IL NUOVO CIMENTO VOL. 20 C, N. 6 Novembre-Dicembre 1997

Observations of MHD turbulence in the solar wind (*)(**)

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(ricevuto il 4 Aprile 1996; approvato il 24 Maggio 1996)

Summary. — Within the past decade there has been a remarkable production of papers, both theoretical and observational, that have shed some light on the intriguing and unsolved problem of the MHD turbulence in the solar wind. *In situ* observations by spacecraft between 0.3 and several AUs, remote sensing of plasma conditions at the basis of the corona and numerical simulations have shown that solar-wind turbulence undergoes a dramatic evolution as the wind expands into the interplanetary space, depending on the region of the solar wind where it is observed. Within high-velocity streams we observe a high level of turbulent energy, relevant presence of outward Alfvén waves with a rather flat energy spectrum, near equipartition between kinetic and magnetic energy, weak density fluctuations and high proton temperature. Within low-velocity regions we record a low level of turbulent energy, almost total absence of Alfvénic correlation, predominance of magnetic over kinetic energy, near-Kolmogorov spectrum of the fluctuations, strong density fluctuations and lower proton temperature. After all, these features identify two types of turbulence: a well developed turbulence in slow wind and an Alfvénic turbulence in fast wind. Although the scientific community has agreed upon many points which are at the basis of the temporal and radial evolution of the turbulence, there are still many open issues which are of fundamental importance for a complete understanding of the phenomenon. In this review we will report about the most important observations performed in the solar wind in the past few years and we will try to interpret them in the light of the latest theories and computer simulations.

PACS 96.50.Ci – Solar wind plasma.
PACS 96.60 – Solar physics.
PACS 01.30.Cc – Conference proceedings.

1. – Introduction

The heliosphere provides an excellent laboratory to study turbulence in a supersonic magnetofluid. There is no other laboratory on Earth where we can access

(*) Paper presented at the VII Cosmic Physics National Conference, Rimini, October 26-28, 1994.
(**) The author of this paper has agreed to not receive the proofs for correction.

such a wide range of spatial and temporal scales. *In situ* measurements have revealed the presence of fluctuations ranging from tenths of a second to years and spatial scales between kilometers and tens of astronomical units. As a matter of fact, there is no measurable parameter of the solar wind which can be thought of as a constant. There are two major questions that arise from the observations. The first one is about the source of these fluctuations. The answer should be searched at the surface of the Sun, where a highly variable and inhomogeneous environment generates structures on scales ranging from seconds to months which are continuously modified and destroyed. This activity is reflected in some way in the solar-wind parameters but exactly how the variability that we measure in the interplanetary space is transferred through the initial layers of the solar atmosphere represents the second and largely still unanswered question. *In situ* measurements and remote sensing, computer simulations and modelling have provided a large amount of data, observations and new ideas which have shed some light on this intriguing problem; however, a precise answers will come only by future missions like Solar Probe that will observe the solar wind right at the basis of the corona, where the wind comes from.

In this review we will briefly report on what we have learned in the past few years both from observations and computer simulations about solar-wind turbulence. For the sake of simplicity, MHD turbulence in the solar wind can be seen as made of incompressive and compressive fluctuations and we will treat them separately, although we are aware that these two types of fluctuations couple to each other in a very complicated way because of the inhomogeneity of the interplanetary medium and the non-linearity of these fluctuations.

2. – Incompressive turbulence

The spectra obtained from the first observations of interplanetary fluctuations performed by Mariner 2 in 1962 [1] were surprisingly similar to the velocity spectra obtained for isotropic, homogeneous fluid turbulence [2]. This suggested Coleman [1] to formulate his heuristic model in which the kinetic energy stored in the streams was released via stream-shear instability to generate large-scale Alfvén waves directed both outward and inward with respect to the Sun. The energy associated with these waves would then cascade towards shorter scales through a hierarchy of Alfvén waves to be finally dissipated by ion-cyclotron damping. On the other hand, this *turbulent* picture was in contrast with observations by Belcher and Davis [3] who suggested that the solar wind was permeated mainly by a superposition of persistent and coherent outwardly propagating Alfvén waves. Moreover, the *turbulence* point of view was firmly based on the evidence that the spectral index of the inertial range of the spectrum was of Kolmogorov type and the superposition of non-interacting Alfvén waves could not lead to this result. The way-out was found in a mechanism successively called Dynamic Alignment. The idea was that the absence of non-linear interactions was the product of a relaxation of a turbulent process which started with an initial asymmetry in outward (Z^+, hereafter) and inward (Z^-, hereafter) modes. Given this asymmetry, it was impossible to reach a stationary state keeping the same unbalance of the two modes, since the transfer rate of energy $\Pi^{\pm}$ along the spectrum was the same for Z^+ and Z^-,

$$\Pi^{\pm} = \frac{de^{\pm}}{dt} \approx \frac{k}{B_0} e^- e^+ , \tag{1}$$

e^+ and e^- being the energy of the two modes at the same wave number k and B_0 the background field intensity. The final, relaxed Alfvénic population would be made of e^+ modes only, provided the non-linear time be much shorter than the convection time. If we start with an e^+ spectrum well developed over many decades and only a few e^- modes at low k with $e^+ \gg e^-$, the interaction is such that the e^- spectrum evolves acquiring higher wave numbers and transferring energy along the spectrum towards the dissipation range. In practice, the majority species sends the minority one to higher wave numbers to be dissipated. Matthaeus *et al.* [4] showed that the appearance of an inertial range populated by the minority species was extremely rapid compared to the convection time, while the spread of the majority species to higher k was very slow. This suggested a possible scenario which could explain the preponderance of outward modes in the solar wind. As a matter of fact, the very large scales are a reservoir of energy for the interplanetary fluctuations. Considering the stream interfaces as fluctuations in velocity and magnetic field, they can be looked at as a reservoir of negative cross-helicity which would produce a cascade of positive cross-helicity at smaller scales. Then, it was implicitly accepted that the interplanetary turbulence is not of solar origin but it is created beyond the critical point. However, this view clashed with the experimental evidence provided by Bruno *et al.* [5] about the presence of Alfvénic fluctuations with period longer than the transit time from the Sun to the observer which proved the solar origin of the observed fluctuations. The same authors [6] also noticed that Alfvénic fluctuations experienced a loss of Alfvénicity with increasing radial distance and that the Alfvén ratio (see definition in subsect. **2**·2) tended to values considerably lower than unity, which is not expected for pure Alfvén waves. Even these observations were in clear contradiction with predictions from the Dynamic Alignment model.

Helios observations provided a unique opportunity to study the radial evolution of the fluctuations in the inner solar system. The tendency of the spectral index towards a Kolmogorov index with increasing radial distance [6, 7] was recognised by Tu *et al.* [8] as evidence that non-linear processes were active. This strongly influenced the formulation of a new theory in which both inward and outward Alfvén modes, present in the solar wind in different amount, non-linearly interact producing the observed energy cascade. This model, which took into account effects due to WKB wave propagation (or geometric optic approximation) and those due to non-linear interactions between outward and inward modes, represents the first step towards a unification of the *turbulence* point of view of Coleman and the *wave* point of view of Belcher and Davis. This new theoretical approach has been the trigger and the standpoint for many papers [9], which have shed further light on the nature and evolution of interplanetary incompressive fluctuations.

2·1. *The statistical approach.* – Matthaeus and Goldstein [10] made the first effort to interpret how the spectra of magnetic (**b**) and velocity (**v**) fluctuating fields were related to the wind macroscopic structure. Since the detailed behaviour of **b** and **v** as functions of space and time is analytically impossible to obtain when an MHD fluid is in a turbulent state, they adopted a statistical approach. They suggested to use the three known rugged invariants eqs. (2)-(4) of the ideal MHD equations (*i.e.* in the hypothesis of magnetic diffusivity μ and kinematic viscosity ν equal to zero):

1) the energy per unit density

$$E = (1/2) \int (\mathbf{v}^2 + \mathbf{b}^2)\, \mathrm{d}^3 x\,, \tag{2}$$

which measures the total fluctuating energy associated with velocity and magnetic field in Alfvén units,

2) the cross-helicity

$$H_\mathrm{c} = (1/2) \int \mathbf{v} \cdot \mathbf{b}\, \mathrm{d}^3 x\,, \tag{3}$$

which measures the degree of correlation between the magnetic field expressed in Alfvén units and the velocity fluctuations,

3) the magnetic helicity

$$H_\mathrm{m} = \int \mathbf{A} \cdot \mathbf{B}\, \mathrm{d}^3 x\,, \tag{4}$$

which is a measure of the degree of kinkness of the magnetic-field lines and $\mathbf{A}$ is the magnetic vector potential such that $\mathbf{B} = \nabla \times \mathbf{A}$.

The use of the cross-helicity H_c normalised to the total energy E became of particular interest [11, 12]. This parameter, named σ_c, can vary between -1 and $+1$ giving a direct measurement of the preponderance either of outward modes ($\sigma_\mathrm{c} > 0$) or of inward modes ($\sigma_\mathrm{c} < 0$) once the magnetic sectors have been rectified accordingly [11, 12]. A study of the radial evolution of σ_c performed on Helios and Voyager data showed a clear tendency towards lower values with increasing heliocentric distance [11, 12]. This was taken as evidence of an increasing production of inward modes due to Kelvin-Helmholtz plasma instability at stream shears. However, Bavassano and Bruno [13] showed that a decrease of σ_c at hourly frequencies is most of the time closely related to magnetic field and/or density enhancements, concluding that in these cases the presence of convected structures in the solar wind would be responsible for the anomalous depletion of the Alfvénic correlation. In other words, they suspected that the observed decrease of σ_c in correspondence of compressive structures convected by the wind should have been ascribed to a loss of Alfvénicity in the outward fluctuations rather than to a local generation of inward modes. To solve the controversy it was necessary to look at the behaviour of the two opposite Alfvénic modes separately, as suggested by Grappin *et al.* [14]. This distinction has been made possible by the introduction of the Elsässer variables [15], although this exactly holds only within the framework of a homogeneous and incompressible medium. These variables were used for the first time within the solar-wind context by Dobrowolny *et al.* [16, 17] and Veltri *et al.* [18] for theoretical studies and by Grappin *et al.* [19, 14], Grappin [20] and Goldstein *et al.* [21] in numerical simulation. Successively, Marsch and Mangeney [22] re-formulated the compressible MHD equations in terms of Elsässer variables and, lately, Marsch and Tu [23] further developed their theory. Finally, Grappin *et al.* [24] and successively Tu *et al.* [25] fully adopted these variables to analyse Helios plasma and magnetic-field data.

2.2. *Elsässer variables and parameters definition.* – The Elsässer variables are defined as $\mathbf{Z}^{\pm} = \mathbf{V}_{\mathrm{sw}} \pm \mathbf{V}_{\mathrm{A}}$, where $\mathbf{V}_{\mathrm{sw}}$ is the solar-wind velocity, $V_{\mathrm{A}} = |\mathbf{B}|^2/\sqrt{4\pi\varrho}$ is the Alfvén velocity, and $|\mathbf{B}|$ and ϱ are the magnetic-field intensity and the proton number density, respectively. If we indicate the fluctuating part of the Elsässer variables as $\delta\mathbf{Z}^{\pm} = \delta\mathbf{V}_{\mathrm{sw}} \pm \delta\mathbf{V}_{\mathrm{A}}$ then $\delta\mathbf{Z}^{+}$ and $\delta\mathbf{Z}^{-}$ would be associated with outward and inward Alfvén modes, respectively.

If we indicate with $e_i^+(f)$ and $e_i^-(f)$ the second-order moments in the Fourier space of the i-th component of $\delta\mathbf{z}^+$ and $\delta\mathbf{z}^-$, respectively, we can define the following quantities:

the total energy spectrum

$$e(f_j) = \frac{1}{2} \sum_{i=x,\,y,\,z} (e_i^+(f_j) + e_i^-(f_j)) \,, \tag{5}$$

the cross-helicity spectrum

$$e^{\mathrm{c}}(f_j) = \frac{1}{2} \sum_{i=x,\,y,\,z} (e_i^+(f_j) - e_i^-(f_j)) \,, \tag{6}$$

the normalised cross-helicity

$$\sigma_{\mathrm{c}}(f_j) = e^{\mathrm{c}}(f_j)/e(f_j) \,, \tag{7}$$

and, finally, the Alfvén ratio spectrum

$$r_{\mathrm{A}}(f_j) = E_v(f_j)/E_b(f_j) \,, \tag{8}$$

where $E_v(f_j)$ and $E_b(f_j)$ are the traces of the spectral tensors of velocity and field, respectively.

2.3. *Elsässer spectra.* – The possibility of studying separately the behaviour of e^+ and e^- triggered a statistical study on the normalised cross-helicity σ_{c} by Bruno and Bavassano [26] at hourly frequencies. They suggested that compressible structures, progressively built up by the dynamic interaction between slow and fast streams, act destructively on the Alfvénic correlation decreasing the value of σ_{c}. In other words, the radial depletion of the normalised cross-helicity is due to a loss of outward modes rather than to an increasing production of inward modes. These results strongly reduced the role of e^- modes in the evolution of the turbulence, as is observed at low frequencies.

Grappin *et al.* [27] and Tu and Marsch [28] have studied in detail the behaviour of the spectral index of these modes across the stream structure and have found that the behaviour of outward and inward modes strongly depends on the stream structure and also on the heliocentric distance [25, 26].

At short heliocentric distances and within high-speed streams the spectrum of e^+ shows a rather flat spectral index, around -1, at low frequencies, say less than a few times 10^{-4} Hz, and a steeper index close to the Kolmogorov one at higher frequencies. The e^- spectrum behaves in the opposite way. It shows a Kolmogorov-type index at low frequencies and a rather flat index, larger than -1, at higher frequencies. When these spectra are plotted together, as in fig. 1, they form an inclined lozenge with e^+ lying above e^- [29]. Moving from fast to slow wind the lozenge tends to close and resemble a

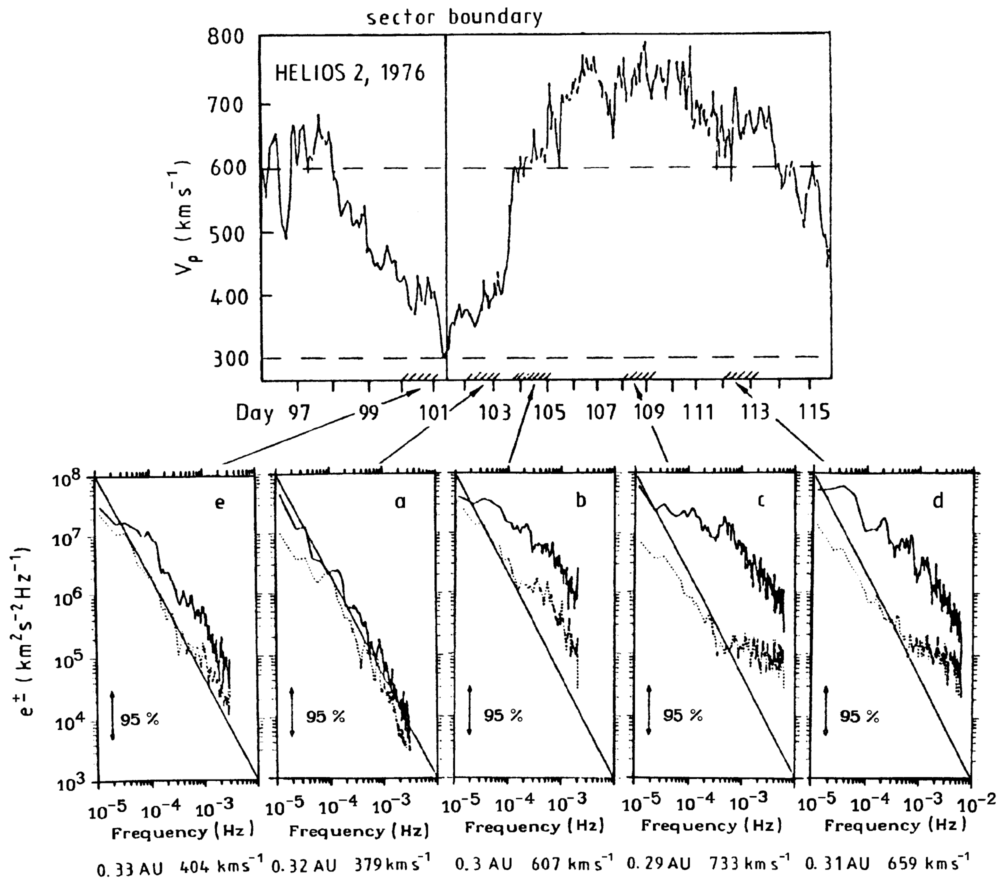

Fig. 1. – Helios data. Sequence of five spectra of e^+ and e^- across slow and fast plasma at 0.3 AU (after Tu *et al.* [25]).

fully developed Kolmogorov spectrum. The spectrum of e^+ is the one that experiences the largest evolution while the spectrum of e^-, except for the high-frequency range, is quite constant. A statistical study was performed by Tu and Marsch [28] on a large number of spectra taken at 0.4 and around 0.8 AU within slow and fast wind. For scales larger than $\sim 2 \cdot 10^6$ km they found that e^- spectra are statistically independent of wind velocity and heliocentric distance, with a spectral index near $-5/3$. This peculiarity suggested the idea of a *universal background spectrum* closely resembling a fully developed turbulence. However, the smaller scales do experience some stream dependence. As a matter of fact, within fast wind and at short distances from the Sun the spectral index is rather flat. Quite a different behaviour was shown by e^+ modes, which experience a strong dependence on both distance and speed. Spectra are much higher in fast wind and the radial evolution would indicate that the *universal* spectrum could be the final state. Moreover, their study confirmed the radial tendency for r_A to assume values around 0.5, indicating a predominance of magnetic over kinetic energy. These observations unambiguously proved that the radial decline of σ_c is mostly due to

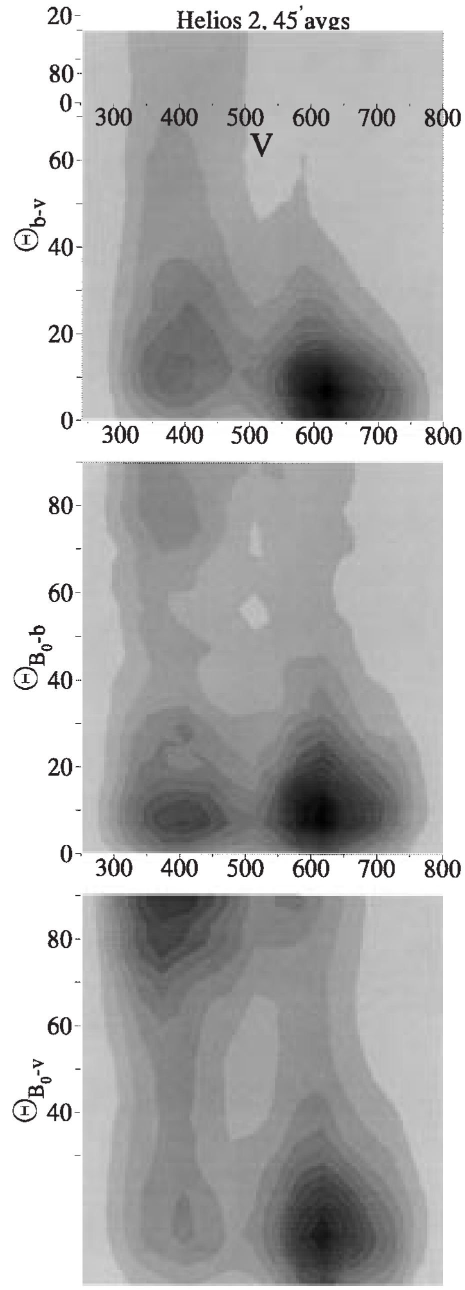

Fig. 2. – Helios data. Contour lines of minimum variance angles *vs.* solar-wind speed. The angle Θ_{VB} measures the separation between velocity and field minimum variance directions. The angle Θ_{bB_0} indicates the angular separation between the field minimum variance direction and the direction of the average background field B_0. The angle Θ_{vB_0} indicates the angular separation between the velocity minimum variance direction and the direction of B_0 (after Klein *et al.* [30]).

a loss of e^+ modes rather than to an increasing local production of e^- modes as previously suggested [12, 13].

2·4. *Magnetic field-velocity fluctuation decoupling.* – What kind of interaction is then established when Alfvén waves non-linearly couple with static structures convected by the wind is one of several issues still open in MHD turbulence. The hypothesis has recently been advanced [30-32] that one of the effects of this interaction could be a decoupling between field and velocity fluctuations. This phenomenon would cause the destruction of the Alfvénic correlation which would consequently contribute to the depletion of the normalised cross-helicity σ_c. In fig. 2 we show three-dimensional histograms of minimum-variance angles *vs.* solar-wind speed obtained by Klein *et al.* [30] for the low-frequency range (45 minute averages) using Helios data from 0.3 to 1 AU. There is a clear tendency for the minimum-variance directions of field and velocity fluctuations to be aligned to each other and to lie along the average background field direction mostly within fast wind. In slow wind the distribution spreads out between 0° and 90°, especially for velocity data. In summary, they found not only a strong tendency for field and velocity fluctuations to decouple in slow, high-β plasma throughout the entire distance range, but also a trend to decouple within fast wind with increasing distance. It was also confirmed that during the wind expansion the field minimum-variance direction remains along the spiral while the velocity minimum-variance direction tends to be radial [33]. Compressive fluctuations were found particularly efficient in decoupling field and velocity fluctuations by Bruno and Bavassano [31] who, in a successive paper, studied the hourly frequency range.

However, even non-compressive structures have been recognised to have an important role in the observed decrease of σ_c and r_A by Tu and Marsch [34] in a case study of very low cross-helicity. These structures, named *magnetic-field directional turnings* (MFDT), are characterised by kinky variations of the magnetic-field direction that keep the field intensity constant. MFDTs, typically incompressive, could also be found in fast wind but the original amplitude, close to the Sun, should be much lower than the Alfvénic one. With the wind expansion the amplitude of the Alfvénic fluctuations should damp according to the WKB and the turbulent cascade effect, while the amplitude of MFDTs should not change much. Thus, at larger heliocentric distances MFDTs would become more efficient and could strongly contribute to the observed σ_c and r_A decrease. Tu and Marsch [35] suggested that the fluctuations in the solar wind are mainly composed of Alfvén waves of coronal origin, intermingled with convected static structures (MFDTs) with, in addition, inward Alfvén waves of local origin generated by shear instability [11, 12] and/or parametric decay instability [29, 20].

2·5. *Turbulence as a mixture of* 2D *convected structures and Alfvén waves.* – Matthaeus *et al.* [36], using 16 months of ISEE 3 data, were able to build a two-dimensional correlation function producing the *Maltese cross*-like contour plot shown in fig. 3. In this plot, constructed under the assumption of rotational symmetry, Alfvénic turbulence characterised by $\mathbf{k} \parallel \mathbf{B}_0$ is represented by contours elongated parallel to $r_\perp$ which dominate at small separations on contours that are elongated parallel to $r_\parallel$ and that dominate at large separations. In other words, the correlation function is due to two populations of fluctuations: Alfvénic fluctuations characterised by larger correlation length transverse to $\mathbf{B}_0$ and 2D turbulence with larger correlation length in planes parallel to $\mathbf{B}_0$. Oughton and Matthaeus [37] were the first ones to suggest that the solar-wind fluctuations can be modelled as a mixture of Alfvén waves

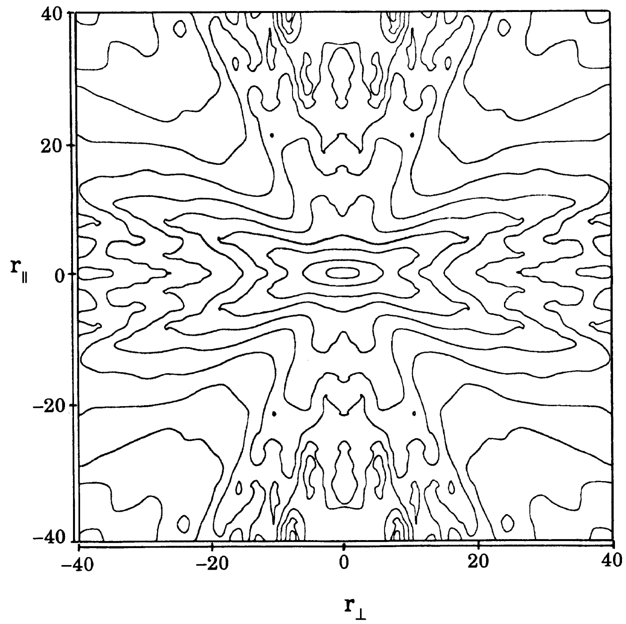

Fig. 3. – ISEE 3 data. Contour plot of the 2D correlation function built from 15′ averages of the IMF fluctuations taken at 1 AU. The separations in $r_\perp$ and $r_\parallel$ are in units of 10^5 km. Alfvénic fluctuations are represented by contours elongated parallel to $r_\perp$ and dominate at short separations. 2D turbulence is represented by contours elongated parallel to $r_\parallel$ and dominate at large separations (after Matthaeus *et al.* [36]).

and 2D turbulence, but Tu and Marsch [38, 35] were the first ones to combine in a single model the effects of a turbulence made of these two components, namely, Alfvén waves of solar origin and convective uncompressive structures. In this model the angular effect due to Parker spiral on the expansion of the wind is an important ingredient to explain the radial behaviour of both σ_c and r_A [38]. Depending on the distance from the source and on the speed regime of the wind, the observer samples data preferentially either along the field lines or across them. The first condition would be favourable to the detection of Alfvén waves, the second one would be favourable to detect convective structures that are a specific kind of 2D turbulence [35, 38]. The following two important assumptions were made in order to apply this model: *a*) there are no inward modes, *b*) flow tube structures and also non-linear interactions are absent. The next step [39] was to extend Tu's WKB-like model in order to include both spectrum equations for e^+ and e^- and solve them self-consistently. However, they found that the energy cascade process due to non-linear interactions and WKB effect could only produce Dynamic Alignment and could not create and sustain the flat part of the e^- spectrum. In other words, since negative cross-helicity injected at low frequency would not be transferred to the high-frequency range, e^+ and e^- spectra would diverge. At this point local sources had to be invoked, and the parametric decay instability of circularly polarised Alfvén waves [25, 39] seemed to be a good candidate. This instability consists of the decay of circularly polarised Alfvén waves (even incoherent) forward propagating, into backward propagating Alfvén waves and forward sound-like waves. Promising results were obtained from the model and it was shown that this mechanism is able to sustain the flat part of the e^- spectra.

From *in situ* measurements performed at 0.3 AU we infer that the energy of e^- contained at small scales (*i.e.* at frequencies ω between 10^{-3} and 10^{-2} Hz) is in excess by about 1 decade with respect to the *Universal Background Spectrum* [25]. We may want to compare the time required by parametric decay to produce such an excess with the transit time required to reach the location of the observer at 0.3 AU. Marsch and Tu [39] provide the following expression for the maximum growth rate in a plasma with $0.5 < \beta < 1.0$:

$$\gamma = (\mathrm{rat})\, \omega_0 b \,, \tag{9}$$

where rat is a constant ratio varying between 0.1 and 0.2, b is the relative amplitude of the pump wave, with a reasonable value around 0.1 and ω_0 is the frequency of the pump wave in the plasma rest frame. For a pump wave propagating in the radial direction, $\omega_0 \sim \omega/5$, with ω measured in the s/c reference system and, consequently, $\gamma \sim 2 \cdot 10^{-5}\,\mathrm{s}^{-1}$. Then, the order of magnitude of the time required for the production of the observed e^- excess would be

$$\Delta t = \ln\left(\frac{e^-}{e_0^-}\right) \cdot \frac{1}{\gamma} \sim 1 \text{ day} \,, \tag{10}$$

which is comparable with the transit time between the Alfvénic point and 0.3 AU.

However, the non-local interaction between e^- modes at high frequency and e^+ modes at low frequency, as suggested by Grappin [21] in numerical simulations, could also do the job and produce the observed flattening. For this author non-linear interactions are non-local in k-space and the modes of one type at small scales ($e^\pm$) are continuously created by modes of the opposite type at large scales ($E^\mp$), *i.e.* at scales about 1 decade larger. From the closure equation adopted by Grappin [20],

$$\frac{\mathrm{d}e^\pm}{\mathrm{d}t} \sim \frac{k}{B_0} E^\mp e^\pm \,, \tag{11}$$

we can obtain the order of magnitude for this time interval from

$$\Delta t = \ln\left(\frac{e^-}{e_0^-}\right) \cdot \frac{B_0}{kE^+} \,. \tag{12}$$

If we set the ratio $e^-/e_0^- \sim 10$, $B_0 = 10^2\,\mathrm{km\,s^{-1}}$, $E^+ \sim 10^2(\mathrm{km\,s^{-1}})^2$, we obtain $\Delta t \sim 2 \cdot 10^5\,\mathrm{s}$, which is again comparable with the transit time of about a day. However, also this value has to be considered just as a rough estimate of Δt.

2.6. *Turbulence generated at velocity shear.* – As predicted by Coleman [1] and as shown in computer simulations, interplanetary velocity shears are a reservoir of kinetic energy which could be released via plasma instabilities as Alfvénic fluctuations of both signs [40-45]. This phenomenon is currently being interpreted by part of the community as the main cause of the radial decay of the normalised cross-helicity throughout the entire inertial range of the fluctuations. These simulations were based on a 2D incompressible, periodic MHD code and the initial conditions consisted in a narrow low speed stream surrounded by two high-speed streams and the magnetic and velocity shear layers were determined by the 6 lowest wave numbers. A population of pure Alfvén waves characterised by a rather flat spectrum were added at scales smaller than those defining the shear layers and some random fluctuations were added to initiate the non-linear coupling. The initially dominant e^+ fluctuations rapidly evolve towards a steeper spectrum, while e^- modes at higher frequencies are continuously added to the original spectrum. The shear-driven turbulence provides almost the same energy input at large scales for e^+ and e^-. On the contrary, while the dissipation rate for e^- at small scales equals the energy input at large scales, the dissipation rate for e^+ is greatly enhanced by the higher level of power at small scales. As a consequence, the e^- spectrum reaches a quasi-steady state that is successively approached by e^+. However, the model seems unable to reproduce the flattening of e^- modes at high frequencies. In another simulation, where they used a compressible code [43] in the presence of a velocity shear, Roberts and collegues obtained a persistent anti-correlation between magnetic-field intensity and number density within the high-velocity stream suggesting the presence of pressure-balanced structures as predicted by the N.I. theory. They also found a clear correlation between z^- and density fluctuations, as already observed by other authors [13, 27] in Helios data. They concluded that density fluctuations observed in the solar wind are then essentially a by-product of the incompressible evolution of the fluctuations rather than due to convected structures. However, the stream-shear mechanism fails to reproduce some of the observations to the extent that Marsch and Tu [39] and Bavassano and Bruno [46] found essentially no correlation between field and density fluctuations inside high-velocity streams and a slight negative correlation in low-velocity regions just in opposition to the results obtained by Roberts *et al.* in their simulations [43].

3. – Compressive turbulence

Compressive fluctuations are an important ingredient of interplanetary MHD turbulence. Although their weight in the turbulence energy budget of the solar wind is much less than that of incompressible fluctuations [46], they play an important role in the evolution of the turbulence itself, but their specific role is still uncertain. Some investigators think they strongly contribute in reducing the *Alfvénicity* of the fluctuation, while some others think they rather are a by-product of the non-linear evolution of non-compressive fluctuations. It is clear that a good understanding of their nature is necessary for a complete knowledge of the MHD fluctuations in the solar wind.

Magnetic-field intensity and number density power spectra are quite similar in many respects [23]. Larger fluctuations are found in slower wind, where the spectral index is closer to $-5/3$ and no radial evolution is observed. In contrast, in fast streams the turbulence level increases with distance and a flattening of the spectrum is observed at high frequencies and fades away as the radial distance increases. It should be noted that there is some similarity to the e^- spectrum, which would suggest a similar origin for these fluctuations. As previously proposed [13], the e^- spectrum may become more and more connected to compressive fluctuations as the wind expands. There are three kinds of candidates which could be responsible for such compressive fluctuations: static-pressure–balanced structures, fast magnetosonic waves and the fluctuations described by the NI-MHD theory [47]. The study of the correlation between the thermal pressure P_k and the magnetic pressure P_b is useful to understand the nature of these fluctuations. A positive correlation would be indicative of the presence of compressive structures, like interaction regions or fast-mode magnetosonic waves, a negative correlation would suggest the presence of pressure-balanced structures (PBSs), since slow-mode magnetosonic waves, which would have the same kind of correlation, are strongly and quickly damped right after a few cycles away from the source. The occurrence rate of PBSs has been found to increase with heliocentric distance and in the outer heliosphere firstly by Vellante and Lazarus [48]. A strong dependence on stream structure and solar activity has also been found [46]. Near the Sun Alfvénic intervals are clearly dominant on PBS although less evident during solar maximum. Close to 1 AU the Alfvénic intervals are still a majority only at low solar activity and in fast wind, while at low speed they are overcome by PBS. At high solar activity PBS are generally dominant.

The spectral similarity between e^- and compressive fluctuations has brought Marsch and Tu [39] to formulate a model in which field intensity and plasma density fluctuations are assumed to derive from PBSs related to small flow tubes and from fast magnetosonic waves produced by residual pressure imbalances between adjacent flow tubes. The growth of large-scale compressive fluctuations would be due to a lateral interaction between flow tubes deriving from the spiral configuration of the field geometry. A successive cascade process would produce compressive fluctuations at smaller scales. Results from this superposition model have shown to be quite consistent with observations.

3·1. *Nearly incompressible theory.* – A different way to look at compressive fluctuations is given in the Nearly Incompressible (NI) theory introduced by Montgomery *et al.* [47] and Shebalin and Montgomery [49] and further developed by Matthaeus and coworkers in the past few years [50-55]. The basic ideas descend on the acustic-wave generation by vortical flows found by Lighthill [56] and the connection found by Kleiderman and Majda [57] between low turbulent Mach number and incompressible fluidodynamics. In this theory the compressible MHD equations are expanded about small turbulent Mach numbers to approach the solutions of the incompressible MHD equations. In this approach compressive fluctuations are non-linearly driven by sources related to small deviations from incompressibility and, if heat conduction is allowed, two different types of magnetofluids arise from the theory. However, in both types of fluid, density and temperature spectra are predicted to be similar.

The NI theory predicts the existence of a clear correlation between temperature T and density N. Such a correlation is expected to be positive or negative depending on the character of the fluid which is due to the relative amplitude of density, temperature and thermal pressure and by their scaling with turbulent Mach number M. For a fluid characterised by strong fluctuations in temperature and density, a negative N-T correlation is expected and, moreover, the thermal pressure should scale as M^2, while density and temperature should scale linearly with M. Such a fluid is called *heat flux dominated* or *HFD fluid*. In this type of fluid the situation is such that thermal pressure fluctuations are balanced by anticorrelated density and temperature fluctuations. On the other hand, in a fluid where these fluctuations are all on an equal footing and scale as M^2, the above N-T correlation is positive, provided that acoustic and incompressible pressure fluctuations are correlated with temperature fluctuations, leading to a recovery of the *pseudosound* relations [47, 51]. This kind of fluid is called *heat flux modified* or *HFM fluid*. These small density fluctuations should ride parasitically on the back of the incompressible turbulence [50].

3˙2. *Tests for the* NI *theory.* – Voyager data were used by Matthaeus *et al.* [55] to test the validity of NI theory. They chose hourly averages with $M < 1.0$ between 1 and 7 AU and plotted density fluctuations $\delta\varrho$ as a function of ϱM^2 in a log-log scale, as shown in the left panel of fig. 4. The clear linear dependence visible in the data distribution is completely lost once we look at the distribution of the normalised density fluctuations $\delta\varrho/\varrho$ *vs.* M^2 in the same scale. These results suggest that $\delta\varrho$ fluctuations are organised by values of ϱ rather than M^2, since the range of variability of M is not large enough to influence this distribution. Other papers [29, 51, 52] have reported about cases of agreement between theoretical predictions and data analysis, however, an extensive applicability of the NI theory to the solar-wind data has not been found, yet [9]. Bavassano *et al.* [58] and Bavassano and Bruno [59] have performed a careful analysis based on the study of the correlation coefficients between density and temperature fluctuations for time scales of 45 minutes and 3 hours within the inner heliosphere. It was found that: 1) relative density fluctuations and sonic Mach number much less than unity (*e.g.* ~ 0.1) are very seldom encountered in the solar wind; 2) out of almost 5000 intervals of 45 minutes each, less than 2.5% of cases showed a significant correlation/anticorrelation ($|\varrho_{NT}| \geqslant 0.8$) between density and temperature fluctuations; 3) anticorrelations are mostly found in slow wind while correlations do not depend much on the flow speed; 4) no evidence for radial dependence of the correlations or anticorrelations is found; 5) for those few intervals where the above condition on $|\varrho_{NT}|$ was satisfied and had $\delta N/N$ and M small enough to make the theory applicable, no scaling of $\delta N/N$ with M or M^2 was found at all.

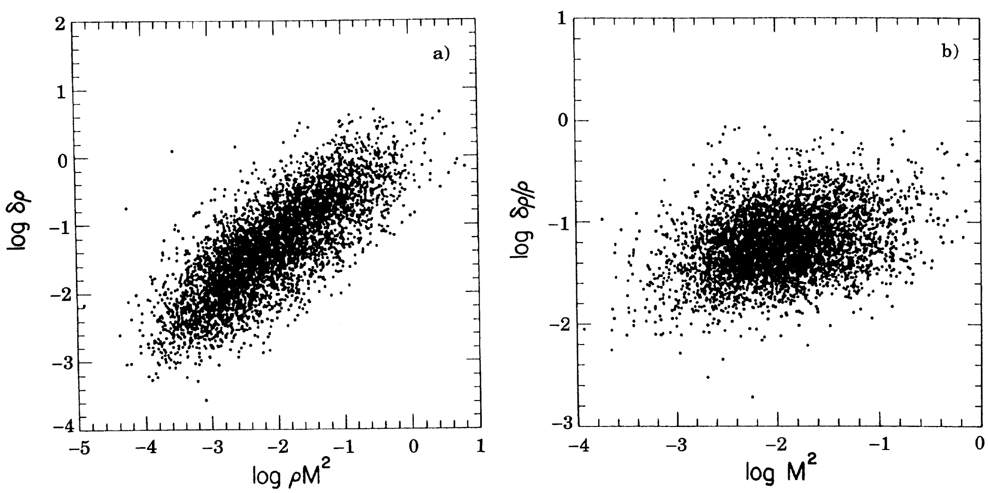

Fig. 4. – Voyager data. Left panel: scaling of $\delta\varrho$ *vs.* ϱM^2 computed for 1-hour intervals between 1 and 7 AU. The linear scaling which characterises these data in log-log scale fades away when relative number density fluctuations $\delta\varrho/\varrho$ are plotted *vs.* M^2 as shown in the right panel (after Matthaeus *et al.* [55]).

As suggested by Marsch and Tu [39] the solar wind is not homogeneous enough to make the NI theory applicable. From our point of view the model proposed by Tu and Marsch [35] in which density, temperature and field intensity fluctuations should derive from pressure-balanced structures related to small flow tubes imbedded in the large streams, and from fast magnetosonic waves due to residual pressure imbalances between adjacent flow tubes, seems more realistic.

4. – Conclusions

The MHD turbulence in the solar wind has been the object of a remarkable number of studies within the past decade. Although theoretical and observational works have brought several new ideas in this field, there are still many questions which need to be answered in order to fully understand the nature and evolution of the MHD fluctuations.

There have been important advances in understanding the origin of these fluctuations and the Sun has been recognised to be the main source. The so-called *wave* point of view and the *turbulence* point of view finally coexist within the frame of a single theory, called WKB-like. The introduction of Elsässer variables has largely changed the way to look at Alfvénic fluctuations and allowed to study several aspects of the *inward* and *outward* contributions separately. The presence of nonlinear turbulence interactions in the wind fluctuations has been fully accepted and the spectral transfer equations for *inward* and *outward* modes have been included in the WKB-like model and solved separately. Models which represent the wind fluctuations as due to two components like outward Alfvén waves and two-dimensional convected, incompressible structures have given promising results. The study of the correlations between several compressible parameters and their spectra have helped to establish the nature of compressive fluctuations in the solar wind and new theories like the *nearly incompressible* one provide a new class of compressive fluctuations besides the pressure-balanced structures and magnetosonic waves. Numerical simulations have shown the importance of velocity shears in generating turbulence locally in the wind.

However, there are many other points which need to be understood. For example, we still do not know the exact mechanism responsible for the observed evolution of *inward* and *outward* Alfvén modes. Only speculative solutions have been given to the problem of the radial evolution of the Alfvén ratio. Which is the cause that generates the flat part of the spectrum of the *inward* modes? Are these modes really *inward* propagating Alfvén waves or rather the spectral signature of convective structures? Which is the role of compressive and noncompressive structures in the evolution of the Alfvénic turbulence? What is the relevance of locally generated turbulence by velocity shear? Are the compressive fluctuations one of the causes for the radial depletion of the normalised cross-helicity or rather the product of the turbulent evolution of uncompressive fluctuations?

These and many other outstanding issues on the argument need to be answered and the lack of very desired missions like Solar Probe makes this task very difficult to accomplish. However, some help can certainly come from computer simulations once the codes reach an adequate resolution in fully three dimensions. In the mean time, modelling efforts must be carefully and continuously checked against observations which should remain the only testing ground of any theory.

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This paper is based on an invited talk given at the VII GIFCO Meeting held in Rimini (Italy) in 1994. I wish to thank Prof. A. EGIDI and Prof. F. MARIANI for their kind invitation. I would also like to acknowledge many enlightening discussions with Dr. B. BAVASSANO.

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