Single spacecraft identification of the bow shock orientation and speed: A comparison between different methods (*)(**)

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Summary. — We examine 33 bow shock crossings by IMP8 and compare different methods to calculate the bow shock normal direction and speed using single spacecraft measurements. We find that the mixed equation by Abraham-Shrauner combined with the mass flux conservation equation and the minimum-variance technique applied to a limited set of the Rankine-Hugoniot conservation equations give very similar results that are in good agreement with theoretical predictions. The solutions obtained by the velocity coplanarity theorem are reliable only for nearly perpendicular shocks, while poor results are obtained for such cases from the magnetic coplanarity theorem. We also suggest that in some cases the time resolution of plasma measurements (about 60 s) may be too low to resolve the density behaviour close to the bow shock and to allow definite evaluation of the shock parameters.

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1. – Introduction

An important aspect of bow shock (BS) as well as of interplanetary shocks research is determining the normal direction ($\widehat{n}$) and speed (V_s, directed along $\widehat{n}$) of the shock front. In the last several years, different methods have been developed to calculate these two parameters from single spacecraft measurements [1-6] and several papers [2-7] also compared the results obtained by different methods; however, this kind of analysis has been performed only for a few events, and mostly for inter-

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planetary or synthetic shocks. In this note, focusing our attention on 33 BS crossings identified in the IMP8 observations between 1974 and 1978 [8], we statistically compare the shock parameters obtained by different methods and check their reliability for different shock geometries by calculating the deviations between the observed parameters and the values expected from the Rankine-Hugoniot (R-H) conservation equations.

2. – An analysis of different methods

The problem of determining $\hat{n}$ and V_s theoretically consists in finding a suitable frame of reference in which the full set of the R-H conservation equations

(1) mass flux $$[\varrho U_n] = 0\,,$$

(2) magnetic-field normal component $$[B_n] = 0\,,$$

(3) electric-field tangential component $$B_n[\overline{U}_t] - [\overline{B}_t U_n] = 0\,,$$

(4) momentum tangential component $$\varrho U_n[\overline{U}_t] - \frac{1}{4\pi} B_n[\overline{B}_t] = 0\,,$$

(5) momentum normal component $$\left[\varrho U_n^2 + P + \frac{1}{8\pi} B_t^2\right] = 0\,,$$

(6) energy $$\left[\frac{U^2}{2} + \frac{\gamma}{\gamma - 1}\frac{P}{\varrho} + \frac{B^2}{4\pi\varrho} - \frac{B_n \overline{U}\cdot\overline{B}}{4\pi\varrho U_n}\right] = 0\,,$$

is satisfied for a stationary shock. In these equations $\overline{B}$ is the magnetic field vector, $\overline{U}$ is the plasma bulk velocity in the shock frame ($\overline{U} = \overline{V} - \hat{n}V_s$, where $\overline{V}$ is the bulk velocity in the inertial frame), P is the total plasma pressure, ϱ is the mass density, γ is the ratio of specific heats, subscripts n and t indicate, respectively, the normal and tangential component to the shock front and the square brackets indicate the variation across the shock of a given quantity, $[A] = A_2 - A_1$, in which subscripts 1 and 2 identify, respectively, the upstream and downstream side of the shock.

As matter of fact, due to limitations of the experimental observations, $\hat{n}$ and V_s are often estimated by a reduced set of the R-H equations which does not require definite knowledge of the whole set of solar-wind parameters. Equations (2), (3) and (4) allow estimation of $\hat{n}$ by means of the magnetic coplanarity theorem (MC) [1],

$$\hat{n} = \pm \frac{(\overline{B}_1 \times \overline{B}_2) \times (\overline{B}_2 - \overline{B}_1)}{|(\overline{B}_1 \times \overline{B}_2) \times (\overline{B}_2 - \overline{B}_1)|}\,, \tag{7}$$

the velocity coplanarity theorem (VC) [7],

$$\hat{n} = \pm \frac{\overline{V}_2 - \overline{V}_1}{|\overline{V}_2 - \overline{V}_1|}\,, \tag{8}$$

and the equation by Abraham-Shrauner (AS) [3, 7]

$$\hat{n} = \pm \frac{[(\overline{B}_2 - \overline{B}_1) \times (\overline{V}_2 - \overline{V}_1)] \times (\overline{B}_2 - \overline{B}_1)}{|[(\overline{B}_2 - \overline{B}_1) \times (\overline{V}_2 - \overline{V}_1)] \times (\overline{B}_2 - \overline{B}_1)|}\,. \tag{9}$$

It is easy to see that the reliability of those methods depends on the angles Θ_{Bn} (between $\hat{n}$ and B_1) and Θ_{BB} (between B_1 and B_2). Indeed, both the MC and AS equations become singular when Θ_{Bn} approaches 0°, *i.e.* for parallel shocks (eqs. (3) and (4) show this in the case $\overline{B}_2 - B_1 \rightarrow 0$). Besides, as discussed by Colburn and Sonett [1], MC is singular also for $\Theta_{BB} \rightarrow 0$. As for VC, it is expected to be an exact technique only for exactly perpendicular or parallel shocks; however, it progressively improves [7] with increasing Θ_{Bn} and Alfvenic Mach number M_a ($M_a = V_1/V_{a1}$, $V_{a1} = B_1/(4\pi\varrho_1)^{1/2}$).

Given $\hat{n}$, the shock speed can be calculated from the mass flux conservation equation (1), which gives

$$V_s = \frac{\hat{n}\cdot(\varrho_2 \overline{V}_2 - \varrho_1 \overline{V}_1)}{\varrho_2 - \varrho_1} . \tag{10}$$

Smith and Burton [5] (SB) have proposed an alternative equation for the calculation of the shock speed V_s' in the upstream solar-wind frame; it is based on the conservation of the electric field in the shock frame:

$$V_s' = \frac{|(\overline{V}_2 - \overline{V}_1) \times \overline{B}_2|}{|(\overline{B}_2 - \overline{B}_1)|} . \tag{11}$$

In this case the shock normal speed is easily determined by means of

$$V_s = V_s' + \overline{V}_1 \cdot \hat{n} . \tag{12}$$

As pointed out earlier, none of the previous equations use the whole set of the R-H conditions for determining the shock orientation and speed. This aspect, together with the effects of the uncertain determination of the upstream and downstream plasma parameters (which always requires an averaging procedure over subjective time scales) lead to different solutions when different methods are used.

A different procedure has been developed by Viñas and Scudder [4] and then improved by Szabo [6] for the calculation of the shock parameters. It is expected to be much less affected by the experimental uncertainties and its reliability is not expected to depend on the shock geometry; moreover, it also provides an estimate of the shock parameters uncertainties. The basis of the technique is a minimum-variance analysis (MV). It consists in finding the $\hat{n}$ direction which minimizes the quantity,

$$\chi^2 = \sum_{i=1}^{N_{\text{data}}} \sum_{j=1}^{N_{\text{equat}}} \frac{Y_j(X_i)^2}{\sigma_{ij}^2} , \tag{13}$$

where X_i are individual measurements of the plasma and magnetic-field parameters on the upstream and downstream sides of the shock, Y_j are the left-hand terms of eqs. (2)-(6) in which eq. (10) has been used to eliminate the shock speed V_s (the expected values of Y_j are of course zero) and σ_{ij} are their standard deviations due to the experimental uncertainties. Once $\hat{n}$ has been determined, V_s is calculated by the same minimum-variance technique applied to eq. (1). In the present analysis, we used the reduced set of the R-H equations (1)-(4) [9] to calculate the shock parameters by means of the MV technique (and in particular by means of the numerical codes by Szabo [6]), since the electron temperature is not experimentally determined.

The MV technique is the only method which optimizes the agreement between its solutions and the predictions of the R-H conservation equations; conversely, the other methods assume some relationships derived from the R-H equations to be exactly verified and then use them to compute the shock parameters.

We check the reliability of the different methods by comparing the agreement of their solutions with the theoretical predictions; using the V_s and $\hat{n}$ parameters obtained from each method, we calculated the normalized deviations of the left-hand terms of equations (1)-(4) from the null values predicted for the ideal case:

$$\{\varrho U_n\} = \frac{2[\varrho U_n]}{\varrho_1 U_{n1} + \varrho_2 U_{n2}} , \tag{14}$$

$$\{B_n\} = \frac{2[B_n]}{|\overline{B}_1| + |\overline{B}_2|} , \tag{15}$$

$$\{B_n \overline{U}_t + \overline{B}_t U_n\} = \frac{2\,|B_n[\overline{U}_t] - [\overline{B}_t U_n]\,|}{|B_n \overline{U}_{t1} - \overline{B}_{t1} U_{n1}| + |B_n \overline{U}_{t2} - \overline{B}_{t2} U_{n2}|} , \tag{16}$$

$$\{\varrho U_n \overline{U}_t - B_n \overline{B}_t /4\pi\} = \frac{2\,|\varrho U_n [\overline{U}_t] - B_n[\overline{B}_t]/4\pi|}{|\varrho U_n \overline{U}_{t1} - B_n \overline{B}_{t1}/4\pi| + |\varrho U_n \overline{U}_{t2} - B_n \overline{B}_n /4\pi|} . \tag{17}$$

3. – Data analysis and experimental results

We examined the same set of 36 bow shock crossings by IMP8 (whose orbit is approximately circular around the Earth, at a geocentric distance of the order of 35 R_e and with a period of the order of 13 days) selected for a previous study of the BS motion [8]. The time resolution of the plasma measurements [10] was of the order of 30 s in the first months of the mission and then decreased to approximately 60 s; the experimental errors are of the order of 3% for the plasma velocity and 5% for the density. The magnetic-field data [11] consist of 15 s average values and variances of the three components, computed from higher-resolution data. In our analysis, the magnetic-field measurements have been averaged with the same time resolution as the plasma data. Three events were discarded from the original set [8], due to large variations of the magnetic-field components: the computed average values were significantly different from the higher-resolution ones closest to the BS crossing. This study thus included 33 events.

For the application of the MV technique, two or three data points on each side of the shock have been used (their number has been chosen so that they are subjectively part of the same stationary conditions); the weighted average values of the same experimental data have been used to calculate the shock parameters by means of eqs. (7)-(12). In the present investigation, $\hat{n}$ is positive for the outward direction; it implies that negative values of the BS speed V_s indicate motion toward Earth.

Figure 1(a) shows the histogram of the Θ_{Bn} (with $\hat{n}$ determined by means of the MV technique) and the Θ_{BB} angles for the 33 events. As can be seen, in most cases Θ_{Bn} is greater than 60°, and the magnetic-field rotation across the shock is smaller than 20°. This result indicates that for our data set the BS is usually a quasi-perpendicular structure. (As a matter of fact, 9 events occur in the dusk side of the magnetosphere,

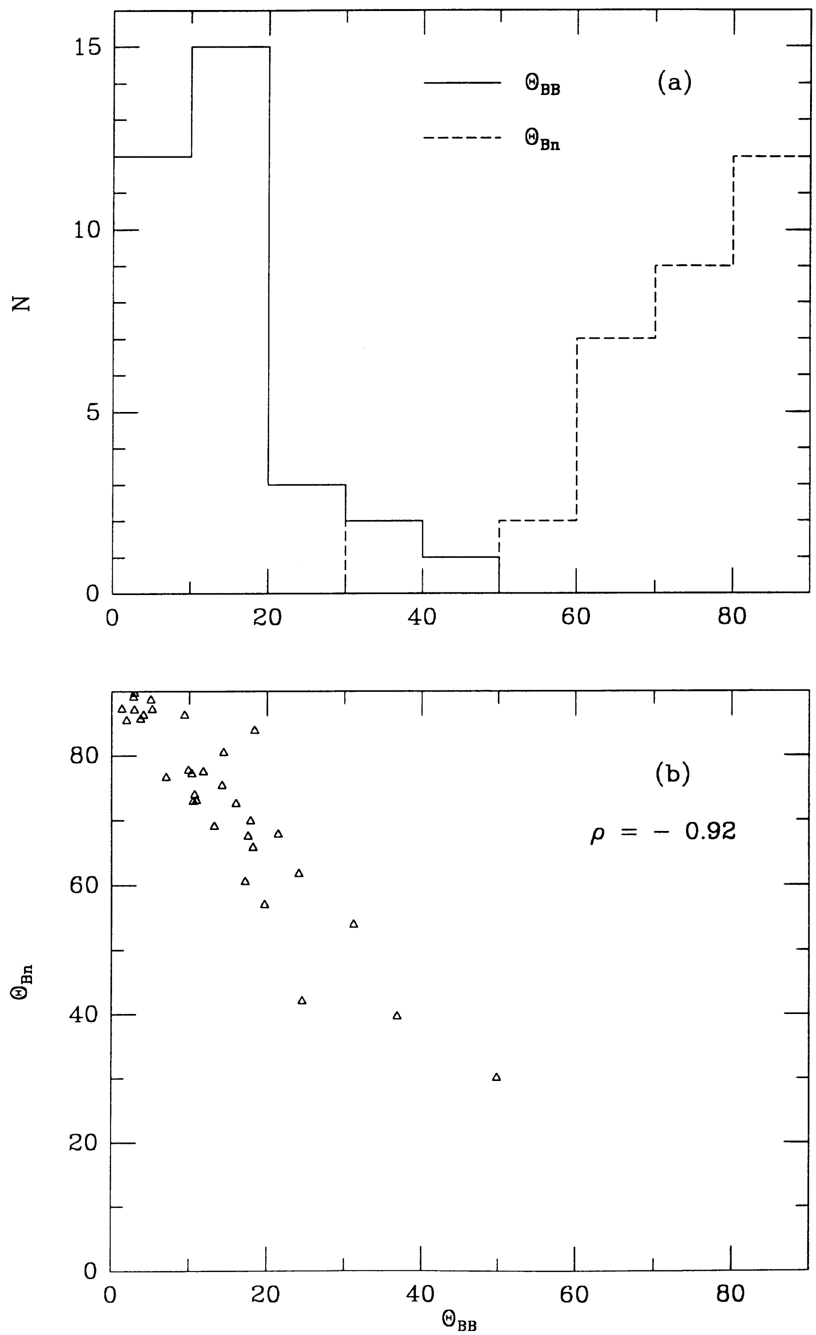

Fig. 1. – (a) Histograms of the angles Θ_{BB} (between B_1 and B_2) and Θ_{Bn} (between B_1 and $\widehat{n}$ estimated by means of the MV technique). (b) Θ_{Bn} *vs.* Θ_{BB}. Both angles are in degrees.

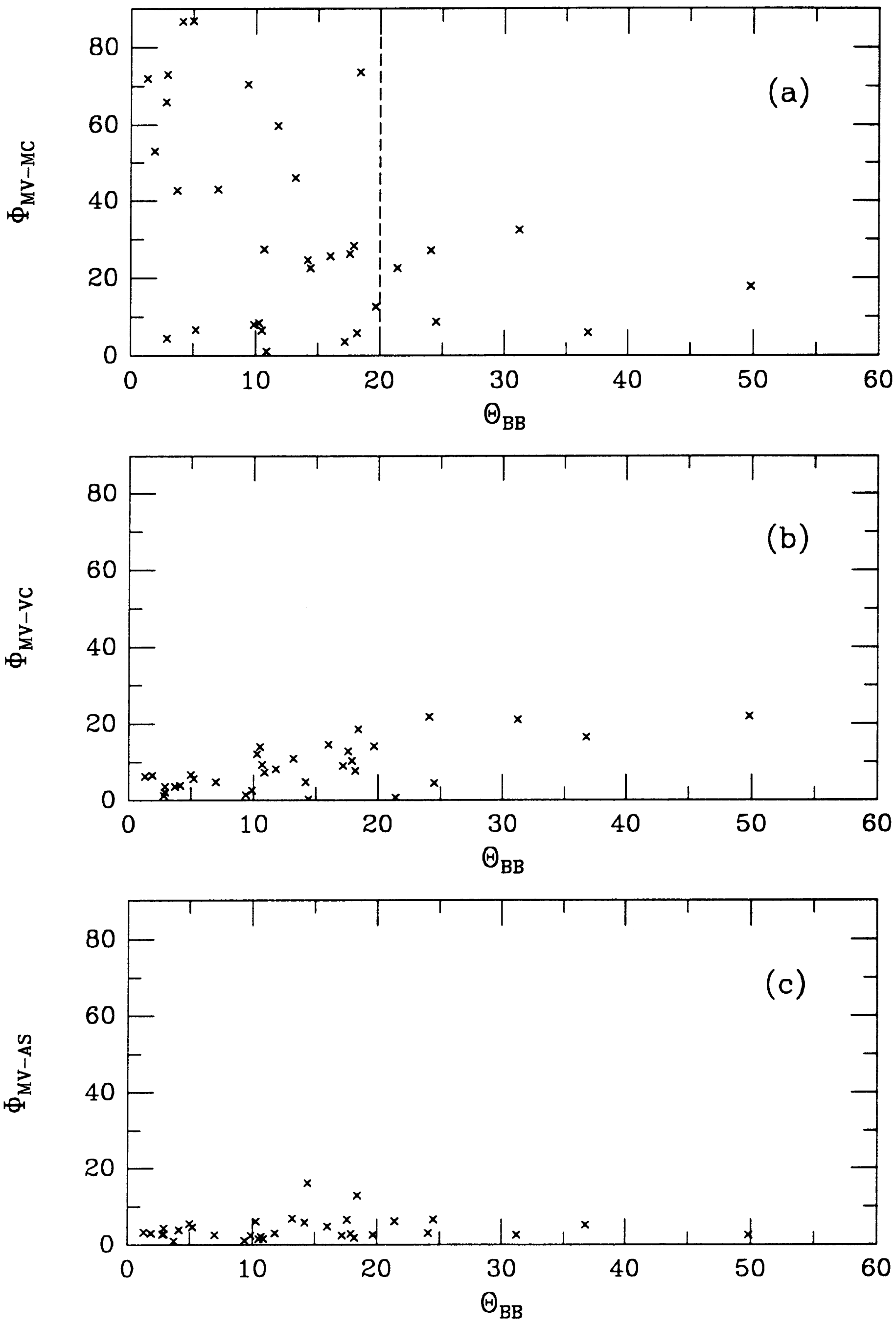

Fig. 2. – Angles between the shock normal directions calculated from the MV technique and the ones calculated from the MC (a), the VC (b) and the AS (c) equations (indicated, respectively, with $\Phi_{\mathrm{MV-MC}}$, $\Phi_{\mathrm{MV-VC}}$ and $\Phi_{\mathrm{MV-AS}}$) *vs.* Θ_{BB}. All the angles are in degrees.

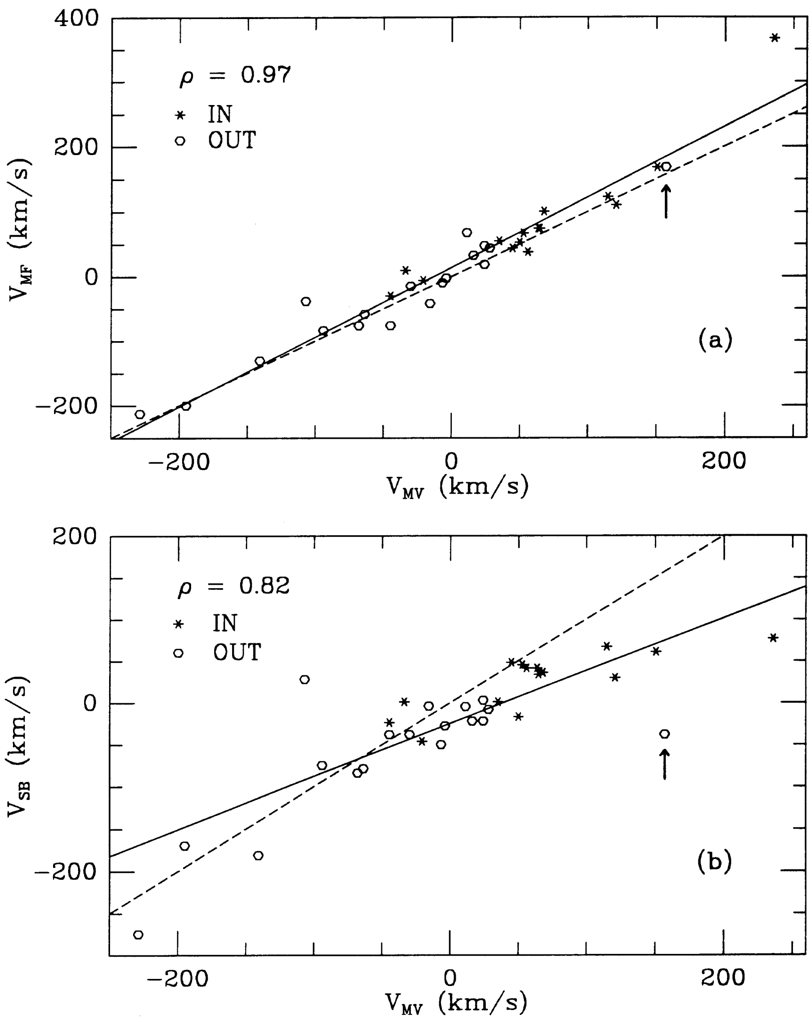

Fig. 3. – (a) Shock normal speeds (V_{MF}) calculated from the mass flux conservation equation (using $\widehat{n}$ from the AS equation) *vs.* the ones calculated by means of the MV technique (V_{MV}); the solid and the dashed line indicate, respectively, the best linear fit ($V_{\mathrm{MF}} = 1.08V_{\mathrm{MV}} + 14$) and the line $V_{\mathrm{MF}} = V_{\mathrm{MV}}$. (b) Shock normal speeds (V_{SB}) calculated from the SB equation (using $\widehat{n}$ from the AS equation) *vs.* V_{MV}; the solid and the dashed line indicate, respectively, the best linear fit ($V_{\mathrm{SB}} = -25 + 0.63V_{\mathrm{MV}}$) and the line $V_{\mathrm{SB}} = V_{\mathrm{MV}}$.

where under usual conditions the bow shock is expected to be a quasi-parallel structure; this feature is extensively discussed by Lepidi *et al.* [8], and is interpreted in terms of a rotation of the upstream magnetic field close to the bow shock.) It is well known that when the variation of the magnetic-field strength across a shock is significant, the R-H equations predict an anticorrelation between Θ_{Bn} and Θ_{BB}; in the present case, the ratio B_2/B_1 typically ranges between 2.0 and 3.0 [8], and the expected anticorrelation between Θ_{Bn} and Θ_{BB} clearly emerges in fig. 1(b) (the correlation coefficient is -0.92).

Figure 2 shows the differences of the angles between the shock normal directions calculated from the MV technique and those calculated from the MC, the VC and the AS equations (indicated, respectively, by $\Phi_{\mathrm{MV-MC}}$, $\Phi_{\mathrm{MV-VC}}$ and $\Phi_{\mathrm{MV-AS}}$) *vs.* Φ_{BB}. The results for the MC method (fig. 2(a)) show a much larger spread of values than in the other cases: it implies that the MV and the MC methods may well provide very different estimates of the shock orientation. Moreover, the experimental observations also show that the greatest differences (up to 90°) and spread of values occur for $\Theta_{BB} < 20°$ (dashed line in fig. 2(a)), *i.e.* when the MC method is expected to provide highly uncertain results. Note that the normals calculated with the VC equation are close to the ones calculated with the MV technique (fig. 2(b)), particularly when the rotation of the magnetic field across the shock is small, *i.e.* at high Θ_{Bn} values (fig. 1(a)), when VC is expected to provide more confident results. In particular, the angle $\Phi_{\mathrm{MV-VC}}$ is always smaller than 25°, and decreases to less than 7° for $\Theta_{BB} < 10°$ (12 cases). Lastly, the angle $\Phi_{\mathrm{MV-AS}}$, which does not show any dependence on Θ_{BB}, is greater than 7° for only 2 out of 33 events; we found that the angular differences $\Phi_{\mathrm{MV-AS}}$ are in most cases of the same order of magnitude as the uncertainties on the shock normal directions calculated by means of the minimum-variance technique.

Figure 3(a) shows a comparison between the bow shock normal speeds calculated from the mass flux conservation (V_{MF}) (in which $\hat{n}$ is determined by the AS method) and the ones calculated by means of the MV technique (V_{MV}). As expected from the results of the previous paragraph, the agreement between the two estimates of the shock velocity is very good: the correlation coefficient is 0.97, and the best linear fit of the experimental points is very close to unity ($V_{\mathrm{MF}} = 1.08\,V_{\mathrm{MV}} + 14$). Note also that the sign of the velocities tends to be positive and negative, respectively, for the inward and the outward crossings. Figure 3(b) shows the results obtained comparing the V_{SB} values independently calculated by means of SB equation (11) and (12) (in which $\hat{n}$ is determined by the AS method) with the shock speeds obtained by the MV technique. The spread of values is somewhat larger, and large differences between the two velocities are observed in some cases (especially for V_{MV} greater than a few tens of km/s). Nevertheless, the correlation between the two speeds is fairly good ($\varrho = 0.82$), and also V_{SB} tends to be positive and negative, respectively, for the inward and the outward crossings. In the only case in which the sign of both V_{MF} and V_{MV} clearly conflicts with the observed direction of the shock crossing (the outward crossing with $V_{\mathrm{MF}} = 168$ km/s and $V_{\mathrm{MV}} = 157$ km/s, arrow in fig. 3(a)), the SB method provides a velocity direction ($V_{\mathrm{SB}} = -38$ km/s, arrow in fig. 3(b)) consistent with an outward crossing. Since SB is the only method for determining the shock speed which does not contain the plasma density, this result could be interpreted as indicating an inaccurate density determination close to the bow shock, possibly due to insufficient time resolution of the plasma data.

The top panel of fig. 4 shows the average values over the whole set of 33 events and the r.m.s. deviations of the normalized parameters defined in eqs. (14)-(17), using the V_{s} and $\hat{n}$ estimates obtained from different methods. For each method we show with different symbols the data points corresponding, from left to right (see left panel in fig. 4), to the normalized deviations from the conservation of the normal magnetic field, the mass flux, the tangential electric field and the tangential momentum. Obviously, the smallest values correspond to the best agreement with theoretical predictions. In the MC, AS and AS + SB case $\{B_{\mathrm{n}}\} = 0$ by definition, and the same argument holds for $\{\varrho U_{\mathrm{n}}\}$ in the MC, VC and AS case; so, we did not show these points in fig. 4. Other horizontal subpanels in fig. 4 show the results obtained considering the events with

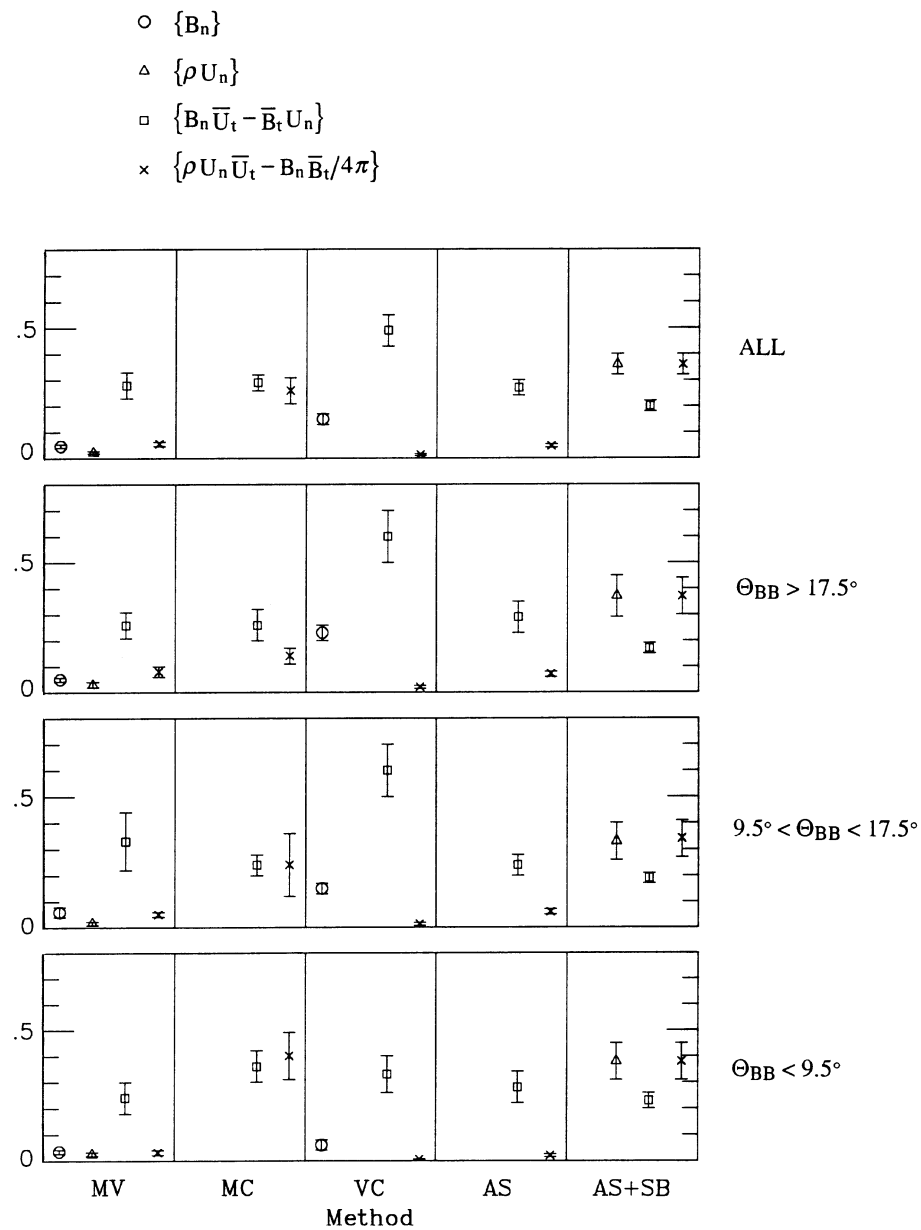

Fig. 4. – Average values and r.m.s. deviations over the 33 events of the normalized parameters defined in eqs. (14)-(17), calculated using the V_s and $\hat{n}$ estimates obtained from different methods: MV technique, combination of mass flux conservation equation with MC, VC and AS and, lastly, combination of SB with AS equation. The data points shown with different symbols for each method correspond, from left to right, to the normal magnetic field, the mass flux, the tangential electric field and the tangential momentum. The four horizontal strips contain, from top to bottom, the results obtained from the whole set of events (33), from the ones with $\Theta_{BB} > 17.5°$ (11), from the ones with $9.5° < \Theta_{BB} < 17.5°$ (11) and, lastly, from the ones with $\Theta_{BB} < 9.5°$ (11).

$\Theta_{BB} > 17.5°$, the ones with $9.5 < \Theta_{BB} < 17.5°$ and the ones with $\Theta_{BB} < 9.5°$ (11 for each). The results in fig. 4 show that in general MV and AS have the best correspondence with the R-H conditions and the experimental results for these methods are not dependent on the shock geometry. In addition, at small rotations of the magnetic field across the shock (*i.e.* at $\Theta_{BB} < 9.5°$, which corresponds to $\Theta_{Bn} > 76°$, fig. 1), VC also provides good results. The reliability of the MC method is not as good as for other methods, as is reasonable to expect from theory for the present quasi-perpendicular events, and it does increase with increasing Θ_{BB}. Lastly, the results of the right panel show that caution should be adopted before using the bow shock speed determined by the SB equation, since, at least for the present case, the R-H equations are more poorly satisfied. Nevertheless, this method (eq. (11)) is based on the electric-field conservation in the shock frame; indeed, the results in fig. 4 show that it provides the best correspondence with the conservation of the tangential electric field which, for other methods, is hardly satisfied.

4. – Conclusions

In the present investigation we analyzed 33 crossings of the Earth's bow shock by IMP8 and compared different methods for the identification of the shock orientation and speed from the single spacecraft measurements. Our results mostly correspond to quasi-perpendicular shock structures: indeed $\Theta_{Bn} > 60°$ for more than 80% of our events. The results of our investigation can be summarized as follows:

a) We confirm the high anticorrelation between Θ_{BB} and Θ_{Bn} which is predicted from the R-H equations when, as in the present case, the variation of the magnetic field strength across the shock is significant.

b) The minimum-variance technique [4, 6] applied to a limited set of the R-H equations and the combination of the Abraham-Shrauner equation [3, 7] with the mass flux conservation equation give very similar shock normal directions (their angular difference is always smaller than 7°) and speeds. In addition, the solutions obtained from both these methods are in good agreement with theoretical predictions of the R-H conservation equations. We suggest that, at least for quasi-perpendicular structures, these are the two methods which provide the best estimates of the shock parameters.

c) The combination of the velocity coplanarity with the mass flux conservation equation is as reliable as the minimum-variance technique only for Θ_{BB} values smaller than approximately 10°. This result agrees with theoretical predictions which suggest that the velocity coplanarity method is expected to give accurate solutions only for perpendicular and nearly perpendicular shocks [7].

d) The magnetic coplanarity method provides better results with increasing Θ_{BB}, consistent with theoretical predictions [1]; nevertheless, its reliability is never as good as the one provided by the minimum variance technique.

e) The results obtained for the bow shock speed by the Smith and Burton equation are in general not as good as those provided by the minimum-variance method and the mass flux conservation equation.

f) In the only case (corresponding to an outward crossing) in which both the minimum-variance method and the mass flux conservation equation provide a shock velocity whose sign clearly conflicts with the observed direction of the shock crossing, the Smith and Burton equation provides a sign for the shock velocity consistent with the expected shock motion. It is very interesting to recall the results obtained by Lottermoser and Luhr [12]: they analyzed 33 BS crossings and found only two cases (both outward) in which the mass flux conservation equation gave a speed greater than 100 km/s that conflicted with the observed direction of the shock crossing, as in the present case. In both cases, the Smith and Burton equation provided a sign for the shock velocity consistent with an inward motion of the shock surface. Since the Smith and Burton equation is the only method which does not use the plasma density, we suggest that this result might be interpreted in terms of the time resolution of the plasma data, which could be too low for an accurate determination of the density behaviour close to the bow shock.

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REFERENCES

[1] COLBURN D. C. and SONETT C. P., *Space Sci. Rev.*, **5** (1966) 439.
[2] LEPPING R. P. and ARGENTIERO P. D., *J. Geophys. Res.*, **76** (1971) 4349.
[3] ABRAHAM-SHRAUNER B., *J. Geophys. Res.*, **77** (1972) 736.
[4] VIÑAS A. F. and SCUDDER J. D., *J. Geophys. Res.*, **91** (1986) 39.
[5] SMITH E. J. and BURTON M. E., *J. Geophys. Res.*, **93** (1988) 2730.
[6] SZABO A., *J. Geophys. Res.*, **99** (1994) 14739.
[7] ABRAHAM-SHRAUNER B. and YUN S. H., *J. Geophys. Res.*, **81** (1976) 2097.
[8] LEPIDI S., VILLANTE U., LAZARUS A. J., SZABO A. and PAULARENA K., *J. Geophys. Res.*, **101** (1996) 11107.
[9] WILKERSON T. D., *Solar Phys.*, **6** (1969) 44.
[10] BELLOMO A. and MAURETIC A., *MIT Plasma Experiment on IMP H and J Earth Orbited Satellite*, MIT Center for Space Research, Technical Report TR-78-2, 1978.
[11] LEPPING R. P., LAZARUS A. J., MORIARTY L. J., MILLIGAN P., KENNON R. S., MCGUIRE R. E. and MISH W. H., *IMP-8 solar wind magnetic field and plasma data in support of Ulysses, Jupiter encounter: 13-31 January 1992*, Laboratory for Extraterrestrial Physics, NASA/Goddard Space Flight Center, Greenbelt, MD, International Report, December 25, 1992.
[12] LOTTERMOSER R. F. and LUHR H., *Geophys. Res. Lett.*, **21** (1994) 2055.