High-order velocity structure functions and anomalous scaling laws in the interplanetary space(*)

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Summary. — The analysis of velocity fluctuations in the interplanetary space plasma reveals the existence of anomalous scaling exponents for the high-order velocity structure function. In this paper we analyze two examples coming from measurements of the fluctuations by both the Helios and the Voyager satellite. We show that the scaling exponents can be fitted by the existing intermittency models and, by introducing the singularity spectra $D(h)$, we show that the turbulence has a multifractal structure.

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The most interesting aspect of fully developed turbulence is the existence of universal scaling behavior of small-scale fluctuations [1]. Turbulent fluctuations on the spatial scale ℓ in the inertial range ($L \gg \ell \gg \eta$, where L is the energy injection scale and η the dissipative scale) reach an equilibrium state characterized by a continuous flux of energy from L up to η which can be viewed in the real space as an energy cascade generated by the breakdown of eddies at different length scales (Richardson's energy cascade), described by a fragmentation process. The above-mentioned universal behavior of fully developed turbulence refers to observations of all moments of the fluctuations of velocity field $\vec{V}(\vec{x})$, which show a power law dependence on ℓ and the scaling exponents are universal. Kolmogorov [2] conjectured the existence of the universal state in which the longitudinal velocity difference $\delta V_\ell = [V(\vec{x}+\vec{\ell}) - V(\vec{x})] \cdot \vec{e}_x$ and the energy transfer rate per unit mass ϵ_ℓ are related through

$$\epsilon_\ell \sim \frac{\delta V_\ell^3}{\ell} \,. \tag{1}$$

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This is also true in Magnetohydrodynamic (MHD) turbulence providing the Alfvén effect of the large-scale magnetic field does not take place [3]. When the magnetic field is strong enough the nonlinear interactions are slowed down due to the Alfvén effect [3]. This is due to the fact that the nonlinear time $\tau_\ell \sim \ell/\delta V_\ell$ is greater than the Alfvén time $\tau_{\rm A} \sim \ell/C_{\rm A}$, being $C_{\rm A} = B_0/(4\pi\rho)^{1/2}$ (B_0 is the magnetic field of the largest scale L and ρ is the constant plasma mass density). Then the energy transfer rate $\delta V_\ell^2/\tau_\ell$ is lowered by a fraction $\tau_{\rm A}/\tau_\ell$, and

$$\epsilon_\ell \sim \frac{\delta V_\ell^4}{C_{\rm A}\ell}\,. \tag{2}$$

As an order of magnitude estimate, from equations (1) and (2) we obtain the following relation between the average energy transfer rate and the q-th power of the averaged velocity differences (say the *structure function* in the oldest language of turbulence theory)

$$\langle \delta V_\ell^q \rangle \sim \ell^{qh^*} \langle \epsilon_\ell^{qh^*} \rangle\,, \tag{3}$$

$h^* = 1/3$ in the fluid-like case (that is when relation (1) is effectively at work) or $h^* = 1/4$ in the Iroshnikov-Kraichnan case (the MHD case).

It can be shown that [4] MHD equations are invariant under the scaling transformations $\ell \to \ell'\lambda^{-1}$ and $\vec{v} \to \vec{v}'\lambda^{-h}$ ($\vec{v}$ represents the velocity field, $\lambda > 0$ and $h > 0$ can assume any positive value), providing the time t, the total (magnetic plus kinetic) pressure P and the magnetic field $\vec{B}$ scale as $t \to t'\lambda^{h-1}$, $P \to P'\lambda^{-2h}$, $\vec{B} \to \vec{B}'\lambda^{-h}$. Two interesting consequences of the invariance of MHD equations under these scaling transformations are: 1) The velocity difference δV_ℓ is singular, that is $(\delta V_\ell/\ell^h) \neq 0$ in the limit $\ell \to 0$. Then we expect to obtain a scaling law where δV_ℓ scales as ℓ^h, where h is arbitrary. 2) From the relations (1) and (2) it can be found $\epsilon_\ell \to \epsilon'_\ell \lambda^{1-h/h^*}$ and the requirement that ϵ_ℓ is invariant under the scaling transformations fixes a single value for h, say $h = h^*$. Obviously in this case the energy transfer rate is statistically independent of the scale length, and its spatial average is equal to the mean dissipated energy $\langle\epsilon_\ell\rangle \sim \epsilon_{\rm D}$. From (3) we then obtain a scaling law where the scaling exponents ξ_q are given by $\langle \delta V_\ell^q \rangle \sim \ell^{\xi_q} = \ell^{qh^*}$. The universality of this scaling law, where the scaling exponents are linear with respect to q, strongly depends on the hypothesis of statistical homogeneity of the energy transfer rate. In presence of intermittency this hypothesis is not verified. Experiments carried out by measuring the velocity structure function in the interplanetary space plasma do not support the prediction that the scaling exponents are linear in q [5]. Rather an anomalous scaling law where ξ_q is a nonlinear function of q is found.

Landau firstly pointed out that, if the hypothesis $\langle\epsilon_\ell\rangle \sim \epsilon_{\rm D}$ is not satisfied (as effectively is the case in the presence of intermittency), the universality of the linear scaling law qh^* cannot exists. Measurements on turbulent fluid flows show that ϵ_ℓ is a statistically fluctuating quantity. Then, introducing the scaling law $\langle\epsilon_\ell^q\rangle \sim \ell^{\tau(q)}$, a universal state can be obtained providing $\xi_q = qh^* + \tau(qh^*)$. Some models have been built up in order to take into account the intermittency (see, for example, the review by Meneveau and Sreenivasan [6]). A recent theory which better describes intermittency in fluid flows is the multifractal theory [7].

Since the MHD equations (like the Navier-Stokes equations) are invariant for *any* value of h, we follow Frisch and Parisi [7] which proposed a scenario for the existence of an entire spectrum of singularities with variable strengths h for the velocity gradients. In this framework the entire field can be covered with interwoven subsets each with a different singularity h, which is a spatially random variable (Hölder exponent), and a variable dimension $D(h)$ (this is the meaning of the word *multifractal*). It can be found that

$$h(q) = \frac{\mathrm{d}\xi_q}{\mathrm{d}q}; \qquad D(h) = qh + 3 - \xi_q. \tag{4}$$

The curve $D(h)$ represents the singularity spectrum, that is the whole set of fractal dimensions related to the various singularities with strengths h, and describes the multifractal structure of turbulence. From the knowledge of ξ_q the convex curve $D(h)$ can be obtained providing each singularity is determined by the local slope of the scaling exponents.

Simple models describing the scaling exponents ξ_q have been built up in the last years. These models derive from some fragmentation processes in the Richardson's framework, and the links between them and the original equations are not clear. The first model in the multifractal framework is the random-β-model by Benzi *et al.* [8]. The model is obtained by assuming that the space-filling factor for the offsprings in the Richardson's cascade is given by a random variable β. The probability of occurrence of a given β is assumed to be a bimodal distribution where the eddies fragmentation process generates either space-filling eddies with probability x or planar sheets with probability $(1-x)$ (providing $0 \le x \le 1$). By using this model it can be found

$$\xi_q = qh^* - \log_2\left[1 - x + x(1/2)^{1-qh^*}\right]. \tag{5}$$

The p-model [9, 7] consists in an eddies fragmentation process for ϵ_ℓ described by a two-scale Cantor set with equal partition intervals. An eddy at the length scale ℓ with a measure ϵ_ℓ breaks down into two eddies at the scale $\ell/2$. Even in this case it is assumed that the distribution of occurrence of the two offsprings is a bimodal distribution, that is the daughter eddies are generated according to the probabilities p and $(1-p)$ (randomly chosen with $0 < p < 1$). From this model

$$\xi_q = 1 - \log_2\left[p^{qh^*} + (1-p)^{qh^*}\right]. \tag{6}$$

The parameters x and p in equations (5) and (6) are free parameters not fixed by the model, and their values can be obtained by a fit on the data. She and Leveque [10] introduced a new model which does not contain arbitrary parameters. Recently the model has been extended to the MHD case [11]. The model is based on two statistical hypothesis on the q-th moments of the energy transfer rate and leads to the relation

$$\xi_q = qh^*(1-2h^*) + C\left[1 - \left(1 - \frac{2h^*}{C}\right)^{qh^*}\right]. \tag{7}$$

C is the codimension of the most intermittent dissipative structures ($C = 2$ in the case of filaments). Grauer *et al.* [11] assume that in the MHD case these structures are planar sheets, thus $C = 1$.

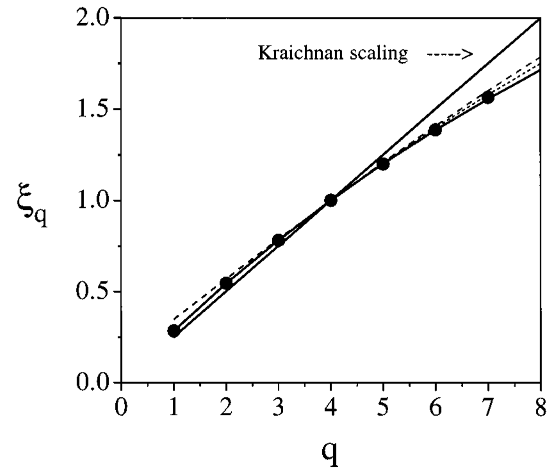

Fig. 1. – We show the scaling exponents ξ_q *vs.* q relative to the data collected by the Helios satellite (symbols). The full line corresponds to the fit on the data obtained with the p-model, the dashed line corresponds to the fit obtained with the random-β-model and the dotted line corresponds to the fit obtained with the model by She and Leveque. The linear curve refers to the classical Iroshnikov-Kraichnan scaling law.

A large amount of information on the large amplitude fluctuations of the velocity field and the magnetic field in the Solar Wind have been extracted from the *in situ* spacecraft measurements. In fact the Solar Wind represents the greatest *laboratory* which allows us to study the characteristic properties of MHD turbulence for a very large range of length scales. To verify the relevance of intermittency models to the Solar Wind turbulence, we use velocity measurements $V(t)$ collected in two data sets. The first set contains measurements of the Helios 2 spacecraft from the 46-th day up to the 48-th day of the mission when it was at about 0.6 AU (Astronomical Units) in a low-speed stream. The second set refers to the measurements of the Voyager satellite at about 8.5 AU. The scaling exponents ξ_q have been calculated using the relation

$$\langle \delta V_\tau^q \rangle = \langle |V(t+\tau) - V(t)|^q \rangle \sim \tau^{\xi_q} \,. \tag{8}$$

The usual Taylor's hypothesis is used to relate fluctuations at the time scale τ to fluctuations at the length scale $\ell = \tau V_{\mathrm{SW}}$ (where V_{SW} is the Solar Wind speed). In fig. 1 we show the scaling exponents of the q-th order velocity structure function obtained from the data by Helios. Superimposed we show the best fit obtained from the random-β-model and the p-model with $h^* = 1/4$, and the parameters $x \simeq 0.16$ and $p \simeq 0.73$. We show also the MHD model by Grauer *et al.* [11], with $C = 1$. As can be seen, the agreement between the various models and the measured scaling exponents is very good. A small difference is visible for the random-β-model. Our results probably imply that the Solar Wind turbulence, in the particular period we have examined, is in an intermittent MHD state which can be described by the Iroshnikov-Kraichnan model corrected by the presence of intermittency. In fig. 2 we show the scaling exponents obtained from the data by the Voyager spacecraft (previously examined by Burlaga [5]). In this case the best fit is obtained with

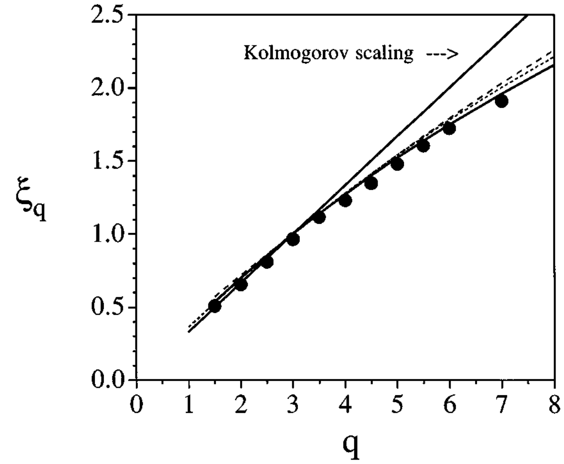

Fig. 2. – We show the scaling exponents ξ_q *vs.* q obtained by Burlaga when analyzing the data from the Voyager satellite (symbols). The full line corresponds to the fit on the data obtained with the p-model, the dashed line corresponds to the fit obtained with the random-β-model and the dotted line corresponds to the fit obtained with the model by She and Leveque. The linear curve refers to the classical Kolmogorov scaling law.

the fluid-like relations $h^* = 1/3$, providing $x \simeq 0.13$ and $p \simeq 0.72$. On the other hand, the model by She and Leveque [10] fits almost well the data. Really in this case the p-model appears to be the better one, mainly for the higher values of q. Our results mean that the Solar Wind turbulence examined at 8.5 AU is in a fluid-like state [5]. From the knowledge of the scaling exponents and from the relations (4), we have calculated the singularity spectra $D(h)$ for both the data sets. These spectra are reported in fig. 3. As can be seen, both the data sets are compatible with the existence of a whole set of scaling exponents h

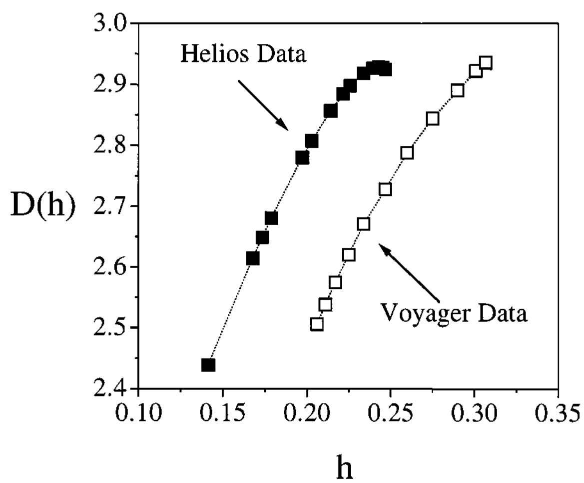

Fig. 3. – We show the singularity spectra $D(h)$ *vs.* h calculated by using the data relative to both the Helios satellite (black squares) and the Voyager satellite (white squares).

each of decreasing dimension $D(h)$, thus confirming the multifractal structure of the Solar Wind turbulence. The MHD case appears to be more singular, because the minimum value of h in the first set is lower. However, this is due to the different scaling relations. The curves $D(h)$ do not extend towards lower values since we used only the first seven structure functions, in order to minimize the errors in the linear fits which give us the scaling exponents [5].

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