IL NUOVO CIMENTO VOL. 21 C, N. 1 Gennaio-Febbraio 1998

Current collection by a highly positive body moving in the ionospheric plasma(*)

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(ricevuto il 24 Marzo 1997; approvato il 29 Luglio 1997)

Summary. — In this paper we derive an interesting feature of the space charge region surrounding a positively charged body moving in a magnetoplasma and, precisely, the fact that, for potentials of the body in excess of a certain value, at least in an inner region close to the body, the electron collection (and the structure of the self-consistent potential) is isotropic. This is used to derive current-voltage characteristics for such a situation. These theoretical characteristics are then convincingly compared with those obtained from the analysis of the data obtained during the recently flown TSS-1R mission.

PACS 94.20 – Physics of the ionosphere.

1. – Introduction

The problem of determining the current-voltage characteristics of a charged probe is an old one in plasma physics dating back to the basic work of I. Langmuir [1]. Although consistent theories have been developed for the case of a stationary probe and in the absence of a magnetic field [2], no such theories exist when the plasma is magnetized and, in addition, the probe is moving. This is, however, the realistic situation to consider for artificial satellites in space and, in particular, this problem has become more important in relation to tethered satellite systems, like the TSS-1 mission flown in 1992 and reflown again in 1996 under the name TSS-1R [3].

In this paper, through an analysis of the momentum equation of the plasma electrons, we first derive a result which appears to be quite interesting in comparison with the recently acquired data on the current-voltage (I-V) characteristics from the TSS-1R mission [4, 5]. What we show is that, for potentials of the charged body in excess of a certain value, the collection of electrons to the body, at least in a certain region closest to the

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body, becomes isotropic.The limiting potential Φ_ϵ is defined, in order of magnitude, by the inequality

$$\frac{e\Phi_\epsilon}{KT_\mathrm{e}} \sim \left(\frac{r_\mathrm{s}}{a_\mathrm{e}}\right)^2 ,$$

where a_e is the electron Larmor radius, T_e the electron temperature and r_s the satellite radius.

More precisely, in the region, surrounding the charged body, where the local self-consistent potential Φ is greater than Φ_ϵ, the variation in the electrostatic potential is isotropic. On the other hand, for $\Phi < \Phi_\epsilon$, the potential field acquires cylindrical symmetry, as channeling of electrons along magnetic lines starts having a dominant role.

The plan of the paper is the following. We demonstrate the above statements in sect. **2**. In sect. **3** we to derive the I-V characteristics, for the case $\Phi_0 > \Phi_\epsilon$ (Φ_0 being the satellite potential). These characteristics are quite similar (but not identical) to those of the completely spherical problem [6].

In sect. **4** we show I-V characteristics which have been obtained from TSS-1R data [4, 5] and we compare those with our theory. The comparison is indeed quite convincing so that one of the main features of the data, *i.e.* the closeness of the experimental results with the prediction of an isotropic collection, is given a possible explanation.

2. – Isotropic collection in the region of high fields

The electron momentum equation is written as

$$nm_\mathrm{e}\frac{\mathrm{d}\mathbf{V}_\mathrm{e}}{\mathrm{d}t} = -ne(\mathbf{E} + \mathbf{V}_\mathrm{e} \times \mathbf{B}) - \nabla p_\mathrm{e} + nf_\mathrm{e} , \tag{1}$$

where

$$f_\mathrm{e} = -\frac{m_\mathrm{e}}{\tau_\mathrm{e}}(\mathbf{V}_\mathrm{e} - \mathbf{V}_\mathrm{i})$$

and τ_e is the electron-ion collision time [7].

It is convenient to write for the electric field $\mathbf{E}$ appearing in eq. (1)

$$\mathbf{E} = E_0\hat{i}_y - \nabla\Phi , \tag{2}$$

where Φ now is the potential induced by the charged body and $\mathbf{E}_0$ is the field connected to the relative motion between the body and the ionosphere (which we assume with velocity V_0 in the x-direction)

$$E_0 = -V_0 B . \tag{3}$$

In this way the momentum equation (1) contains the effect of the plasma velocity flow and, on the other hand, through its collisional term, it is coupled to the ion problem.

It can be shown [8] that velocity flow and/or collisions are possibly important only in the region

$$\tilde{\Phi} < 1 , \tag{4}$$

where we introduce, from now on,

$$\tilde{\Phi} \equiv \frac{e\Phi}{KT_{\mathrm{e}}} \,. \tag{5}$$

In addition, it can also be shown that, in region (4), the so-called pre-sheath region , the effect of velocity flow dominates over collisions if

$$V_0 > \frac{r_{\mathrm{s}}}{\tau_{\mathrm{e}}} \,,$$

a condition which is verified for TSS parameters. In this case the electron problem becomes completely collisionless [9].

For potentials

$$\tilde{\Phi} > 1 \tag{6}$$

the electron momentum equation (1) reduces therefore to

$$nm_{\mathrm{e}}\frac{\mathrm{d}\mathbf{V}_{\mathrm{e}}}{\mathrm{d}t} = -ne(-\nabla\Phi + \mathbf{V}_{\mathrm{e}} \times \mathbf{B}) - \nabla p_{\mathrm{e}} \,. \tag{7}$$

In this regime there is no effect of the velocity flow and no coupling to the ions through collisions. In reality, ions enter the region $\tilde{\Phi} > 1$ and are repelled from the region $\tilde{\Phi} > \tilde{\Phi}_{\mathrm{v}}$ where $\tilde{\Phi}_{\mathrm{v}} \sim 5$ volt is the voltage equivalent of the directed velocity flow (this cannot be described, however, in terms of a fluid model). In addition, in regions of non-neutrality, the electron problem (7) is of course still coupled to the ions through Poisson's equation. In dimensionless terms, the electron momentum equation (7) can be rewritten as

$$\epsilon\big(\tilde{V}_{\mathrm{e}} \cdot \tilde{\nabla}\big)\tilde{V}_{\mathrm{e}} = \epsilon\tilde{\nabla}\psi - (\tilde{V}_{\mathrm{e}} \times \mathbf{h}) \,. \tag{8}$$

To arrive at this, we have introduced the following definitions:

$$\psi = \tilde{\Phi} - \ln \tilde{n}_{\mathrm{e}}$$

with the electron density $\tilde{n}_{\mathrm{e}}$ normalized to the density n_0 of the unperturbed ionosphere

$$\tilde{n}_{\mathrm{e}} = \frac{n_{\mathrm{e}}}{n_0} \,.$$

Furthermore, we have introduced

$$\tilde{\mathbf{V}}_{\mathrm{e}} = \frac{\mathbf{V}_{\mathrm{e}}}{v_{\mathrm{the}}} \,,$$

$$\epsilon = \frac{a_{\mathrm{e}}}{r_{\mathrm{s}}} \,,$$

$$\mathbf{h} = \frac{\mathbf{B}}{B} \,,$$

with v_{the} the electron thermal velocity. As far as space variations are concerned, we have defined dimensionless variables

$$\tilde{x} = \frac{x}{r_{\mathrm{s}}}\,, \qquad \tilde{y} = \frac{y}{r_{\mathrm{s}}}$$

in the perpendicular direction and

$$\tilde{z} = \frac{z}{L_z}$$

in the parallel direction (as the length scales perpendicular and parallel to $\mathbf{B}$ are in principle different). Corresponding to this, $\tilde{\nabla}$ is a dimensionless gradient

$$\tilde{\nabla} = \tilde{\nabla}_{\perp} + \tilde{\nabla}_{\parallel}$$

with

$$\tilde{\nabla}_{\perp} \equiv \frac{\partial}{\partial \tilde{\mathbf{x}}_{\perp}} \qquad \tilde{\nabla}_{\parallel} \equiv \frac{r_{\mathrm{s}}}{L_z}\frac{\partial}{\partial \tilde{z}}\mathbf{h}\,.$$

Referring to TSS, with a typical ionospheric temperature at the Shuttle orbit $T_{\mathrm{e}} \sim 0.2\ \mathrm{eV}$, we find for the parameter ϵ appearing in eq. (8)

$$\epsilon = 4.4 \times 10^{-2}\,.$$

With reference to eq. (8), let us suppose that the inertia term dominates over the term $\tilde{V}_{\mathrm{e}} \times \mathbf{h}$ which describes electron magnetization, and see what this implies. To lowest order then, the problem is not affected by the magnetic field

$$\big(\tilde{V}_{\mathrm{e}} \cdot \tilde{\nabla}\big)\tilde{V}_{\mathrm{e}} = \tilde{\nabla}\psi \tag{9}$$

so that we are forced to suppose isotropy, *i.e.* the same scale length in every direction ($r_{\mathrm{s}} \sim L_z$). From (9) we derive, in order of magnitude, the scaling

$$\tilde{V}_{\mathrm{e}} \sim \psi^{1/2}\,.$$

The ratio between the inertia and the magnetization term in the original equation (8) then becomes

$$\frac{\epsilon\big(\tilde{V}_{\mathrm{e}} \cdot \tilde{\nabla}\big)\tilde{V}_{\mathrm{e}}}{\tilde{V}_{\mathrm{e}} \times \mathbf{h}} \sim \epsilon\psi^{1/2}$$

and our hypothesis of inertia dominating is therefore equivalent to

$$\epsilon^2\psi > 1\,. \tag{10}$$

In conclusion, in this high-field regime, we have a lowest-order electron problem which is isotropic with no effect of the background magnetic field.

Conversely, and still with reference to eq. (8), if we say that electron magnetization dominates over inertia, we obtain, from (8),

$$\epsilon\tilde{\nabla}\psi = (\tilde{V}_\mathrm{e} \times \mathbf{h}) . \tag{11}$$

From this we derive

$$\tilde{\mathbf{V}}_{\mathrm{e}\perp} \sim \epsilon\psi$$

and, *a posteriori*, the condition to neglect inertia is

$$\epsilon\big(\tilde{V}_\mathrm{e} \cdot \tilde{\nabla}\big)\tilde{\mathbf{V}}_{\mathrm{e}\perp} < \epsilon\nabla_\perp\psi$$

or

$$\epsilon\big(\tilde{V}_\mathrm{e} \cdot \tilde{\nabla}\big) < 1 .$$

We must separate here perpendicular and parallel components. For the perpendicular term the condition

$$\epsilon\big(\tilde{\mathbf{V}}_{\mathrm{e}\perp} \cdot \tilde{\nabla}\big) < 1$$

amounts to

$$\epsilon^2\psi < 1 , \tag{12}$$

i.e. the opposite of (10). For the parallel velocity, we must likewise show that

$$\epsilon\tilde{V}_{\mathrm{e}z}\nabla_\parallel < 1 , \tag{13}$$

when $\tilde{V}_{\mathrm{e}z} < \tilde{\mathbf{V}}_{\mathrm{e}\perp}$, this is verified under condition (12). Supposing instead

$$\tilde{V}_{\mathrm{e}z} > \tilde{V}_{\mathrm{e}\perp}$$

we obtain, from the parallel momentum equation,

$$\tilde{V}_{\mathrm{e}z} \sim \psi^{1/2} ,$$

so that (13) leads again to condition (12). Thus the equation of motion in the form (11) requires condition (12). In other words, electron magnetization dominates over inertia in the regime (12) of low and intermediate fields whereas the opposite is true for high fields (see (10)).

An objection to the previous derivation could be raised on the basis of the fact that different components of $\tilde{\mathbf{V}}_{\mathrm{e}\perp}$ appear in the inertia term and the $(\tilde{V}_\mathrm{e} \times \mathbf{h})$ term in eq. (8). Let us then repeat the same reasoning looking at all the components of eq. (8):

$$\epsilon\big(\tilde{V}_\mathrm{e} \cdot \tilde{\nabla}\big)\tilde{V}_{\mathrm{e}x} = \epsilon\frac{\partial\psi}{\partial\tilde{x}} - \tilde{V}_{\mathrm{e}y} ,$$

$$\epsilon\big(\tilde{V}_{\rm e}\cdot\tilde{\nabla}\big)\tilde{V}_{{\rm e}y} = \epsilon\frac{\partial\psi}{\partial\tilde{y}} - \tilde{V}_{{\rm e}x}\,,$$

$$\epsilon\big(\tilde{V}_{\rm e}\cdot\tilde{\nabla}\big)\tilde{V}_{{\rm e}z} = \epsilon\frac{\partial\psi}{\partial\tilde{z}}\,.$$

Combining the first two equations, we obtain

$$\big[1+\epsilon^2\big(\tilde{V}_{\rm e}\cdot\tilde{\nabla}\big)\big(\tilde{V}_{\rm e}\cdot\tilde{\nabla}\big)\big]\tilde{V}_{{\rm e}y} = \epsilon\frac{\partial\psi}{\partial\tilde{x}} + \epsilon^2\big(\tilde{V}_{\rm e}\cdot\tilde{\nabla}\big)\frac{\partial\psi}{\partial\tilde{y}} \tag{14}$$

and a similar equation could be obtained for $\tilde{V}_{{\rm e}x}$.

Now suppose

$$\epsilon\big(\tilde{V}_{\rm e}\cdot\tilde{\nabla}\big) > 1\,. \tag{15}$$

Then, if

$$\frac{\partial}{\partial\tilde{x}} \sim \frac{\partial}{\partial\tilde{y}}\,,$$

eq. (14) reduces to

$$\big(\tilde{V}_{\rm e}\cdot\tilde{\nabla}\big)\tilde{V}_{{\rm e}y} = \frac{\partial\psi}{\partial\tilde{y}}$$

and we have similar equations for the other two components with no effect of the magnetic field on the electron motion. Then, for every component,

$$\tilde{V}_{\rm e} \sim \psi^{1/2}$$

and condition (15) reverts to the high-field condition (10). If, conversely,

$$\epsilon\big(\tilde{V}_{\rm e}\cdot\tilde{\nabla}\big) < 1 \tag{16}$$

and if

$$\frac{\partial}{\partial\tilde{x}} \sim \frac{\partial}{\partial\tilde{y}}\,,$$

eq. (14) reduces to

$$\tilde{V}_{{\rm e}y} = \epsilon\frac{\partial\psi}{\partial\tilde{x}}\,.$$

Similarly,

$$\tilde{V}_{{\rm e}x} = -\epsilon\frac{\partial\psi}{\partial\tilde{y}}$$

and the electrons are clearly magnetized. Again (16) implies

$$\epsilon^2\psi < 1\,.$$

3. – Current collection for high fields

When the satellite potential $\tilde{\Phi}_0$ is such that

$$\tilde{\Phi}_0 > 1/\epsilon^2 \,, \tag{17}$$

we have seen in the previous section that, in the region

$$\frac{1}{\epsilon^2} \leq \tilde{\Phi} \leq \tilde{\Phi}_0 \tag{18}$$

the current collection is isotropic so that $\tilde{\Phi} = \tilde{\Phi}(r)$ where r is the radius in spherical coordinates. On the other hand, for

$$\tilde{\Phi} < \frac{1}{\epsilon^2}$$

the electrons will be channeled by the magnetic field, while the $\tilde{\Phi} < 1$ region will be also influenced by the velocity flow.

It is worth recalling that these conclusions and, in particular, the presence of an inner isotropic region for high fields, appear to be supported, at least qualitatively, by the results of a simulation model [10], where the derived equipotential surfaces, in an inner region close to the body, appear indeed to be circular.

An overall solution for the current collected by the satellite (as a function of $\tilde{\Phi}_0$) requires a self-consistent solution of the overall potential region surrounding the satellite. Here we will do much less than that and solve the self-consistent Poisson equation only for the inner high-field region. As a consequence, the current that we derive will contain a parameter which is essentially the input current at the boundary of this region.

Let us introduce

$$\tilde{\Phi}_\epsilon \equiv \frac{1}{\epsilon^2} \tag{19}$$

and indicate with r_ϵ the distance at which the potential, starting from the value $\tilde{\Phi}_0$ at the surface of the sphere, drops to $\tilde{\Phi}_\epsilon$, *i.e.*

$$\tilde{\Phi}(r = r_\epsilon) \equiv \tilde{\Phi}_\epsilon \,. \tag{20}$$

The current to be calculated is that from a spherical diode of outer radius r_ϵ and inner radius r_s. The emitter is at r_ϵ where the potential is $\tilde{\Phi}_\epsilon$ and the collector is at r_s where the potential is $\tilde{\Phi}_0$.

Poisson's equation is written as

$$\frac{\mathrm{d}^2\tilde{\Phi}}{\mathrm{d}\tilde{r}^2} + \frac{2}{\tilde{r}}\frac{\mathrm{d}\tilde{\Phi}}{\mathrm{d}\tilde{r}} = A^2\tilde{n}_\mathrm{e} \tag{21}$$

with

$$A = \frac{r_\mathrm{s}}{\lambda_\mathrm{d}} \,, \tag{22}$$

λ_d being the electron Debye length.

From current conservation

$$\tilde{r}^2 \tilde{n}_e \tilde{V}_r = \tilde{I}\,, \tag{23}$$

where

$$\tilde{I} = \frac{I}{I_0} = \frac{I}{n_0 e v_{the}}$$

is the current normalized to the electron thermal current. On the other hand, indicating with $\tilde{V}_{r\epsilon}$ the value of the radial velocity at r_ϵ, we can write

$$\tilde{V}_r^2 = 2(\tilde{\Phi} - \tilde{\Phi}_\epsilon) + \tilde{V}_{r\epsilon}^2\,.$$

However, we may assume

$$V_{r\epsilon} \sim V_{z\epsilon}^*\,,$$

where $V_{z\epsilon} \sim (2\tilde{\Phi}_\epsilon)^{1/2}$ is the value of the parallel velocity reached at the end of the region $1 < \tilde{\Phi} < \tilde{\Phi}_\epsilon$ [8]. With that

$$\tilde{V}_r^2 \sim 2\tilde{\Phi}$$

and eq. (21) can be written, using (23), as

$$\frac{d^2\tilde{\Phi}}{d\tilde{r}^2} + \frac{2}{\tilde{r}}\frac{d\tilde{\Phi}}{d\tilde{r}} = \frac{A^2 \tilde{I}}{(2\tilde{\Phi})^{1/2}}\,. \tag{24}$$

Starting from here we can parallel the treatment of the spherical diode problem [5]. We introduce

$$\gamma = \ln \hat{r} \tag{25}$$

with

$$\hat{r} = \frac{r}{r_\epsilon} \tag{26}$$

and, in place of $\tilde{\Phi}$, the new unknown $\alpha^2(\gamma)$ defined through

$$\tilde{I} = \frac{4}{9{A_\epsilon}^2}\frac{\tilde{\Phi}^{3/2}}{\alpha^2(\gamma)} \tag{27}$$

with

$$A_\epsilon = \frac{r_\epsilon}{r_s} A\,. \tag{28}$$

Then the equation derived for α is

$$3\alpha\left[\frac{d\alpha}{d\gamma} + \frac{d^2\alpha}{d\gamma^2}\right] + \left(\frac{d\alpha}{d\gamma}\right)^2 = 1\,. \tag{29}$$

This is exactly the Langmuir equation, but the boundary conditions for α^2 are different and, precisely,

$$\alpha^2(\gamma = 0) = \frac{4}{9{A_\epsilon}^2 \tilde{I}} \tilde{\Phi}_\epsilon^{3/2} , \qquad \alpha^2(\gamma_s) = \frac{4}{9{A_\epsilon}^2 \tilde{I}} \tilde{\Phi}_0^{3/2} \tag{30}$$

with

$$\gamma_s = \ln\left(\frac{r_s}{r_\epsilon}\right) . \tag{31}$$

The Langmuir condition for $\alpha^2(\gamma = 0)$ was $\alpha^2(\gamma = 0) = 0$ and is obtained from the first of conditions (30) for $\tilde{\Phi}_\epsilon = 0$ which in fact corresponds to the limit of zero magnetic field as it should.

We can write for the solution of eq. (29)

$$\alpha^2(\gamma) = \alpha^2(\gamma = 0) + {\alpha_L}^2(\gamma) , \tag{32}$$

where $\alpha^2(\gamma = 0)$ is given by the first of equations (30) and ${\alpha_L}^2(\gamma)$ is the Langmuir solution.

Let us now see how the current I is actually derived. Writing (27) at $r = r_s$ (*i.e.* $\gamma = \gamma_s$), we obtain

$$\tilde{I} = \frac{4}{9{A_\epsilon}^2} \frac{\tilde{\Phi}_0^{3/2}}{\alpha^2(\gamma_s)} . \tag{33}$$

Using (32) and the first of equations (30), this can be rewritten as

$$\frac{4}{9{A_\epsilon}^2} \tilde{\Phi}_\epsilon^{3/2} + \tilde{I} \left(\frac{r_\epsilon}{r_s}\right)^2 {\alpha_L}^2(\gamma_s) = \frac{4}{9} \tilde{\Phi}^{*\,3/2} , \tag{34}$$

where

$$\tilde{\Phi}^* = \tilde{\Phi}_0 \left(\frac{\lambda_d}{r_s}\right)^{4/3} \tag{35}$$

is recognized as the dimensionless potential which appears in any spherically symmetric theory. The classical spherical case is then represented by

$$\tilde{I} \left(\frac{r_\epsilon}{r_s}\right)^2 {\alpha_L}^2(\gamma_s) = \frac{4}{9} \tilde{\Phi}^{*\,3/2} ,$$

whereas it is the first term in (35) which represents the deviation from the spherical case due to $B \neq 0$. Notice that, comparing this term with the rigth-hand side of (35), we obtain that the magnetic-field term is actually negligible for potentials $\tilde{\Phi}_0$ such that

$$\frac{\tilde{\Phi}_0}{\tilde{\Phi}_\epsilon} \gg 1 , \tag{36}$$

a condition which actually coincides with the regime of high fields we are referring to (eq. (17)).

Let us now introduce into eq. (34)

$$\tilde{I} = I_\epsilon \left(\frac{r_\epsilon}{r_{\rm s}}\right)^2 \tag{37}$$

which expresses the fact that we have a spherical collection from a radius r_ϵ inwards. With that we obtain

$$\frac{4}{9{A_\epsilon}^2}\tilde{\Phi}_\epsilon^{3/2} + \tilde{I}_\epsilon \left(\frac{r_\epsilon}{r_{\rm s}}\right)^4 {\alpha_{\rm L}}^2(\gamma_{\rm s}) = \frac{4}{9}\tilde{\Phi}^{*3/2} \tag{38}$$

which is now an equation for $r_\epsilon/r_{\rm s}$ only.

According to the definition (37), when

$$r_\epsilon = r_{\rm s}\,,$$

i.e. zero thickness of the isotropic region (or, equivalently, $\tilde{\Phi}_0 = \tilde{\Phi}_\epsilon$), the current $\tilde{I}$ has the value I_ϵ. Thus I_ϵ is the input current that we get from the region

$$\tilde{\Phi} < \frac{1}{\epsilon^2}\,,$$

where magnetic-field channeling takes place. Owing to the fact that we are not treating here this part of the problem, I_ϵ has to be considered as a parameter in the following calculations.

Once $r_\epsilon/r_{\rm s}$ is derived as a function of $\tilde{\Phi}_0$ from eq. (38), we can use eq. (37) to obtain the I-V characteristics of the charged body.

4. – Results and comparison with TSS-1R data

Let us first recall some distinctive features of TSS-1R data. Figure 1 (a)-(b) represents I-V characteristics derived [4, 5] from data of various TSS-1R experiments. In fig. 1 (a) the current, normalized to the electron thermal current, is plotted against the dimensionless potential $\tilde{\Phi}$. In fig.1 (b), the same normalized current is plotted against the potential $\tilde{\Phi}^*$, defined by eq. (35) and occurring in any theory of isotropic collection.We see clearly a great scatter of the experimental points in fig. 1 (a), while the points appear to be quite close to a single curve in fig. 1 (b). The data indicate therefore that the normalized current depends essentially on $\tilde{\Phi}^*$ and a model, like that of Parker and Murphy [11] (where $\tilde{I}$ would depend solely on $\tilde{\Phi}$) seems to be excluded from the data.

The model that we have discussed and calculated in the previous sections, where the particle collection becomes isotropic (with respect to the magnetic-field direction) at least in an inner region close to the body, gives, on the other hand, also (through the solution of eq. (38)) a normalized current depending essentially on the parameter $\tilde{\Phi}^*$.

Figure 2 reproduces again the I-V characteristics from the data (as in fig. 1 (b) and, in addition, shows two curves obtained from the solution of eq. (38) for two different values of the parameter I_ϵ. In one case (the lower curve), we have used $I_\epsilon = 4\pi$ which corresponds to the value that would be obtained from the theory of Parker and Murphy [11]

$$\tilde{I} = 2\pi \left[1 + \left(\frac{\tilde{\Phi}_0}{\tilde{\Phi}_\epsilon}\right)^{1/2}\right]$$

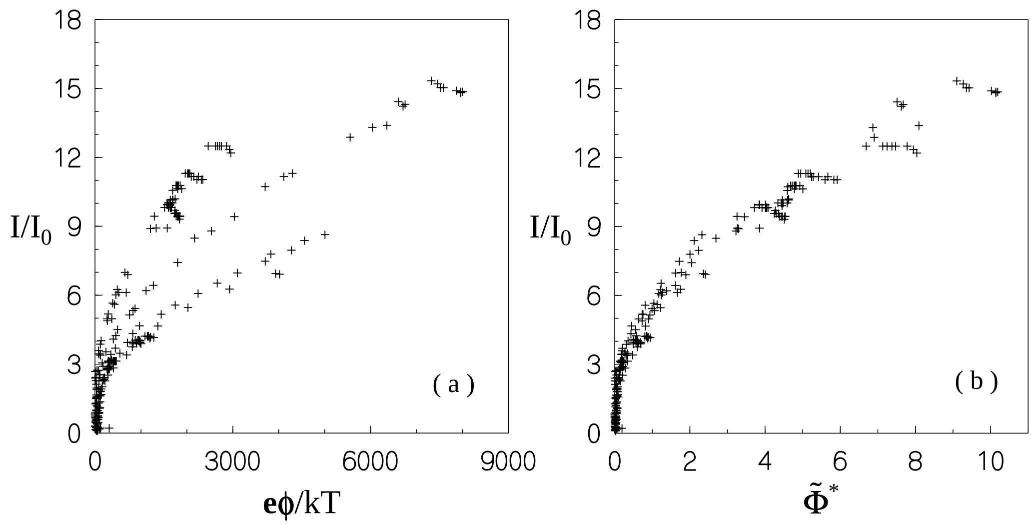

Fig. 1. – I-V characteristics derived from TSS-1R data [4]: (a) normalized current is plotted *vs.* $e\Phi/KT_e$; (b) normalized current is plotted *vs.* the parameter $\tilde{\Phi}^*$ defined in eq. (35).

for $\tilde{\Phi}_0 = \tilde{\Phi}_\epsilon$. The other value used is $I_\epsilon = 12\pi$ with which we obtain theoretical points quite close to the experimental data.

As we stated already, only a detailed treatment of the regime $\tilde{\Phi}_0 < \tilde{\Phi}_\epsilon$ can give us the value of I_ϵ. While such a treatment, including magnetic field and velocity flow, is exceedingly difficult, we have reasons to suppose that values of I_ϵ above the PM value are to be expected. More precisely, we expect an increase of the current collected at the border of the pre-sheath region above the thermal value (corresponding to $I_\epsilon = 2\pi$),

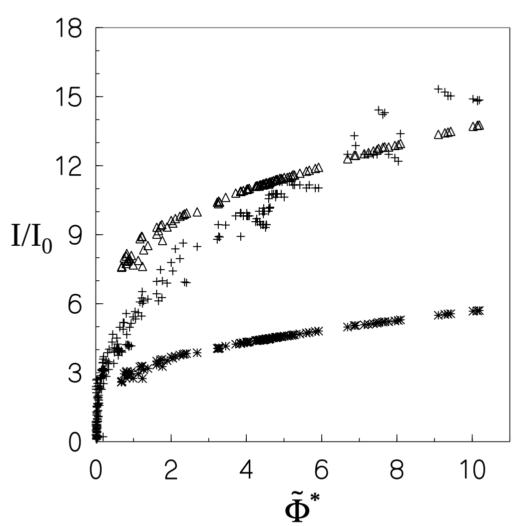

Fig. 2. – Normalized current *vs.* $\tilde{\Phi}^*$: + represents experimental points (same as in fig. 1 (b)). The other symbols are all points calculated from eq. (38) and, precisely : $*$ for $\tilde{I}_\epsilon = 4\pi$, $\triangle$ for $\tilde{I}_\epsilon = 12\pi$.

because the plasma velocity flow results in an extension of the pre-sheath region [12] up the the potential $\tilde{\Phi}_v \sim 5$ volt which is the equivalent of the kinetic energy of the directed flow.

The main point of our comparison between the experimental data and our theory, is however not the adjustment of the parameter I_ϵ, but the fact that we have a theory predicting isotropic collection in an inner region of the sheath close to the charged body (for $\tilde{\Phi} > \tilde{\Phi}_\epsilon$), giving, as a consequence of that, theoretical *I-V* characteristics where the normalized current $\tilde{I}$ depends essentially only on $\tilde{\Phi}^*$, a feature which is quite evident also in the data.

A further point, in the TSS measurements, in support of our model theory, is given by the measurements of the current collected by the Langmuir probe of the experiment RETE, while the TSS satellite was spinning, showing a clear ram-wake effect on the current values but no effects whatsoever related to the magnetic-field direction, *i.e.* no evidence whatsoever of channeling of electrons due to the magnetic field. The important point is that the RETE Langmuir probe was sitting at a distance of ~ 20 cm from the spacecraft surface. This has to be compared with the value of r_ϵ derived from the solution of eq. (39). For the range of densities and potentials of the Langmuir probe measurements we are referring to [4], r_ϵ varies in the range 98–104 cm. This corresponds to a distance from the satellite surface between 18 and 24 cm so that the RETE Langmuir probe is indeed inside the region $\tilde{\Phi}_0 < \tilde{\Phi} < \tilde{\Phi}_\epsilon$. The prediction of our theory that the electron collection starts becoming isotropic in this region, agrees therefore with the absence of any indication of magnetic channeling in the Langmuir probe data as the LP was sitting precisely in that range of potential values.

Finally, we should stress that, when we say that collection is isotropic in an inner sheath region, we want to refer mainly to the absence of magnetic-field channeling. On the other hand, we know that, for the real TSS satellite, as shown by the Langmuir probe measurements, there is a clear ram-wake asymmetry in plasma density (which is simply ignored in our model).

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