $\mathbf{B}_0$, magnetic-field fluctuations are necessarily parallel to $\mathbf{B}_0$. In the latter one, along with incompressibility, it is assumed that the energy in the fluctuations is much less than in the dc magnetic field; both hypotheses do not apply to the solar-wind case.

The presence in the solar wind of magnetic-field fluctuations with both $\mathbf{k}$ and $\delta\mathbf{B}$ perpendicular to $\mathbf{B}_0$ has been also suggested by Tu and Marsch [39]. Analyzing solar-wind data, these authors found static structures satisfying

$$(\mathbf{B}\cdot\nabla)\mathbf{B} = 0, \qquad \delta|\mathbf{B}| = 0.$$

Using *linear wave* theory, it can be immediately shown that these static structures should have both $\mathbf{k} \perp \mathbf{B}_0$ and $\delta\mathbf{B} \perp \mathbf{B}_0$, *i.e.* these fluctuations should belong to the Alfvénic polarization. The contrast between the results by Tu and Marsch [39] and by Carbone *et al.* [30] is only apparent, because:

i) in the nonlinear case (which is the case of solar wind) nothing can be said about the polarization (Alfvénic or magnetosonic) of these structures;

ii) these structures have been observed during an interval of a non-Alfvénic period (almost zero correlation between velocity and magnetic-field fluctuations) where they seem to represent the main component of magnetic-field fluctuations.

A further characterization of the second population of magnetic fluctuations can be obtained considering other observations. In the weak-turbulence framework this population would result from the superposition of slow and fast magnetosonic waves at quasi-perpendicular wave vectors. Actually, slow magnetosonic waves with $\mathbf{k}$ perpendicular to $\mathbf{B}_0$ are characterized by [33]: i) velocity and magnetic-field fluctuations parallel to $\mathbf{B}_0$, ii) vanishing phase velocity (they are stationary in the plasma reference frame) and iii) equilibrium between magnetic and thermal pressure fluctuations. On the contrary, fast waves display a positive magnetic-thermal pressure correlation.

The "compressive" fluctuations observed in the solar wind are characterized by the presence of a distinct anticorrelation between proton density and magnetic-field intensity [20, 34, 40-42], which has been interpreted as being due to the presence of quasi-static pressure-balanced structures. Thus, the magnetic-field fluctuations, which belong to the magnetosonic polarization and have wave vectors quasi-perpendicular to $\mathbf{B}_0$, are probably due to a nearly pressure-balanced structure with a smaller amount of fast magnetosonic waves.

The usefulness of these last considerations is somewhat limited by the fact that the amplitude of magnetic-field fluctuations in the solar wind is of the same order as B_0. Nonetheless, quasi-static pressure-balanced structures, stationary in the plasma reference frame (convected structures), represent also a nonlinear solution of MHD equations; in the limit of small characteristic lengths they are also known as "tangential discontinuities". The departures $\delta\mathbf{B}$ from the mean magnetic field $\mathbf{B}_0$ associated with such structures are parallel to $\mathbf{B}_0$, while their wave vector $\mathbf{k}$ is perpendicular to $\mathbf{B}_0$, just as we find in our polarization analysis for the second population fluctuations.

6. – Conclusions

In conclusion, the considerations of the above section show that a great caution is necessary in analyzing low-frequency fluctuations in the solar wind in terms of superposition of linear modes. Also when nonlinear solutions of MHD waves are identified through a careful examination of the characteristics of different observations, one must keep in mind the fact that the sum of two of such solutions is no more a solution of MHD equations. Decomposing the measured quantities as a superposition of two or more of such nonlinear solutions might be the source of considerable misunderstanding.

In spite of the above-mentioned *caveat*, the concepts derived from both linear and nonlinear analysis of MHD equations, when properly used, remain useful tools in organizing the information obtained from solar-wind measurements of low-frequency fluctuations. In particular, we want to recall that anisotropy analysis on magnetic-field fluctuations, combined with observations of compressive quantities fluctuations, shows that in the solar wind, during Alfvénic periods

i) nonlinear Alfvénic fluctuations represent the most energetic mode;

ii) static-pressure–balanced compressive structures convected by solar-wind velocity are also present; these structures are nothing but slow magnetosonic waves with $\mathbf{k} \perp \mathbf{B}_0$ or nonlinear tangential discontinuities;

iii) comparison with spectra of compressive quantities indicates the existence of a much lower level of high-frequency acoustic waves, propagating along $\mathbf{B}_0$, which could be the result of a parametric instability [27].

* * *

This work is part of a research program, which is financially supported by the Ministero dell'Università e della Ricerca Scientifica, the Consiglio Nazionale delle Ricerche (CNR) (contract 94.00802.CT02), and the Agenzia Spaziale Italiana (ASI) (contract 1994 RS 164).

REFERENCES

[1] COLEMAN P. J., *Astrophys. J.*, **153** (1968) 371.
[2] BELCHER J. W. and DAVIS L., *J. Geophys. Res.*, **76** (1971) 3534.
[3] VELTRI P., *Nuovo Cimento C*, **3** (1980) 45.
[4] ELSASSER W. M., *Phys. Rev.*, **79** (1950) 183.
[5] WOLTJER L., *Proc. Natl. Acad. Sci. USA*, **44** (1958) 833.
[6] DOBROWOLNY M., MANGENEY A. and VELTRI P., *Phys. Rev. Lett.*, **45** (1980) 144.
[7] KRAICHNAN R. H., *Phys. Fluids*, 8 (1965) 1385.

[8] GRAPPIN R., FRISCH U., LEORAT J. and POUQUET A., *Astron. Astrophys.*, **105** (1982) 6.
[9] GRAPPIN R., POUQUET A. and LEORAT J., *Astron. Astrophys.*, **126** (1983) 51.
[10] CARBONE V. and VELTRI P., *Geophys. Astrophys. Fluid Dyn.*, **52** (1990) 153.
[11] GLOAGUEN C., LEORAT J., POUQUET A. and GRAPPIN R., *Physica D*, **17** (1985) 154.
[12] CARBONE V. and VELTRI P., *Astron. Astrophys.*, **188** (1987) 239.
[13] MATTHAEUS W. H. and MONTGOMERY D., in *Statistical Physics and Chaos in Fusion Plasmas*, edited by C. W. HORTON and L. E. REICH (Wiley Interscience, New York) 1984.
[14] GRAPPIN R., *Phys. Fluids*, **29** (1986) 2433.
[15] TING A., MATTHAEUS W. H. and MONTGOMERY D., *Phys. Fluids*, **29** (1986) 3261.
[16] MENEGUZZI M., FRISCH U. and POUQUET A., *Phys. Rev. Lett.*, **47** (1981) 1060.
[17] POUQUET A., MENEGUZZI M. and FRISCH U., *Phys. Rev. A*, **33** (1986) 4266.
[18] GRAPPIN R., *Quelques aspects de la turbulence MHD développée*, These de Doctorat d'Etat en Sciences Physiques, Université de Paris VII, Paris (1985).
[19] ROBERTS D. A., GOLDSTEIN M. L., KLEIN L. W. and MATTHAEUS W. H., *J. Geophys. Res.*, **92** (1987) 12023.
[20] ROBERTS D. A., GOLDSTEIN M. L. and KLEIN L. W., *J. Geophys. Res.*, **95** (1990) 4203.
[21] BAVASSANO B. and BRUNO R., *J. Geophys. Res.*, **94** (1989) 11977.
[22] GRAPPIN R., MANGENEY A. and MARSCH E., *J. Geophys. Res.*, **95** (1990) 8197.
[23] VELLI M., *Astron. Astrophys.*, **270** (1992) 304.
[24] GOLDSTEIN M. L., ROBERTS D. A. and MATTHAEUS H. W., in *Solar System Plasma Physics*, edited by J. H. WAITE JR., J. L. BURCH and R. L. MOORE (AGU, Washington DC) 1989.
[25] ROBERTS D. A., GHOSH S., GOLDSTEIN M. L. and MATTHAEUS W. H., *Phys. Rev. Lett.*, **67** (1991) 3741.
[26] ROBERTS D. A., GOLDSTEIN M. L., MATTHAEUS W. H. and GHOSH S., *J. Geophys. Res.*, **97** (1992) 17115.
[27] VELTRI P., MALARA F. and PRIMAVERA L., in *Solar Wind Seven*, edited by E. MARSCH (Pergamon Press, Oxford) 1992.
[28] GRAPPIN R., VELLI M. and MANGENEY A., in *Solar Wind Seven*, edited by E. MARSCH (Pergamon Press, Oxford) 1992.
[29] BAVASSANO B., DOBROWOLNY M., FANFONI G., MARIANI F. and NESS N. F., *Solar Phys.*, **78** (1982) 373.
[30] CARBONE V., MALARA F. and VELTRI P., to be published in *J. Geophys. Res.*
[31] DOBROWOLNY M., MANGENEY A. and VELTRI P., *Astron. Astrophys.*, **83** (1982) 26.
[32] CARBONE V., VELTRI P. and MALARA F., *Nuovo Cimento C*, **15** (1992) 621.
[33] AKHIEZER A. I., AKHIEZER I. A., POLOVIN R. V., SITENKO A. and STEPANOV K. N., *Plasma Electrodynamics* (Pergamon Press, New York) 1975.
[34] MATTHAEUS W. H., KLEIN L. W., GHOSH S. and BROWN M. R., *J. Geophys. Res.*, **96** (1991) 5421.
[35] MARSCH E. and TU C. Y., *J. Geophys. Res.*, **95** (1990) 11945.
[36] MATTHAEUS W. H., GOLDSTEIN M. L. and ROBERTS D. A., *J. Geophys. Res.*, **95** (1990) 20673.
[37] SHEBALIN J. V., MATTHAEUS W. H. and MONTGOMERY D., *J. Plasma Phys.*, **29** (1983) 525.
[38] MONTGOMERY D., *Phys. Scr.*, **T2/1** (1990) 83.
[39] TU C.-Y. and MARSCH E., *Ann. Geophys.*, **9** (1991) 319.
[40] BURLAGA L., *Solar Phys.*, **4** (1968) 67.
[41] BURLAGA L. and OGILVIE K., *Solar Phys.*, **15** (1970) 61.
[42] VELLANTE M. and LAZARUS A. J., *J. Geophys. Res.*, **92** (1987) 9893.
[43] BURLAGA L., SCUDDER J. D., KLEIN L. W. and ISENBURG P. A., *J. Geophys. Res.*, **95** (1990) 2229.