New calculation of atmospheric neutrino fluxes (*)

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Summary. — We have performed a one-dimensional Monte Carlo calculation of atmospheric neutrino fluxes in the energy range 0.05 GeV–20 GeV including muon polarization effects. It is shown that the calculated ν_μ/ν_e ratio does not appear sufficient to explain the Kamiokande data from sub-GeV to multi-GeV energy region. It is suggested that neutrino oscillations would provide a solution to the anomalous ν_μ/ν_e ratio.

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1. – Introduction

In this paper atmospheric neutrino fluxes are calculated over the energy range from 0.05 GeV to 20 GeV, covering the observational range of underground neutrino detectors.

A detailed calculation of atmospheric neutrino fluxes is very important to evaluate the ratio of muon-neutrino ($\nu_\mu + \bar{\nu}_\mu$) to electron-neutrino ($\nu_e + \bar{\nu}_e$) fluxes and vice versa, denoted hereafter by ν_μ/ν_e or ν_e/ν_μ ratio, observed by many experiments.

It has been shown [1] that the ν_e/ν_μ ratio which removes uncertainties in the overall normalization of the neutrino flux calculation is a good estimator for making a comparison between theory and experiment.

The ν_μ/ν_e ratio, measured by Kamiokande detector, has been found to be smaller, nearby $(60 \pm 7)\%$, than expected [2] at low energies ($\langle E_\nu \rangle \sim 1$ GeV).

The deficit of muon neutrinos compared with prediction which eventually leads to a smaller ν_μ/ν_e ratio, has also been observed in the IMB detector [3-6].

On the contrary, no disagreement with the expected ν_μ/ν_e ratio, averaged over zenith angle and energy, at the same energy region is found by the NUSEX [7, 8] and

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Frejus [9, 10] detectors. Barr *et al.* [11, 12] have found that the increase of the ν_e/ν_μ ratio, including the effect due to muon polarization, does not appear to be sufficient to remove the discrepancy with the Kamiokande data.

On the other hand, Volkova [13] showed that all available experimental atmospheric ν_e/ν_μ ratios are in a good agreement with or do not contradict theoretical expectations if one takes muon polarization effect into account.

Recently, atmospheric ν_μ/ν_e ratio in the multi-GeV region ($\langle E_\nu \rangle \sim 3$–$6$ GeV), depending on zenith angle, unlike the isotropic dependence in the sub-GeV data, has been reported [14].

The observed ν_μ/ν_e ratio showed also a significant deviation from the expected value as well as that at low energies.

Recently, Midorikawa *et al.* [15] have suggested the possibility that these deviations are evidence for ν oscillations.

The question arises as to which effect (muon polarization, neutrino oscillations) is more effective to remove the discrepancy of ν_μ/ν_e ratio between theory and experiment.

In view of this circumstance, we think that it is indispensable to evaluate atmospheric neutrino fluxes at energies around $E_\nu \sim 3.0$ GeV as precisely as possible taking into account muon polarization.

It is, firstly, the aim of this work, to calculate the atmospheric neutrino fluxes as a function of energy and zenith angle taking muon polarization into account, which has been neglected in our earlier paper [16].

Secondly, the effect of neutrino oscillation competing with that of muon polarization is considered using a simple formula proposed by Learned *et al.* [17].

Finally, we compare the ν_μ/ν_e ratio obtained with the Kamiokande multi-GeV data, where the geomagnetic effect may be neglected and we discuss which effect greatly contributes to the ratio to remove the discrepancy.

2. – Muon polarization

2·1. *No muon polarization.* – On the process $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$, by the energy ratio $y(= E_\nu/E_\mu)$ which is the energy of neutrino E_ν to that of muon E_μ, the distribution function for the muon-neutrino in the limit $\beta_\mu \to 1$ is given by [18]

$$f(y) = \frac{5}{3} - 3y^2 + \frac{4}{3}y^3 \tag{1}$$

and that for the anti-electron-neutrino is given by [18]

$$f(y) = 2(1 - 3y^2 + 2y^3), \tag{2}$$

The average value of y for the muon-neutrino becomes 0.32 and that for the anti-electron-neutrino is 0.27.

In our earlier paper [16], the respective distribution function $f(y)$ for ν_μ and $\bar{\nu}_e$, has been used.

2·2. *Muon polarization.* – As a result of parity non-conservation, muons from pion decay $\pi^\pm \to \mu^\pm + \nu_\mu(\bar{\nu}_\mu)$ are produced fully polarized, left-handed (helicity -1) for π^+ decay and right-handed (helicity $+1$) for π^- decay in the rest frame of the parent pion.

In the muon rest frame, the angular distribution of the neutrinos from the decay

$$\mu^{\pm} \rightarrow e^{\pm} + \nu_e(\overline{\nu}_e) + \overline{\nu}_\mu(\nu_\mu)$$

depends upon the direction of the muon spin.

For the decay $\mu^- \rightarrow e^- + \nu_\mu + \overline{\nu}_e$, the distributions of muon-neutrino and anti-electron-neutrino with energy between ε^* and $\varepsilon^* + d\varepsilon^*$ at a zenith angle between θ and $\theta + d\theta$ in the muon rest frame are given by [19]

$$d\Omega(\varepsilon^*, \theta^*)\, d\varepsilon^*\, d\cos\theta^* \propto \varepsilon^{*2} d\varepsilon^*\, d\cos\theta^* \left[3 - 4\frac{\varepsilon^*}{m_\mu}\right](1 - \alpha\cos\theta^*), \tag{3}$$

with $\alpha = (4\varepsilon^* - m_\mu)/(3m_\mu - 4\varepsilon^*)$, and

$$d\Omega(\varepsilon^*, \theta^*)\, d\varepsilon^*\, d\cos\theta^* \propto \varepsilon^{*2} d\varepsilon^*\, d\cos\theta^* \left[1 - 2\frac{\varepsilon^*}{m_\mu}\right](1 + \cos\theta^*), \tag{4}$$

respectively.

Here θ^* is the angle between the spin of the muon and the direction of the neutrino of interest, and ε^* is the energy of the neutrino.

In the laboratory frame, however, the polarization is not complete when the energy of the muon is not so high compared with the muon mass in the rest frame of the pion.

In this case, muons produced in the backward direction in the rest frame of pions could be Lorentz-transformed into those moving forward.

As the energy spectrum of cosmic-ray mesons has a steep slope, the muons of a given energy are preferentially forward, both from forward pions of low energies and from backward pions of higher energies.

Transforming along the momentum direction of the parent pions, the longitudinal polarization along the momentum direction of the $\mu^{\pm}$ with energy E_μ and momentum P_μ in the laboratory frame has been given by [20]

$$P(E_\pi) = \mp\left(\frac{E_\mu E_\mu^*}{P_\mu P_\mu^*} - \gamma\frac{m_\mu^2}{P_\mu P_\mu^*}\right). \tag{5}$$

Here $\gamma = E_\pi/m_\pi$, where $E_\pi(m_\pi)$ is the energy (mass) of the parent pion, $E_\mu(P_\mu)$ is the energy (momentum) of the muon in the laboratory frame, m_μ is the mass of the muon, and $E_\mu^*(P_\mu^*)$ is the energy (momentum) of the muon in the rest of mass frame of the parent pion.

Substituting the mass of the pion and the muon, $E_\mu^*(P_\mu^*)$ becomes

$$E_\mu^* = \frac{m_\pi^2 + m_\mu^2}{2m_\pi} = 109.8\ \text{MeV},$$

$$P_\mu^* = \frac{m_\pi^2 - m_\mu^2}{2m_\pi} = 29.8\ \text{MeV}.$$

Denoting $\chi^* = 2\varepsilon^*/m_\mu$, α is written as

$$\alpha = \frac{4\varepsilon^* - m_\mu}{3m_\mu - 4\varepsilon^*} = \frac{2\chi^* - 1}{3 - 2\chi^*}. \tag{6}$$

Substituting the above α into eqs. (3) and (4), and considering $P(E_\pi)$, for the decay $\mu^\pm \to e^\pm + \nu_e(\bar{\nu}_e) + \bar{\nu}_\mu(\nu_\mu)$, eqs. (4) and (5) are rewritten as: for the muon-neutrino

$$\mathrm{d}\Omega(\chi^*, \theta^*)\,\mathrm{d}\chi^*\,\mathrm{d}\cos\theta^* \propto \chi^{*2}\,\mathrm{d}\chi^*\,\mathrm{d}\cos\theta^*[(3-2\chi^*) \pm (2\chi^*-1)\,P(E_\pi)\cos\theta^*] \tag{7}$$

and for the electron-neutrino

$$\mathrm{d}\Omega(\chi^*, \theta^*)\,\mathrm{d}\chi^*\,\mathrm{d}\cos\theta^* \propto \chi^{*2}\,\mathrm{d}\chi^*\,\mathrm{d}\cos\theta^*(1-\chi^*)(1 \mp P(E_\pi)\cos\theta^*)\,, \tag{8}$$

where the upper and lower signs refer to μ^+ and μ^-, respectively.

In the present paper, the $K^\pm \to \mu^\pm + \nu_\mu(\bar{\nu}_\mu)$ decay has also been considered in the same way.

If the differential energy spectra of primary cosmic rays are well described by an inverse power law in energy as

$$I(E) \propto E^{-\delta},$$

the energy spectra of the parent pions in the atmosphere may be expressed by

$$I(E_\pi) \propto E_\pi^{-\delta} \propto \gamma^{-\delta} \tag{9}$$

in the high-energy limit [21], where $m_\pi \gamma$ is the total energy of a pion of mass m_π.

The allowed energy E_π of pions for a given energy of muons E_μ in the laboratory frame ranges between the limits

$$E_\pi^\pm = \frac{m_\pi}{m_\mu}\,\frac{E_\mu E_\mu^* \pm P_\mu P_\mu^*}{m_\mu}\,. \tag{10}$$

Then, the degree of polarization could be obtained by averaging eq. (5) over the pion spectrum as

$$\bar{\xi} = \frac{\displaystyle\int_{\gamma^-}^{\gamma^+} \xi I(\gamma)\,\frac{\mathrm{d}\gamma}{\sqrt{1-\gamma^2}}}{\displaystyle\int_{\gamma^-}^{\gamma^+} I(\gamma)\,\frac{\mathrm{d}\gamma}{\sqrt{1-\gamma^2}}}\,, \tag{11}$$

where $\gamma^\pm = E_\pi^\pm/m_\pi$ and a factor $1/\sqrt{\gamma^2-1}$ is included to take into account the decay probability of the parent pions.

If the pion spectrum takes a power law as eq. (9) and $\gamma^2 \gg 1$, eq. (11) can be approximately evaluated as

$$\bar{\xi} \simeq 1 + \left(\frac{E_\mu E_\mu^*}{P_\mu P_\mu^*} - 1\right)\left[1 - \frac{\delta-1}{\delta-2}\,\frac{1-(\gamma^-/\gamma^+)^{\delta-2}}{1-(\gamma^-/\gamma^+)^{\delta-1}}\right]. \tag{12}$$

In the high-energy limit $\beta \simeq 1$, the γ^+ and γ^- become 12.5 (44.2) and 7.2 (2.0), respectively, for the decay of the pion (kaon), if only the two-body decay is assumed.

The polarizations $P(E_\pi)$'s in eqs. (7) and (8) have been replaced with the averaged polarization $\bar{\xi}$ in eq. (12) in the present calculation.

When polarization is included, the energy ratio y, defined in subsect. **2**·1, can be estimated from eqs. (7) and (8), where the above-mentioned $\overline{\xi}$ is used instead of $P(E_\pi)$.

The average values of y for muon-neutrino and electron-neutrino, coming from $\pi \to \mu \to e + \nu_\mu + \nu_e (K \to \mu \to e + \nu_\mu + \nu_e)$ decay chain are 0.32 (0.30) and 0.33 (0.39), respectively.

It is interesting to compare these values with those estimaed from eqs. (1) and (2), corresponding to the case where muon polarization is neglected.

From the comparison of the magnitude of y in pion and kaon decay, it is suggested that electron-neutrinos would receive, in particular for kaon decay, much more energy than muon-neutrinos as compared with the unpolarized case.

This, in turn, decreases the ν_μ / ν_e ratio at a fixed neutrino energy since the energy spectrum of cosmic-ray muons decreases with increasing energy.

3. – Method of calculation

3·1. *Primary cosmic-ray spectrum.* – The differential energy spectrum per nucleon of the whole primary is assumed to have the form

$$I(E_0) = 7.16(E_0 + 1.63)^{-3.12} \quad \text{for } E_0 < 10 \text{ GeV} \tag{13}$$

and

$$I(E_0) = 1.87 \; E_0^{-2.7} \quad \text{for } E_0 \geqslant 10 \text{ GeV} \tag{14}$$

in units of $(\text{cm}^2 \text{ s sr GeV/nucleon})^{-1}$.

3·2. *Collision mean free path.* – If we assume that the collision increases with energy as $E^{0.05}$ when E is measured in units of TeV, the collision mean free path λ_i of the particle i is expressed by

$$\lambda_i = \lambda_{0i}(E/\text{TeV})^{-0.05}, \tag{15}$$

where i and λ_{0i} are as follows:

i	$\lambda_{0i}(\text{g cm}^{-2})$
nucleon	80
pion	120
kaon	150

3·3. *Meson production.* – The average number of the whole mesons produced in the nucleon-nucleon interactions, $\langle n_s \rangle_\text{T}$, has been estimated such that

$$\langle n_s \rangle_\text{T} = 3/2 \langle n_s \rangle = 1.32 + 0.66 \ln s + 0.177 \ln^2 s \,, \tag{16}$$

where s is the center-of-mass energy squared in GeV and $\langle n_s \rangle$ is the average number of charged mesons, which is taken from Tasaka *et al.* [22].

Here it is assumed that the energy distribution of the produced particles in the fragmentation region obeys the scaling law and the multiplicity distribution of the secondary particles behaves according to Koba-Nielson-Olesen scaling.

The $\langle n_s \rangle_T$ produced in the pion (kaon)-nucleon interactions is assumed to behave according to eq. (16).

3·4. *K/π ratio.* – The ratio of multiplicity for the kaon to that for the pion produced in the nucleon-nucleon interactions has been assumed such that

$$\frac{K(K^{\pm},\, K^0,\, \overline{K}^0)}{\pi(\pi^{\pm},\, \pi^0)} \simeq \frac{2(Z_{pK^+} + Z_{pK^-})}{1.5(Z_{p\pi^+} + Z_{p\pi^-})} \simeq 0.175\,, \tag{17}$$

which is independent of energy.

The fractional energy moment Z_{ac} (for the inclusive process $ab \to cx$) is taken from ref. [23].

4. – Calculations and results

In the present paper, the ionization losses for pion, muon, kaon have been taken into account. For the relation between density and altitude in the atmosphere, US-standard atmosphere is used.

The muon energy produced from the decay of pion (kaon) in the laboratory frame is calculated from the relation

$$E_\mu = \gamma_{\pi(K)}(E_\mu^* + P_\mu^* \cos\theta^*)\,, \tag{18}$$

where $\gamma_\pi = 7.165 E_\pi$ and $\gamma_K = 2.026 E_K$.

For the three-body decay of the muon, the similar relation is taken as

$$E_\nu = \gamma_\mu(E_\nu^* + P_\nu^* \cos\theta^*)\,, \tag{19}$$

where γ_π is the Lorentz factor of the muon in the laboratory frame.

The various sources of the neutrino which have been adopted in this paper are assumed to be produced by the following decay channels:

$$\pi^{\pm} \to \mu^{\pm}\, \nu_\mu(\overline{\nu}_\mu)\,,$$
$$K^{\pm} \to \mu^{\pm}\, \nu_\mu(\overline{\nu}_\mu)\,,$$
$$\mu^{\pm} \to e^{\pm}\, \nu_e(\overline{\nu}_e)\, \overline{\nu}_\mu(\nu_\mu)\,,$$
$$K^{\pm} \to \pi^{\pm}\, \pi^0 \to \mu^{\pm}\, \nu_\mu(\overline{\nu}_\mu)\, \pi^0\,,$$
$$K^{\pm} \to e^{\pm}\, \nu_e(\overline{\nu}_e)\, \pi^0\,,$$
$$K_{S^0} \to \pi^+\, \pi^- \to \mu^{\pm}\, \nu_\mu(\overline{\nu}_\mu)\,,$$
$$K_{L^0} \to \pi^{\pm}\, e^{\pm}\, \nu_e(\overline{\nu}_e)\,.$$

The calculations have been performed by one-dimensional Monte Carlo method.

The differential flux of muon-neutrinos (sum of muon-neutrinos and anti-muon-neutrinos) is displayed in table I and that of electron-neutrinos (sum of electron-neutrinos and anti-electron-neutrinos) is displayed in table II, where the neutrino fluxes were calculated in the energy region from 0.05 GeV to 20 GeV and for the zenith angles 0°, 30°, 45°, 60°, 75°, 80°, 94°, 87° and 90°.

TABLE I. – *Differential fluxes of muon-neutrinos in units of* $(\text{cm}^2\ \text{s sr GeV})^{-1}$.

Neutrino energy (GeV)	Zenith angle (degrees)				
	0	30	45	60	75
0.05	5.44	5.57	5.72	5.95	6.21
0.1	3.29	3.37	3.47	3.59	3.81
0.2	1.32	1.32	1.41	1.48	1.58
0.3	$6.66 \cdot 10^{-1}$	$6.79 \cdot 10^{-1}$	$7.08 \cdot 10^{-1}$	$7.50 \cdot 10^{-1}$	$8.12 \cdot 10^{-1}$
0.5	$2.25 \cdot 10^{-1}$	$2.36 \cdot 10^{-1}$	$2.42 \cdot 10^{-1}$	$2.61 \cdot 10^{-1}$	$3.04 \cdot 10^{-1}$
1	$3.72 \cdot 10^{-2}$	$3.89 \cdot 10^{-2}$	$4.04 \cdot 10^{-2}$	$4.71 \cdot 10^{-2}$	$5.31 \cdot 10^{-2}$
2	$5.19 \cdot 10^{-3}$	$5.30 \cdot 10^{-3}$	$5.63 \cdot 10^{-3}$	$6.31 \cdot 10^{-3}$	$7.46 \cdot 10^{-3}$
3	$1.67 \cdot 10^{-3}$	$1.79 \cdot 10^{-3}$	$1.89 \cdot 10^{-3}$	$2.16 \cdot 10^{-3}$	$2.44 \cdot 10^{-3}$
5	$3.33 \cdot 10^{-4}$	$3.50 \cdot 10^{-4}$	$4.00 \cdot 10^{-4}$	$4.64 \cdot 10^{-4}$	$5.60 \cdot 10^{-4}$
10	$4.03 \cdot 10^{-5}$	$4.46 \cdot 10^{-5}$	$4.96 \cdot 10^{-5}$	$5.72 \cdot 10^{-5}$	$7.41 \cdot 10^{-5}$
20	$4.79 \cdot 10^{-6}$	$5.24 \cdot 10^{-6}$	$5.90 \cdot 10^{-6}$	$6.91 \cdot 10^{-6}$	$9.05 \cdot 10^{-6}$

	80	84	87	90
0.05	6.29	6.32	6.34	6.35
0.1	3.86	3.90	3.92	3.93
0.2	1.62	1.64	1.65	1.65
0.3	$8.29 \cdot 10^{-1}$	$8.48 \cdot 10^{-1}$	$8.56 \cdot 10^{-1}$	$8.58 \cdot 10^{-1}$
0.5	$3.15 \cdot 10^{-1}$	$3.26 \cdot 10^{-1}$	$3.28 \cdot 10^{-1}$	$3.34 \cdot 10^{-1}$
1	$5.64 \cdot 10^{-2}$	$6.03 \cdot 10^{-2}$	$6.31 \cdot 10^{-2}$	$6.31 \cdot 10^{-2}$
2	$8.29 \cdot 10^{-3}$	$8.97 \cdot 10^{-3}$	$9.05 \cdot 10^{-3}$	$9.06 \cdot 10^{-3}$
3	$2.65 \cdot 10^{-3}$	$2.77 \cdot 10^{-3}$	$2.92 \cdot 10^{-3}$	$2.95 \cdot 10^{-3}$
5	$6.20 \cdot 10^{-4}$	$6.96 \cdot 10^{-4}$	$7.44 \cdot 10^{-4}$	$7.60 \cdot 10^{-4}$
10	$8.60 \cdot 10^{-5}$	$1.02 \cdot 10^{-4}$	$1.14 \cdot 10^{-4}$	$1.18 \cdot 10^{-4}$
20	$1.11 \cdot 10^{-5}$	$1.32 \cdot 10^{-5}$	$1.55 \cdot 10^{-5}$	$1.64 \cdot 10^{-5}$

The ratio of the flux of muon-neutrino to that of anti-muon-neutrino is displayed in table III for the same zenith angles as in tables I and II.

Similarly, the ratio of the flux of electron-neutrino to that of anti-electron-neutrino is displayed in table IV.

In order to examine the effect of neutrino oscillations on the ν_μ/ν_e ratio, it would be convenient to use the simple formula, denoted by case (A), proposed by Learned *et al.* [17], such as

$$R_0 = \frac{P_{\mu\mu} + r_0 P_{\mathrm{e}\mu}}{P_{\mathrm{ee}} + (1/r_0)\, P_{\mu\mathrm{e}}}\, \frac{1}{r_0}\,, \tag{20}$$

where P_{ij}'s are the probabilities for neutrino flavor i to oscillate into flavor j, and $1/r_0$ is simply the ν_μ/ν_e ratio in production in the atmosphere.

To compare with Kamiokande data, the ν_μ/ν_e ratio in the presence of oscillations should be modified such that

$$R = N_\mu/N_\mathrm{e}, \tag{21}$$

TABLE II. – *Differential fluxes of electron-neutrinos in units of* $(\text{cm}^2\text{ s sr GeV})^{-1}$.

Neutrino energy (GeV)	Zenith angle (degrees)				
	0	30	45	60	75
0.05	3.03	3.13	3.33	3.63	3.94
0.1	1.62	1.72	1.82	2.02	2.22
0.2	$6.16\cdot10^{-1}$	$6.66\cdot10^{-1}$	$7.17\cdot10^{-1}$	$8.08\cdot10^{-1}$	$9.29\cdot10^{-1}$
0.3	$3.03\cdot10^{-1}$	$3.23\cdot10^{-1}$	$3.53\cdot10^{-1}$	$4.04\cdot10^{-1}$	$4.85\cdot10^{-1}$
0.5	$1.01\cdot10^{-1}$	$1.11\cdot10^{-1}$	$1.21\cdot10^{-1}$	$1.41\cdot10^{-1}$	$1.82\cdot10^{-1}$
1	$1.62\cdot10^{-2}$	$1.82\cdot10^{-2}$	$2.02\cdot10^{-2}$	$2.52\cdot10^{-2}$	$3.23\cdot10^{-2}$
2	$1.92\cdot10^{-3}$	$2.32\cdot10^{-3}$	$2.52\cdot10^{-3}$	$3.33\cdot10^{-3}$	$4.54\cdot10^{-3}$
3	$5.05\cdot10^{-4}$	$5.65\cdot10^{-4}$	$6.36\cdot10^{-4}$	$8.78\cdot10^{-4}$	$1.31\cdot10^{-3}$
5	$8.48\cdot10^{-5}$	$9.69\cdot10^{-5}$	$1.01\cdot10^{-4}$	$1.51\cdot10^{-4}$	$2.52\cdot10^{-4}$
10	$6.70\cdot10^{-6}$	$8.50\cdot10^{-6}$	$1.03\cdot10^{-5}$	$1.44\cdot10^{-5}$	$2.54\cdot10^{-5}$
20	$5.54\cdot10^{-7}$	$7.04\cdot10^{-7}$	$8.63\cdot10^{-7}$	$1.23\cdot10^{-6}$	$2.38\cdot10^{-6}$
	80	84	87	90	
0.05	3.94	3.94	3.94	3.94	
0.1	2.32	2.32	2.32	2.32	
0.2	$9.69\cdot10^{-1}$	$9.99\cdot10^{-1}$	1.01	1.01	
0.3	$5.05\cdot10^{-1}$	$5.25\cdot10^{-1}$	$5.35\cdot10^{-1}$	$5.45\cdot10^{-1}$	
0.5	$1.92\cdot10^{-1}$	$2.02\cdot10^{-1}$	$2.02\cdot10^{-1}$	$2.12\cdot10^{-1}$	
1	$3.43\cdot10^{-2}$	$3.73\cdot10^{-2}$	$3.94\cdot10^{-2}$	$3.94\cdot10^{-2}$	
2	$5.05\cdot10^{-3}$	$5.55\cdot10^{-3}$	$5.65\cdot10^{-3}$	$5.75\cdot10^{-3}$	
3	$1.51\cdot10^{-3}$	$1.72\cdot10^{-3}$	$1.82\cdot10^{-3}$	$1.82\cdot10^{-3}$	
5	$3.03\cdot10^{-4}$	$3.73\cdot10^{-4}$	$4.14\cdot10^{-4}$	$4.14\cdot10^{-4}$	
10	$3.15\cdot10^{-5}$	$3.82\cdot10^{-5}$	$4.57\cdot10^{-5}$	$4.90\cdot10^{-5}$	
20	$3.29\cdot10^{-6}$	$4.47\cdot10^{-6}$	$5.80\cdot10^{-6}$	$6.81\cdot10^{-6}$	

TABLE III. – *The ratio of the flux of muon-neutrino to that of anti-muon-neutrinos.*

Neutrino energy (GeV)	Zenith angle (degrees)								
	0	30	45	60	75	80	84	87	90
0.05	1.11	1.12	1.13	1.11	1.06	1.11	1.09	1.16	1.18
0.1	1.12	1.13	1.14	1.12	1.06	1.13	1.09	1.15	1.18
0.2	1.13	1.13	1.16	1.12	1.06	1.14	1.09	1.14	1.18
0.3	1.14	1.14	1.17	1.12	1.07	1.15	1.09	1.14	1.18
0.5	1.18	1.14	1.19	1.11	1.07	1.15	1.09	1.15	1.17
1	1.23	1.16	1.24	1.11	1.08	1.16	1.09	1.15	1.17
2	1.31	1.24	1.3	1.12	1.1	1.15	1.11	1.16	1.17
3	1.36	1.3	1.34	1.13	1.11	1.15	1.11	1.16	1.17
5	1.41	1.36	1.38	1.16	1.13	1.14	1.13	1.17	1.15
10	1.45	1.42	1.43	1.22	1.17	1.18	1.16	1.18	1.14
20	1.48	1.46	1.46	1.29	1.24	1.23	1.21	1.2	1.16

TABLE IV. – *The ratio of the flux of electron-neutrino to that of anti-electron-neutrinos.*

Neutrino energy (GeV)	Zenith angle (degrees)								
	0	30	45	60	75	80	84	87	90
0.05	1.31	1.31	1.30	1.25	1.23	1.22	1.22	1.25	1.27
0.1	1.31	1.31	1.30	1.25	1.22	1.21	1.22	1.25	1.26
0.2	1.31	1.31	1.30	1.24	1.22	1.21	1.22	1.25	1.25
0.3	1.33	1.30	1.30	1.24	1.22	1.21	1.22	1.25	1.25
0.5	1.36	1.31	1.30	1.24	1.22	1.21	1.22	1.25	1.24
1	1.31	1.31	1.30	1.24	1.22	1.21	1.22	1.25	1.24
2	1.32	1.31	1.30	1.24	1.22	1.21	1.22	1.24	1.23
3	1.32	1.31	1.30	1.24	1.22	1.21	1.22	1.24	1.23
5	1.33	1.32	1.30	1.25	1.23	1.21	1.22	1.24	1.23
10	1.35	1.34	1.31	1.27	1.24	1.22	1.22	1.23	1.23
20	1.39	1.37	1.33	1.31	1.27	1.24	1.23	1.23	1.23

where N_μ is the number of muon-type events and N_e is the number of electron-type events.

N_μ and N_e can be written as

$$N_\mu = \int \phi_{\nu_\mu} P_{\mu\mu} \sigma_\mu \, \mathrm{d}E + \int \phi_{\bar{\nu}_\mu} P_{\overline{\mu\mu}} \sigma_{\bar{\mu}} \, \mathrm{d}E + \int \phi_{\nu_\mathrm{e}} P_{\mathrm{e}\mu} \sigma_\mu \, \mathrm{d}E + \int \phi_{\bar{\nu}_\mathrm{e}} P_{\overline{\mathrm{e}\mu}} \sigma_{\bar{\mu}} \, \mathrm{d}E \, , \tag{22}$$

$$N_\mathrm{e} = \int \phi_{\nu_\mathrm{e}} P_{\mathrm{ee}} \sigma_\mathrm{e} \, \mathrm{d}E + \int \phi_{\bar{\nu}_\mathrm{e}} P_{\overline{\mathrm{ee}}} \sigma_{\bar{\mathrm{e}}} \, \mathrm{d}E + \int \phi_{\nu_\mu} P_{\mu\mathrm{e}} \sigma_\mathrm{e} \, \mathrm{d}E + \int \phi_{\bar{\nu}_\mu} P_{\overline{\mu\mathrm{e}}} \sigma_{\bar{\mathrm{e}}} \, \mathrm{d}E \, , \tag{23}$$

where ϕ's are the atmospheric neutrino fluxes, and σ_i's are the charged current cross-sections for the neutrinos of flavor i to interact with the material of the detector.

In this paper as a plausible value of P_{ij}'s we take the one recently proposed by Acker *et al.* [24]:

$$P_{\mathrm{ee}} = 1/3 \, , \qquad P_{\mu\mu} = 1/2 \, , \qquad P_{\mu\mathrm{e}} = 1/3 \, . \tag{24}$$

Though σ_i's, in general, have a different energy dependence, it was shown [25] that $\sigma_\mu \simeq \sigma_\mathrm{e}$ and $\sigma_{\bar{\mu}} \simeq \sigma_{\bar{\mathrm{e}}}$ for neutrino energy greater than 200 MeV.

We make one further approximation which simplifies the analysis of Kamiokande multi-GeV data considerably: $\sigma_{\bar{\mu}}/\sigma_\mu \simeq \sigma_{\bar{\mathrm{e}}}/\sigma_\mathrm{e} \simeq 0.43$ at 3.0 GeV region [25].

Using all the above approximations, the expression for R can be obtained by simply replacing r_0 in eq. (20) with r, where

$$r = \frac{\phi_{\nu_\mathrm{e}}(1 + 0.43(\phi_{\bar{\nu}_\mathrm{e}}/\phi_{\nu_\mathrm{e}}))}{\phi_{\nu_\mu}(1 + 0.43(\phi_{\bar{\nu}_\mu}/\phi_{\nu_\mu}))} \, .$$

Here r simply means the ratio of the number of electron-type events to muon-type events if there were no oscillations.

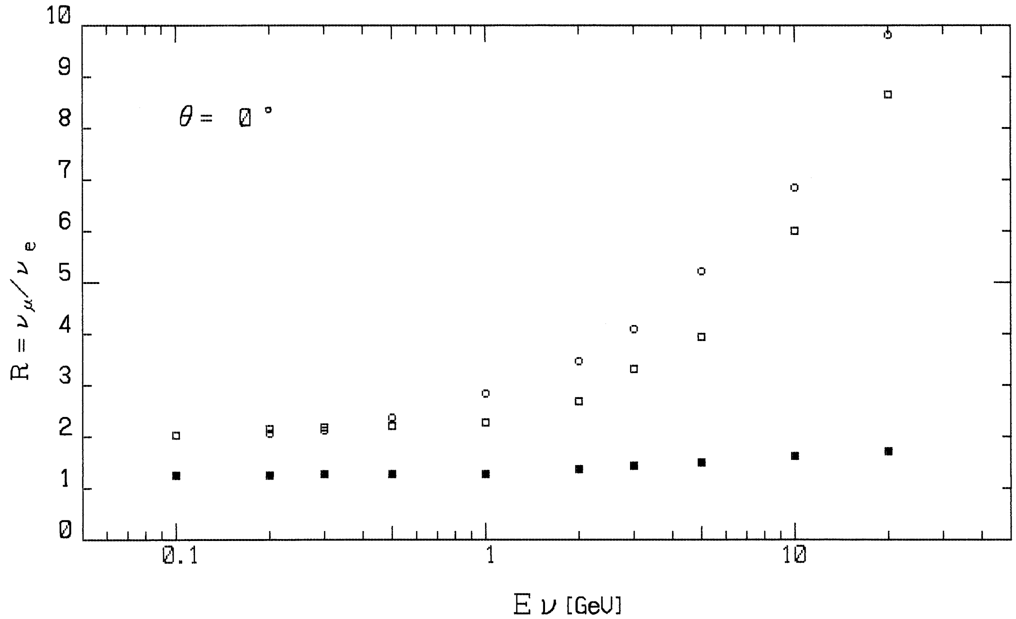

Fig. 1. – Energy distribution of the ν_μ/ν_e ratio for the zenith angle $\theta = 0°$. The abscissa presents neutrino energy E_ν in units of GeV and the ordinate expresses the ν_μ/ν_e ratio. ○ case *a*); □ case *b*); ■ case *c*). See the text for details.

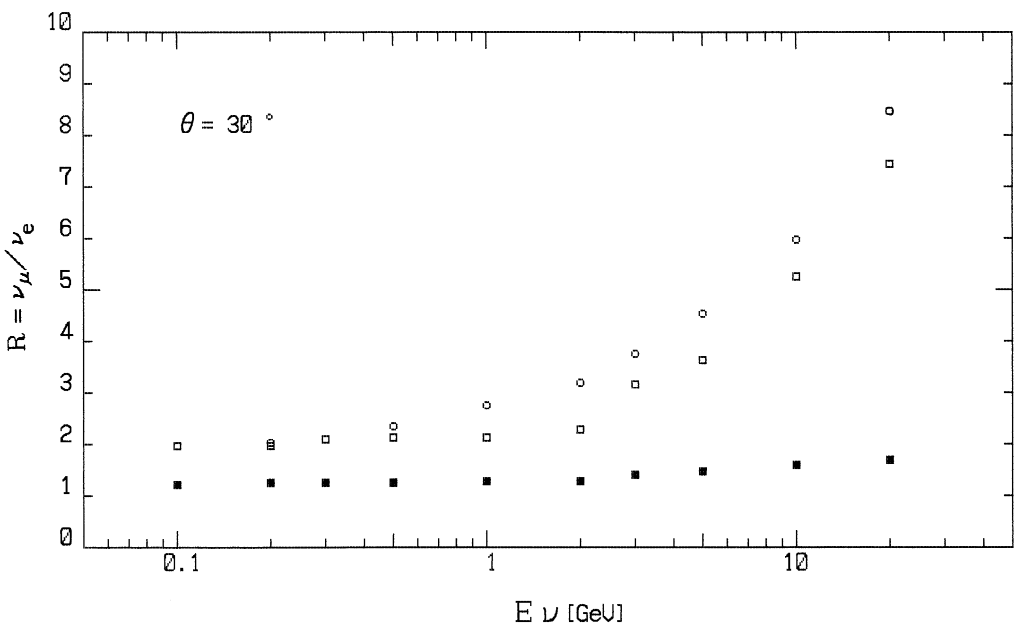

Fig. 2. – As in fig. 1, but for $\theta = 30°$.

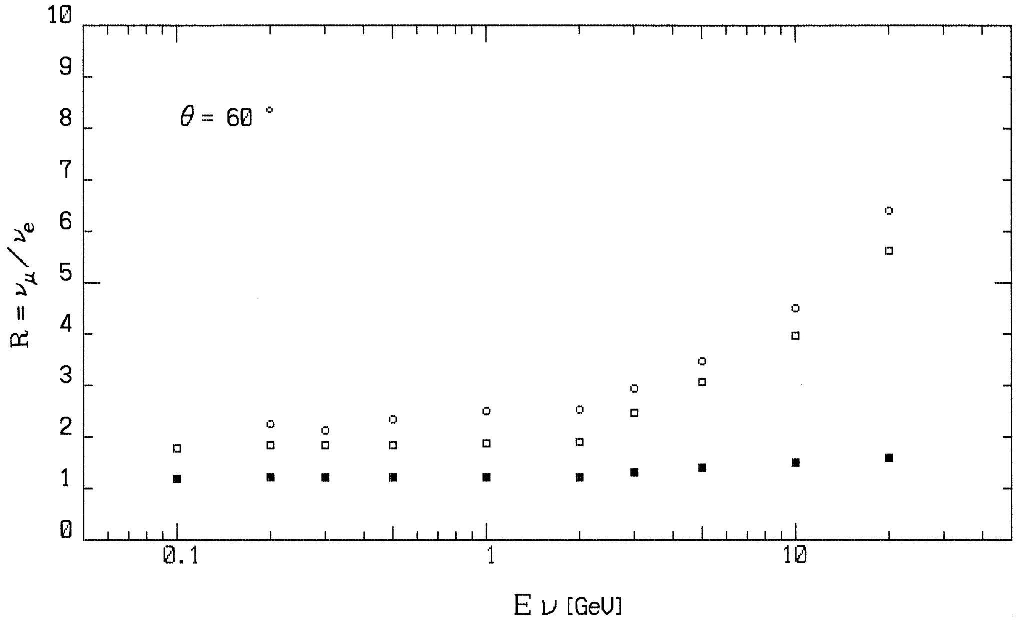

Fig. 3. – As in fig. 1, but for $\theta = 60°$.

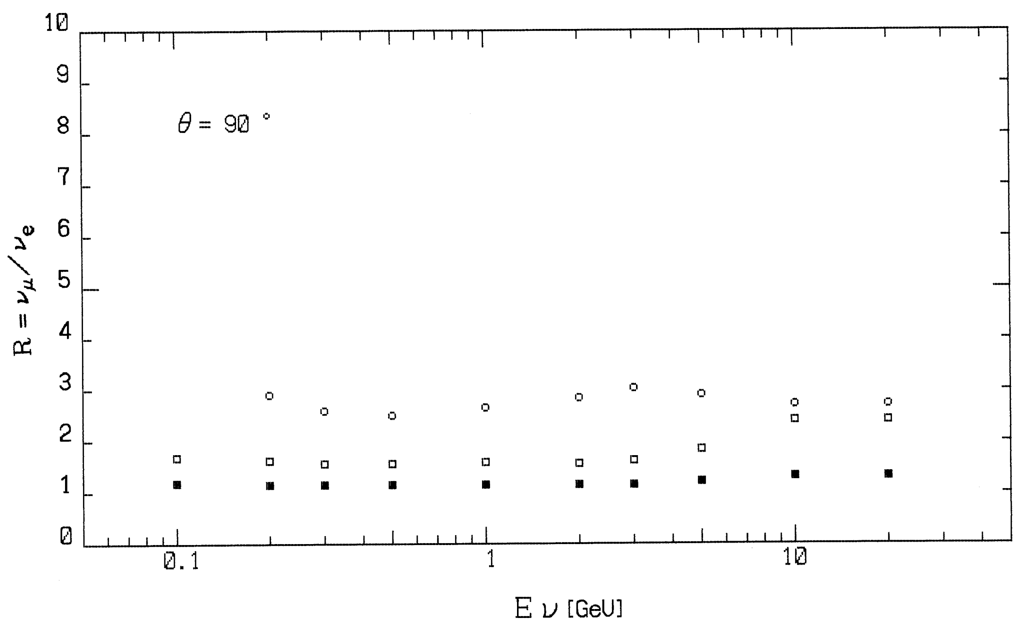

Fig. 4. – As in fig. 1, but for $\theta = 90°$.

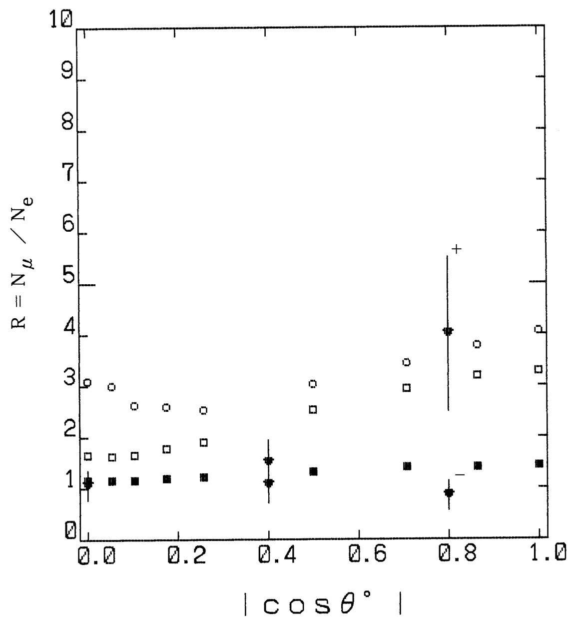

Fig. 5. – Zenith angle distributions of the R ratio at the neutrino energy $E_\nu = 3.0$ GeV. Notations are the same as in fig. 1. For comparison, Kamiokande multi-GeV data [14] are also shown by ▼, with an error bar, where + and − represent the R ratio for the downward-going and upward-going neutrinos, respectively.

The R ratio without oscillations simply reduces to $1/r$.

We consider the following three cases for the analysis of the ν_μ / ν_e ratio:

a) both polarization and oscillations are neglected,

b) only polarization is included,

c) both polarization and oscillations are included.

In figs. 1, 2, 3 and 4 the energy distributions of the N_μ / N_e ratio for the zenith angles $\theta = 0°$, 30°, 60°, 90°, respectively, are shown.

The energy distributions in the cases *a*), *b*), and *c*) are displayed in the respective figure by open circles, squares and full squares, respectively.

In fig. 5 we show the distribution with respect to the cosine of the zenith angle θ at a neutrino energy $E_\nu = 3.0$ GeV, typical value of multi-GeV region, together with the recent Kamiokande data [14] which is denoted by full triangles.

5. – Discussion

It is apparent from figs. 1-4 that the R ratio at energies $E_\nu \sim 3.0$ GeV for the zenith angles $\theta = 0°$, 30°, and 60° becomes about (15–20)% lower than the one calculated ignoring muon polarization effects.

The degree of the decrease amounts to 45% for the zenith angle $\theta = 90°$.

It has been reported [11, 12] that if the effect of muon polarization is included, there is an overall decrease in R by $\sim 20\%$ at $E_\nu \sim 1$ GeV, which is quite consistent with our results except for the case of $\theta = 90°$.

On the other hand, it is found from fig. 5 that the R ratio observed by Kamiokande for $\cos\theta = 0.8$ corresponding to the downward-going neutrinos is in good agreement with the one including only muon polarization (case *b*)).

The observed R ratio for $\cos\theta = -0.8$ corresponding to the upward-going neutrinos contradicts the calculated R ratio including the polarization alone (case *b*)).

If we take into account neutrino oscillations, the observed R ratio is consistent with the calculated R ratio (case *c*)).

The plots show, however, that the calculated value at $\cos\theta = 0$ and ± 0.4 is not in accord with the Kamiokande data.

As pointed out by Bugaev *et al.* [26], this may be caused by misidentifying the particle species of single-ring events.

To summarize, the ν_μ/ν_e ratio, including muon polarization, can well explain the Kamiokande data for downward-going neutrinos, while it cannot explain for upward-going neutrinos.

From this, it is concluded that a possible explanation of the small ν_μ/ν_e ratio and its zenith-angle–dependence might be sought in neutrino oscillations.

In order to draw a definite conclusion about the discrepancy between theory and experiment, however, it seems that not only do we need more statistics, but also we need more accurate calculation of neutrino oscillation, including matter effects.

The measurements of R in the multi-GeV energy region are of special interest because the effect of dark-matter annihilation, which produce high-energy neutrinos, might affect the R ratio, competing with the effect of neutrino oscillations.

Given Super-Kamiokande experiments now under way, it seems, rather likely that the anomaly of the ν_μ/ν_e ratio in the atmosphere will be clarified in the near future.

* * *

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