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Atmospheric models from GPS data

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Summary. — We modeled the atmosphere of the Earth, to 60 km elevation, with ten layers whose refractivity is given by $N = k_i \exp[-h/h_i]$ with h elevation over the surface of the Earth, k_i and h_i defining the refractivity of the layer. The refractivity is assumed continuous between layers, and this reduces the number of free parameters to 11 only. A programme is designed to generate synthetic travel times and from satellites distributed in space to a station on the Earth, travel times are generated with a standard atmospheric model. Stocastic errors are then assigned to the data. A programme is then designed to invert the synthetic data and to fit the travel time to the given satellite positions. The inverted atmospheric model, obtained with 60 satellite positions, deviates from the standard model by less than 4% of N; however, the deviations are positive and negative, the total effect is at most 0.01 ns in the travel time or 0.1 cm in the length of the path. The discrepancies between the model determined and the standard reference model are therefore almost irrelevant in the determination of the coordinates of the points on the surface of the Earth. The distribution of N is then used to infer tentatively the atmospheric water vapor content, the pressure and the temperature. The discrepancies resulting in the comparison obtained by fixing with standard models two of the 3 parameters, are not relevant and very encouranging for the use of GPS data to infer physical properties of the atmosphere. The discrepancy in the model water vapor content and that computed with the inversion is at most 1.2 mbar. A 10% variation of water pressure in the first 10 km of the atmosphere gives a variation 0.1 cm when measuring the elevation of the observing station. It also seen that restricting the observations to a 10° the arc of the orbit of the satellite, the atmospheric model obtained would not lose accuracy. The model suggested, determined by means of observations to GPS satellites, because of its 11 free parameters, instead of the two parameters of the standard model, will allow a more precise description of the real distribution of N at the time when the observations are taken and will therefore allow more precise corrections for the definition of the coordinates of the points on the surface of the Earth and of the positions of the GPS satellites.

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1. – Introduction

The signals transmitted by the GPS, passing through the atmosphere, and recordered by stations over the surface of the Earth are affected by the variation of the

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index of refraction n. It is therefore of great importance to know the profile of n along the path of the ray in order to correct the recordings and also because it supplies important information about the other parameters of climatic and metheorological interest as the pressure, the temperature and the water vapour content on which n depends (Antonelli P. and Caputo M. 1994, Businger S. *et al.* 1995 and Ware R. *et al.* 1995.

We present here the results of a study of a ten-layers model of n with spherical symmetry in the lower atmosphere. In each layer n is variable and the parameters defining it are determined by means of an inversion method applied to the data received at a single observing station and transmitted by one or more GPS satellites.

The possible comparison of the n resulting from the inversion with the available models of the physical parameters of the atmosphere gives good results. It is also seen that the use of two stations observing simulataneously the same GPS satellites may possibly improve the results, but only if the accuracy of the observed time signals is significantly improved.

2. – Definition of the model

Let us consider the GPS satellite in the point P and an observing station in a point A of the surface of the Earth assumed spherical with radius r_0 and center in O as shown in fig. 1. Obviously, the points O, P and A are in a plane; when one uses more than one station and m satellite positions $P_1 P_2 P_3 \dots P_{m-1}$, P_m, then the plane is generally not the same; however, since the important geometric parameters are the angles AOP_j ($j = 1, 2, \dots, m$), and the distance $OP_j = r_j$, the reduction to the same plane is not needed.

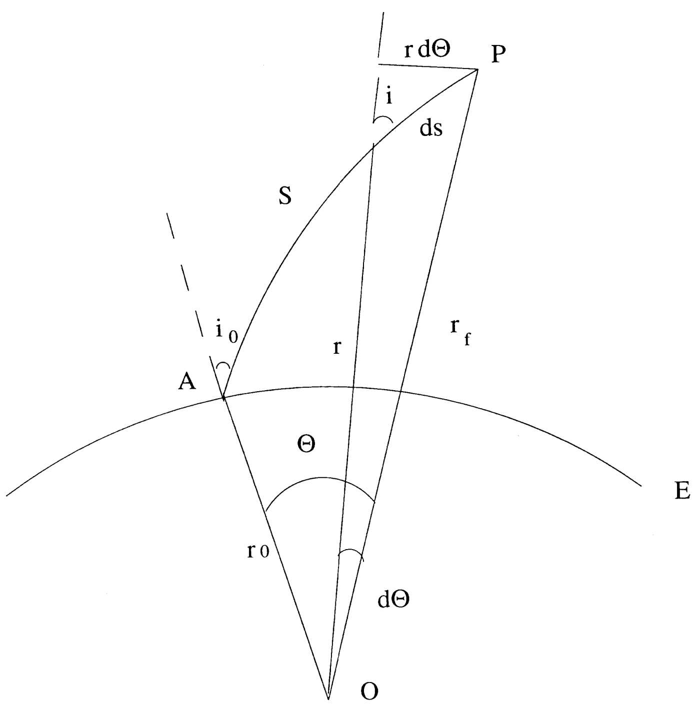

Fig. 1. – Geometry of the path of the signal in the atmosphere.

The signal originating from the satellite in P and travelling to A meets increasing values of n and its velocity is

$$v(r) = c/n(r)\,, \tag{1}$$

where c is the velocity of light and $n(r)$, representing the index of refraction, is a decreasing function of r in each of the ten layers. We shall represent $n(r)$ in each layer as follows:

$$n_i(r) = 1 + k_i \cdot e^{(r-r_0)/H_{s_i}}\,, \qquad r_{i-1} \leqslant r \leqslant r_i\,, \qquad i = 1, \ldots, 10$$

where k_i and H_{s_i} identify the properties of the i-th layer with $H_{s_i} = k_B T/m_i g$; k_B is the Boltzmann constant, T is the temperature, m_i is the mass and g the acceleration of gravity.

In general, instead of $n(r)$, which is very close to unity, the refractivity defined as follows is used:

$$N_i(r) = k_i \cdot e^{(r-r_0)/H_{s_i}}\,.$$

The thickness of the ten layers is scaled appropriately in order to obtain approximately the same integral contribution of the refractivity along the path in all the ten layers.

Finally, the functions $N_i(r)$ have been constrained to continuity through the nine surfaces separating the ten layers setting as condition

$$k_i \cdot e^{-((r_{i-1}-r_0)/H_{s_i})} = k_{i-1} \cdot e^{-((r_{i-1}-r_0)/H_{s_{i-1}})}\,. \qquad i = 2, 3, \ldots, 10\,.$$

Because of the continuity condition, given H_{s_1}, the values of the scale height in the layers above are determined if one assumes that the unknowns to be determined are k_i ($i = 1, 2, \ldots, 10$).

We used also the law of Snell, written as follows, binding the geometry of the ray path to the index of refraction

$$\frac{r \cdot \sin(i)}{v(r)} = \frac{r_0 \cdot \sin(i_0)}{v(r_0)} = \text{const} = p\,, \tag{2}$$

where r is defined in fig. 1, i is the angle which the ray path forms with the normal to the surface of the Earth and the subscript zero indicates that the values are taken at the surface of the Earth.

3. – The synthetic data

The synthetic data have been generated using the proper differential geometry relations between the arc elements shown in fig. 1,

$$\mathrm{d}s^2 = \mathrm{d}r^2 + r^2 \cdot \mathrm{d}\Theta^2\,,$$

$$\mathrm{d}s \cdot \sin(i) = r \cdot \mathrm{d}\Theta\,.$$

Introducing the law of Snell written as in eq. (2), one obtain (Caputo, 1993):

(3)
$$S(i_0) = \int_{r_0}^{r_f} \frac{\mathrm{d}r}{\sqrt{1 - \left(\dfrac{r_0 \cdot v(r) \cdot \sin(i_0)}{r \cdot v(r_0)} \right)^2}} ,$$

(4)
$$\Theta(i_0) = \int_{r_0}^{r_f} \frac{\mathrm{d}r}{r \cdot \sqrt{\left(\dfrac{r \cdot v(r_0)}{r_0 \cdot v(r) \cdot \sin(i_0)} \right)^2 - 1}} ,$$

(5)
$$T(i_0) = \int_{r_0}^{r_f} \frac{\mathrm{d}r}{v(r) \cdot \sqrt{1 - \left(\dfrac{r_0 \cdot v(r) \cdot \sin(i_0)}{r \cdot v(r_0)} \right)^2}} ,$$

where S is the length of the path travelled by the signal, T is the time of flight of the signal (time to cover the distance from P to A), Θ is the angle AOP shown in fig. 1, and r_f is the radius of the sphere defining the upper surface of the tenth layer.

Equations (3), (4) and (5) are of the type

(6)
$$\vec{d} = G(\vec{m}) ,$$

where $\vec{d}$ represents the set of observable data (in our case T and, indirectly, Θ), $\vec{m}$ is the set of unknown parameters to be determined, here represented by the k_i to which we must add the angles i_0 since they are determined by the refractivity profile, G is a non-linear functional relation, obtained from (3), (4) and (5), which we linearize.

The synthetic data are produced assuming the atmospheric model of Alnutt (1989)

TABLE I. – *Values of the parameters used to produce the synthetic data.*

i_0 (rad)	δ_i (rad)	r_0 (km)	r_f (km)	k	H_s (km)	ε_T	ε_Θ
0.02	0.133	6370	6430	$3.15 \cdot 10^{-4}$	7.36	10^{-8}	10^{-9}

TABLE II. – *Initial values of the parameters defining the ten-layer model.*

Layer	Upper limit of r	Value of k	Value of H_{s_i} (km)
1	6370.775	$2.65 \cdot 10^{-4}$	7.36
2	6371.641	$3.10 \cdot 10^{-4}$	2.851652
3	6372.624	$3.12 \cdot 10^{-4}$	1.8675982
4	6373.757	$3.11 \cdot 10^{-4}$	2.877694
5	6375.757	$3.18 \cdot 10^{-4}$	2.829254
6	6376.741	$3.20 \cdot 10^{-4}$	2.819647
7	6378.856	$3.12 \cdot 10^{-4}$	2.849826
8	6381.836	$3.13 \cdot 10^{-4}$	2.846895
9	6386.926	$3.21 \cdot 10^{-4}$	2.829717
10	6430.000	$3.22 \cdot 10^{-4}$	2.828246

and the other values listed in table I, where δ_i represents the spacing between the values of the i_0 of the rays and the ε_T and ε_Θ represent the limits of the moduli of the random errors assigned to T and Q. k and H_s (km) are taken from the standard atmospheric model of Allnut (1989).

To simulate the data we use then eqs. (4) and (5) and the values of table I assuming as initial approximating model of the ten-layer atmosphere the values listed in table II.

Other values of δ_i have also been used but the final results remained unchanged.

We then linearized relations (4) and (5), used the initial model defined in table II and used an inversion procedure, based on the least-square method, which is based on the minimization of the cost function C defined as follows:

$$C = \frac{1}{2}\left(\sum_{j=1}^{M} \frac{(\Theta^j - \Theta^j_{\text{sim}})^2}{\varepsilon^{j2}_\Theta} + \sum_{j=1}^{M} \frac{(T^j - T^j_{\text{sim}})^2}{\varepsilon^{j^2}_T}\right). \tag{7}$$

We obtain a set of ten values of k_i, each associated to one layer, and a set of values of the angles i_0, one for each couple of values T and Θ. In the relation (7) M represents the number of data T and Θ used in the inversion. The result is graphically shown in fig. 2.

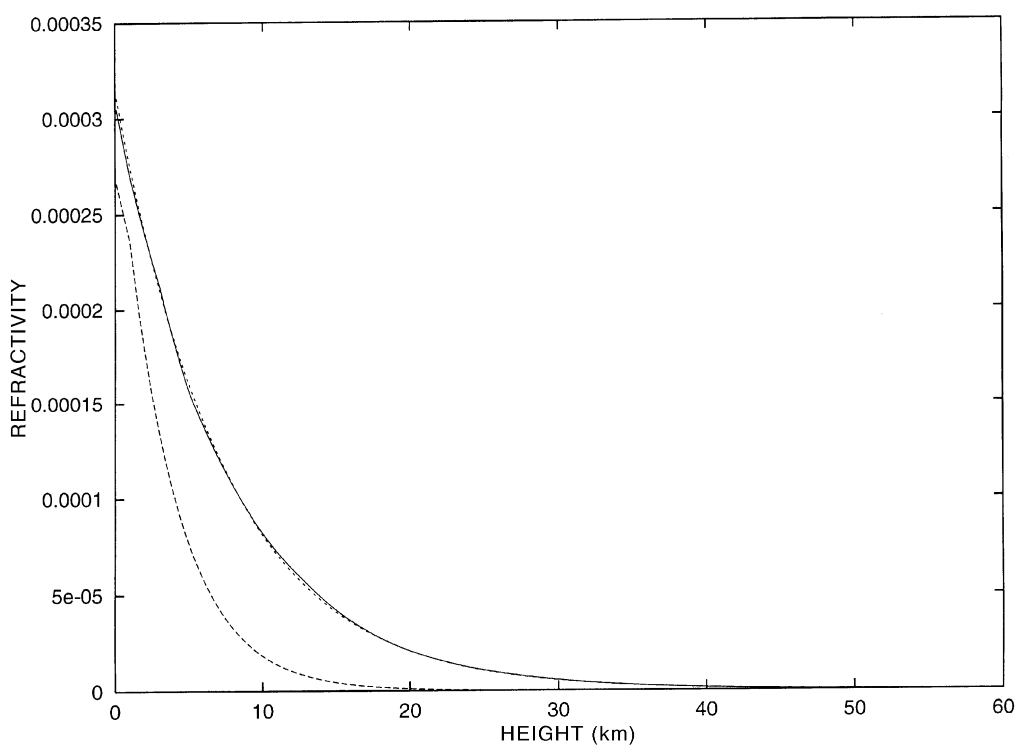

Fig. 2. – Result of the inversion with scale height 7.36 km in the ground layer. The solid line is the model which generated the data, the dashed line near it represents the result of the inversion, the lower dashed line represents the initial layered model.

4. – Results obtained with a single observing station

The possibility to use the GPS times of flight and an inversion procedure for the determination of the refractivity of the atmosphere as been considered also in a previous work of Antonelli and Caputo (1994). However, in the present work the parameters of the initial ten-layer model have been selected in a different manner. We have generalized the method of the selection of the initial parameters, for the initial step of successive approximations, finding the feasibility limits of the method as a function of these initial values. We also found that the initial values of the angles i_0 (see fig. 1) are sufficiently approximated using the angle of the straight line AP connecting the station to the satellite instead of the zenithal angle of the ray path at the station. With these assumptions, which we verified do not limit the accuracy of the results nor the velocity of convergence to the final ten-layer model, the velocity of execution of the programme is much faster.

Assuming known the initial value of the scale height of the first layer from the model of Allnutt (1989), we found that 60 is the optimal number of satellite positions which renders optimal the values of the cost function of the inversion programme. In practice the value of the scale height for the first layer should be observed at the station.

In fig. 3 we show, as a function of the height, the values of the departure of the final refractivity ten-layer model relative to the initial standard model of Allnutt (1989) model; the relative departures are less than 4%, however, since they are positive and negative, their integral effect along the ray path is less than 0.01 ns.

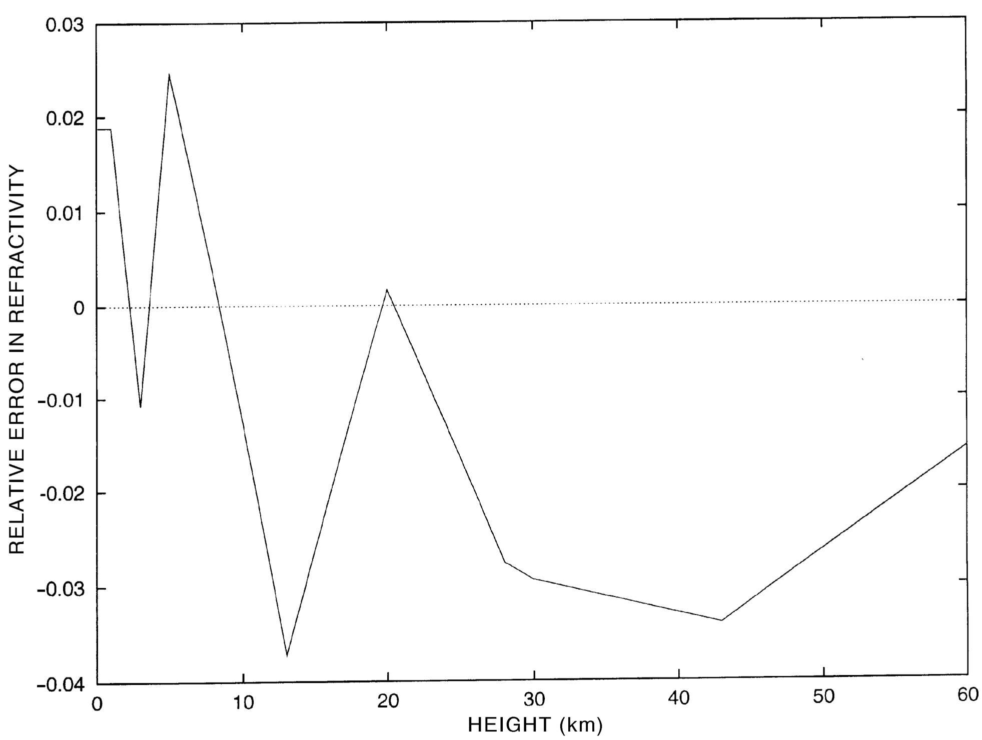

Fig. 3. – Relative error in the refractivity profile obtained from the inversion of the GPS data.

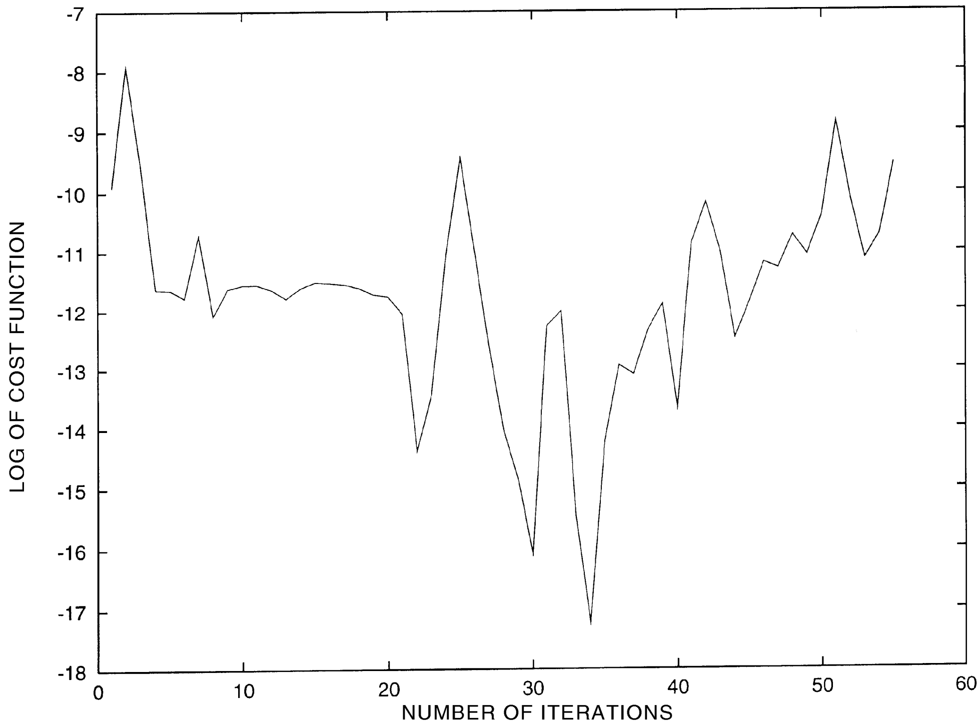

Fig. 4. – The cost function.

The variation of the cost function as a function of the number of iterations is shown in fig. 4. Our experience is that, in general, 10 iterations give an acceptable final ten-layers model of the refractivity of the atmosphere. This velocity of convergence to the final model has been rendered possible by assuming for i_0 the zenithal distance of the direction AP, from the station to the satellite, instead of the zenithal distance of the ray path at A. The computer (DEC Alpha) time to produce the ten-layer model is about 3 minutes and we believe that it is possible to reduce it.

As an indirect check, we interpreted the ten-layered model of the refractivity, resulting from the inversion, as a function of the pressure of the dry air P_d (mbar), of the temperature T (degrees K) and of the water vapor content e (mbar) which are of relavant interest not only in satellite geodesy but also in climatology and physics of the atmosphere.

An empirical relation between the refractivity $N(r)$ and the parameters P_d, T and e has been given by (Smith and Weintraub, 1953)

$$N(r) = a_1 (P_d(r)/T(r)) + a_2 (e(r)/T(r)) + a_3 (e(r)/T^2(r)) . \tag{8}$$

The constants a_i experimentally determined by Smith and Weintraub (1953) have the following values:

$$a_1 = 77.61 \pm 0.01 \text{ K mbar}^{-1},$$

$$a_2 = 72 \pm 9 \text{ K mbar}^{-1},$$

$$a_3 = (3.75 \pm 0.03) \cdot 10^5 \text{ K}^2 \text{mbar}^{-1}.$$

The values of P_d, T and e in the lower atmosphere considered here allow to assume that the totale pressure is the sum of those arising from the dry P_d and wet e components and (8) may be written assuming that $T \cong 273$ K:

$$N(r) = a_1 (P(r)/T(r)) + a_2' (e(r)/T^2(r)), \tag{9}$$

where $a_2' = 3.73 \cdot 10^5 \text{ K}^2 \text{ mbar}^{-1}$.

Moreover, the water vapor pressure is relevant to our study only in the first 7 km of the atmosphere; we may then neglect the second term in the right-hand side member of (9) thus writing

$$N(r) = a_1 (P(r)/T(r)). \tag{10}$$

Using (10), the determination of the refractivity by means of the inversion of the GPS

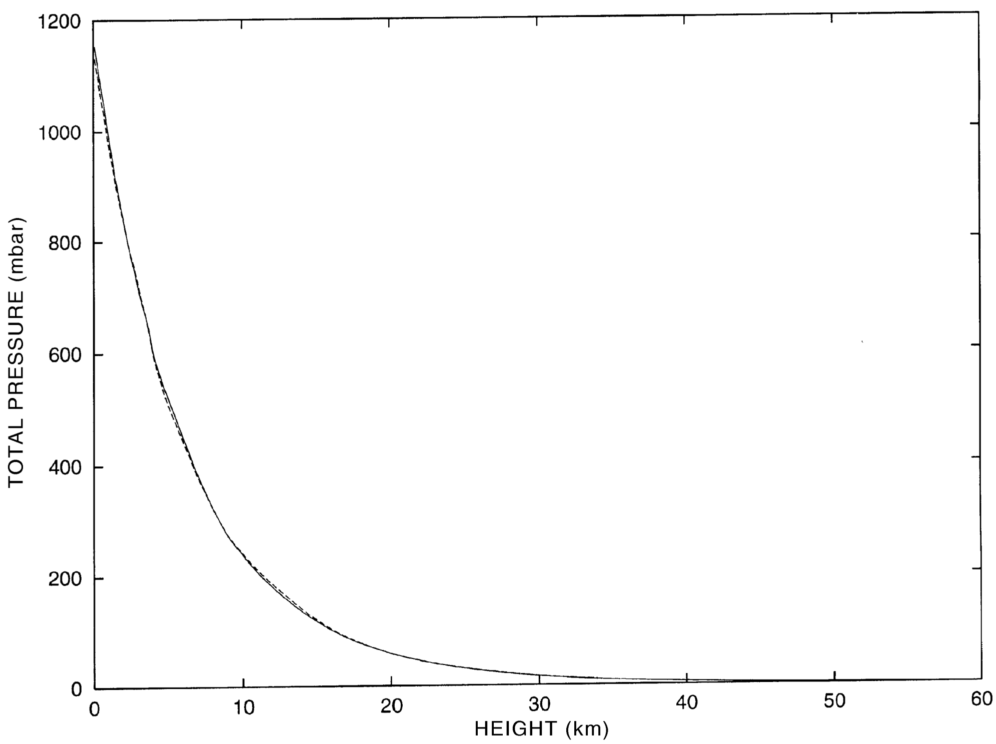

Fig. 5. – Comparison between the the profile of the standard total pressure (solid line) and that obtained from the inversion of the GPS data (dashed line) assuming for the temperature the standard profile and neglecting the contribution of the water vapor pressure.

data and the use of the standard models of $P(r)$ (US Standard Atmosphere 1976) (or $T(r)$) respectively (US Standard Atmosphere 1976), allows to infer $T(r)$ (or $P(r)$, respectively); in fig. 5 we show the departure of $P(r)$, determined from the inverted $N(r)$, relative to the standard model of $P(r)$.

In fig. 6 we show the departure of $T(r)$, determined from the inverted $N(r)$, relative to the standard model of $T(r)$.

Inside the lower atmosphere, the parameter most variable in time is the water vapor content and therefore its pressure $e(r)$; it is measured with various methods with an accuracy of 1 mbar. To compare its standard values with those obtained from the refractivity resulting from the inversion of GPS data, we consider (9) and assume for $P(r)$ and $T(r)$ their standard values. The departure of the standard values of $e(r)$ from those derived from the inverted refraction profile is shown in fig. 7.

From fig. 7 we may tentatively infer that the accuracy of $e(r)$ inferred from GPS observation is sufficient for the correction to apply to the data used for geodetic purposes; in fact a 10% variaton in $e(r)$ causes an error of less than 0.4 cm in the measurement of the length of the ray path.

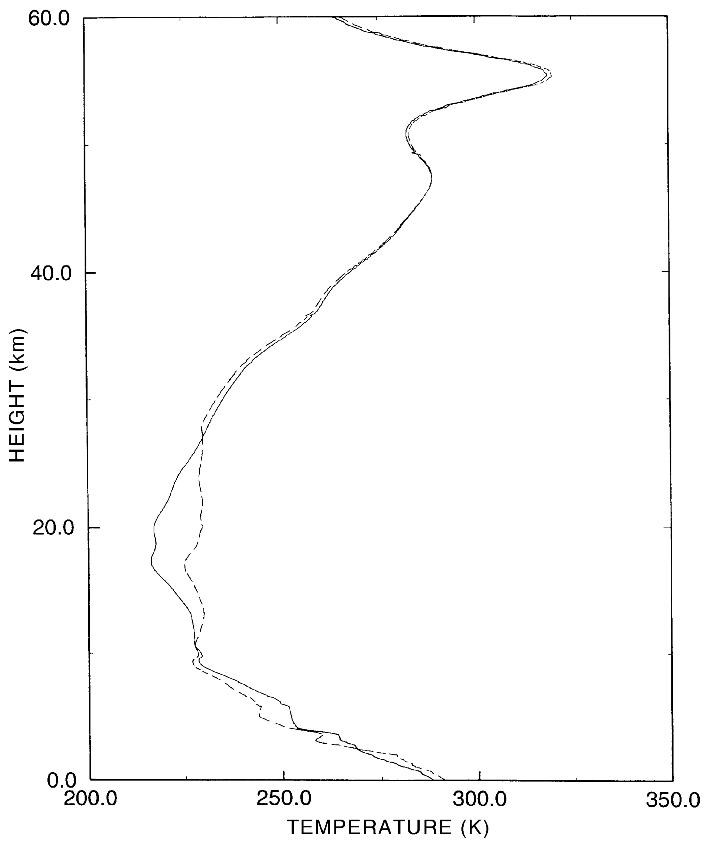

Fig. 6. – Comparison between the profile of the standard temperature (solid line) and its profile obtained from the GPS data (dashed line) assuming for the total pressure the standard values and neglecting the contribution of the water vapor pressure.

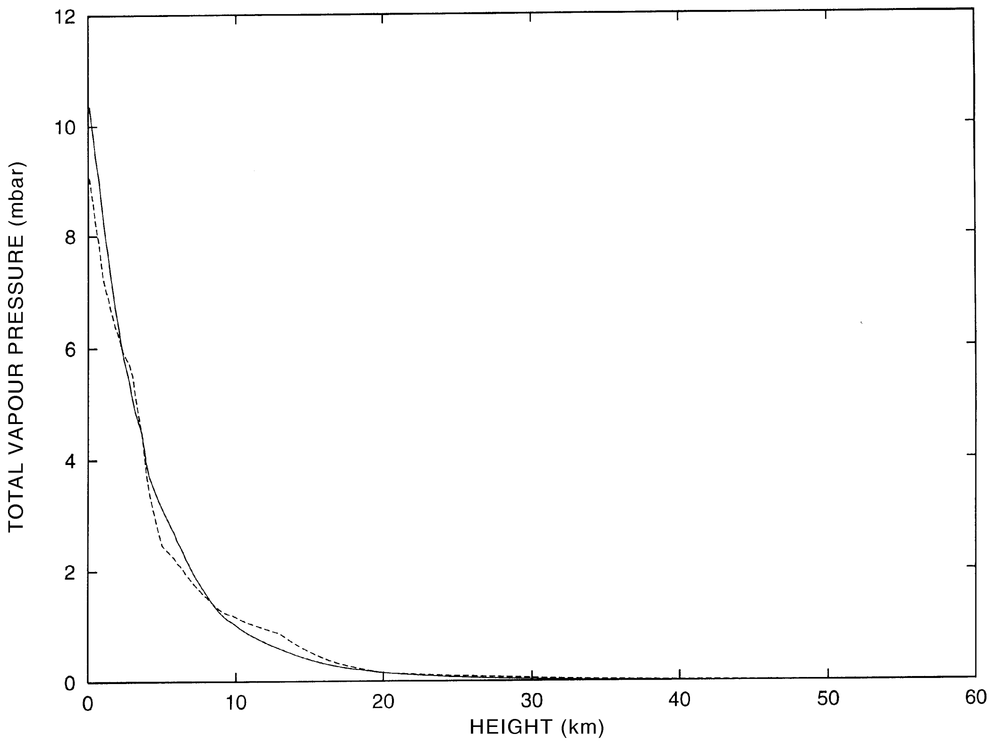

Fig. 7. – Comparison between the profile of the standard water pressure (solid line) and that obtained from the inversion of the GPS data (dotted line) assuming for the total pressure and the temperature the standard profiles.

4·1. *Use of two observing stations.* – One of the most solid limits to the accuracy of the time of flight of the signals from GPS satellites is the error caused by the clocks on the satellite and in the receiver which should be in perfect synchronizaton.

The use of the difference of the time of flight of the signal from a single satellite P to two observing stations separated by some distance allows to eliminate the error related to the satellite clock giving thus a smaller error in the measure of the time of flight which is now of the order of 0.1 ns. This error is larger than the difference of the times of flight from A to P along the straight line from A to P and along the physical path which we find numerically of the order of 0.01 ns, or of the second order in k (Caputo 1993), which is one order of magnitude smaller than the error caused by the clocks.

Because of the size of these quantities, the difference of the times of flight of a signal from a satellite to two different stations i_0 is a measurable quantity and its use to infer the model of the refractivity may be discussed.

The ratio of the paths PA_1 and PA_2 to the corresponding difference of times of flight to A_1 and A_2, to the first order in the values of k_i of the lower layers, is the average velocity of the signal in a number of the lowest layers of the atmosphere depending on the distance of A_1 and A_2. This velocity may be used as an additional constraint for the inversion. However, the use of this average velocity is limited by the following consideration on the accuracy of the measure of the times of flight from P to A_1 and A_2.

In fact we analysed the difference D of the times of flight from P to A_1 and A_2 for several distances A_1A_2 finding that D is an almost linear function of k and that, to record a value of D larger than 1 ns it is necessary that the distance A_1A_2 be larger than 20 km and also that the angles i_0 from A_1 and A_2 to P be wider than 20°.

An alternate method would be to apply the same method used in seismology to infer the velocity of the seismic waves in the interior of the Earth and leading to the well-known Abel's equation; however, also this method suffers from the insufficient accuracy of the the measuments of the times of flight.

We may then conclude that it would be feasible to use D to infer a model of the refractivity only when the system reaches an accuracy in the measure of the times of flight better than 0.01 ns.

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