A note on the testing of a Monte Carlo procedure for evaluating multiple-scattering effects on lidar returns from clouds

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Summary. — A Monte Carlo code for calculating lidar returns from clouds in regime of multiple scattering is tested, by comparing its results pertaining to second order of scattering with those obtained by using an analytic formula developed in a completely different way. For obtaining second order of scattering all the essential parts of the Monte Carlo code are employed. Thus the very good agreement between the results of the two procedures, which have been found in a series of different situations of scattering media, has to be considered as a positive test of the Monte Carlo code reliability.

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1. – Introduction

In general, numerical and theoretical procedures need a series of tests in order to check their reliability.

This note refers to a Monte Carlo procedure developed for calculating the effect of multiple scattering in the lidar technique of sounding turbid atmospheric structures, such as clouds or fogs. It is well known that multiple scattering can make a substantial contribution to lidar returns, when the optical depth of a sounded cloud or fog is greater than a few tenths [1]. This occurs especially when the sounding is carried out from a satellite with a distance of the order of hundreds of kms, in which case the wide size of the cloud part seen by the lidar receiver's Field of View (FOV) allows scattered radiation to follow long paths inside the cloud before being re-scattered towards the receiver [2].

A well reliable procedure for calculating lidar returns from clouds can be considered useful not only for indications of the limit of the "single scattering lidar equation", but

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also because certain systems of inverting lidar data can be tested. In this case one can assume some realistic models of clouds, calculate lidar returns by means of the procedure set-up, and then apply an inversion method, trying to recover some cloud parameters (for instance: size distribution of the scatterers, profile of the extinction coefficient inside the cloud, certain features of the shape for non-spherical scatterers, such as ice particles).

The present versions of a Monte Carlo code is suitable to calculate lidar returns from a cloud, by considering the contributions of different orders of scattering separately. It can provide return power, as well as the components of power relative to parallel and cross-polarisation, when a linearly polarised pulse is emitted by the sounding lidar.

The version of the code, whose check is the object of this note, considers clouds with spherical water droplets. Another version, developed in the framework of a collaboration between the Florence Department of Physics and the Group of Applied Physics of the University of Geneva, can deal with clouds formed with non-spherical simple-shape scatterers (Chebyshev particles).

The essential parts of the code are summarized in refs. [3, 4]. As usual for this type of Monte Carlo code, the continuum of radiation propagation inside the turbid medium is modelled as a discrete series of trajectories, indicated as "photons" in the particular jargon for this type of problem. The trajectories and the points where they are bent by scattering are obtained by means of statistical procedures based on probability laws, which depend on the extinction coefficients and phase functions (or matrices when polarisation is of interest) representing the scattering/absorption properties of the medium. The semianalytic procedure is followed, consisting in obtaining the positions of the scattering points by means of the Monte Carlo scheme, and then calculating the probability for a scattered photon to be received when the scattering points are within the lidar FOV. Variance reduction techniques are used and can be found summarily described in ref. [3].

2. – Previous checks of the code

A series of checks had been already carried out. They were of two kinds.

A check was made by comparing the results of our Monte Carlo code with the numerical results obtained by other researchers taking part in the MUSCLE (Multiple Scattering Lidar Experiments) group. In particular, we can mention the excellent agreement of the Florence and Geneva data with those of Munich (U. Oppel) and Minsk (E. Zege) for a homogeneous cloud at a distance of 1 km from a ground lidar [1]. Further agreement was found with the Minsk data for a stratified cloud sounded by a satellite lidar [5].

A second type of positive check was provided by comparing our numerical data with the results of measurements carried out in the laboratory, where atmospheric turbid layers were simulated by cells with suspensions of latex spheres in water. In this case power and polarisation of the returns were both calculated and measured, with geometry and pulse duration scaled down by a factor of the order of one thousand [4, 6].

3. – Comparison relevant to second order of scattering

As indicated above, the Monte Carlo code provides the contributions to lidar returns (both for power and polarisation) separately for the different orders of scattering.

The present paper aims at obtaining a further check of our Monte Carlo code by comparing the results pertaining to the second order of scattering with those obtained by means of an analytic formalism based on a completely independent scheme.

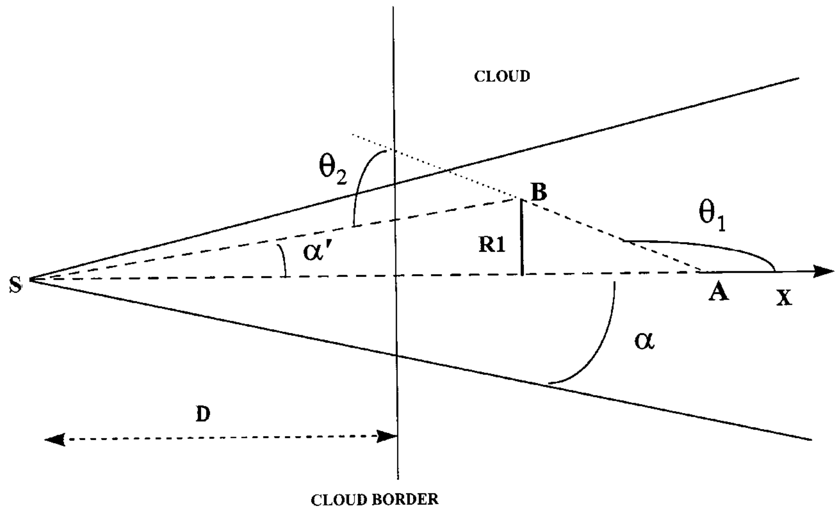

Fig. 1. – Scheme for the calculations of double-scattered received power. S: lidar source. α receiver field of view (FOV) semiaperture. D: distance between the lidar source and the homogeneous cloud border. A: point of first scattering, B: point of second scattering. X: beam axis direction. θ_1, θ_2: angles of first and second scattering, respectively.

Since for obtaining Monte Carlo results pertaining to second scattering all the essential parts of the procedure are applied, the comparison is considered as a real check of the Monte Carlo code validity.

The analytic formalism was developed by us originally for spherical scatterers and was recently extended to a case of non-spherical particles [7].

The formulas for power return from a homogeneous layer of spherical water droplets were derived in ref. [8]. They are here re-presented with reference to fig. 1 depicting the considered geometry. A lidar pulse of small divergence in comparison with the receiver's coaxial FOV of semiaperture a is incident on a cloud of spherical scatterers, with a uniform scattering coefficient. The cloud is placed at a distance D from the sounding apparatus. The formula refers to a very short pulse duration (δ) and to a thin beam. The corrections for a finite-duration pulse, a beam with divergence comparable with the FOV aperture, and a particle albedo different from one can be found in refs. [8] and [9]. Let $p(\theta)$ be the phase function of the medium. Let us indicate as $D21$ the ratio between the contribution to return power due to double scattering and the contribution due to single scattering. With reference to the angles and linear distances shown in the figure, $D21$, as a function of time t from the emission of the pulse, is given as

$$D21(t) = \frac{\sigma}{p(\pi)}\pi c^2 t^2 \int_0^\alpha \mathrm{d}\alpha' \int_D^b \left(\frac{1+\cos\theta_1}{R1^2}\right) p(\theta_1)p(\theta_2)\mathrm{d}x \tag{1}$$

which can be written as

$$D21(t) = \frac{\sigma}{p(\pi)}\pi c^2 t^2 \int_0^\alpha \alpha' \mathrm{d}\alpha' \int_D^b F(x,t,\alpha')p(\theta_1)p(\theta_2)\mathrm{d}x\,, \tag{2}$$

where α is the FOV semiaperture; σ is the volume extinction (pure scattering) coefficient; $p(\theta)$ is the phase function (normalised to one with respect to its integral over the complete solid angle); θ_1 and θ_2 are the angles of first and second scattering , with

$$\theta_2 = \pi - \theta_1 + \alpha' \,,$$

$$b = \frac{ct\left(1 - \frac{2D}{ct}\right)\cos\alpha'}{2\left[1 - \left(\frac{D}{ct}\right)(1+\cos\alpha')\right]}$$

and

$$F(x,t,\alpha') = \frac{2(ct)^{-2}}{1 + 2[(\frac{x}{ct})^2 - (\frac{x}{ct})\cos\alpha'](1+\cos\alpha')(\cos\alpha')^{-2}}\,,$$

$$\pi - \theta_1 = \cos^{-1}\left(\frac{m}{n}\right)\,,$$

$$m = \left[1 - \frac{x}{ct}\left(1 + \frac{1}{\cos\alpha'}\right)\right]^2 - \left(\frac{x}{ct}\right)^2 \frac{(1-\cos^2\alpha')}{\cos^2\alpha'}\,,$$

$$n = \left[1 - \frac{x}{ct}\left(1 + \frac{1}{\cos\alpha'}\right)\right]^2 + \left(\frac{x}{ct}\right)^2 \frac{(1-\cos^2\alpha')}{\cos^2\alpha'}$$

(two misprints of ref. [8] are here corrected).

It can be seen that the singularity in the integrand in the second member of eq. (1) for α' going to zero ($R1$ to zero in eq. (1)) is cancelled out by the integration over α'. This is also shown by a first check carried out by considering a fictitious isotropic phase function $p = \frac{1}{4}\pi$. This allows a simple integration when α is small, so that one can put $\cos\alpha = 1 - \frac{\alpha^2}{2}\sin\alpha = \tan\alpha = \alpha$.

One obtains the simple relation

$$D21 = \frac{1}{4}\pi\sigma\alpha ct\,. \tag{3}$$

For this case fig. 2 shows $D21$, with a comparison between results from eq. (3) and the Monte Carlo code. One sees that the values and the linear behaviour of $D21$ with α are in agreement for the two procedures.

Equations (1), (2) are extended to the consideration of the radiation polarisation. A linearly polarized pulse is assumed to be emitted. To this aim, in the analytic procedure and in the Monte Carlo one, Stokes vectors and scattering matrices are used in place of power and phase function.

We now consider spherical scatterers (water clouds). (For Chebishev particles, and a comparison between Monte Carlo results and those of an analytic formalism which is an extension of that of eqs. (1) to (5), one can see appendix A of ref. [7].)

By considering the scattering plane, the first two elements of the modified Stokes vector I_1 and I_2 indicate the power components normal and perpendicular to this plane, respectively [10]. For spherical scatterers the relationship between the incident (modified)

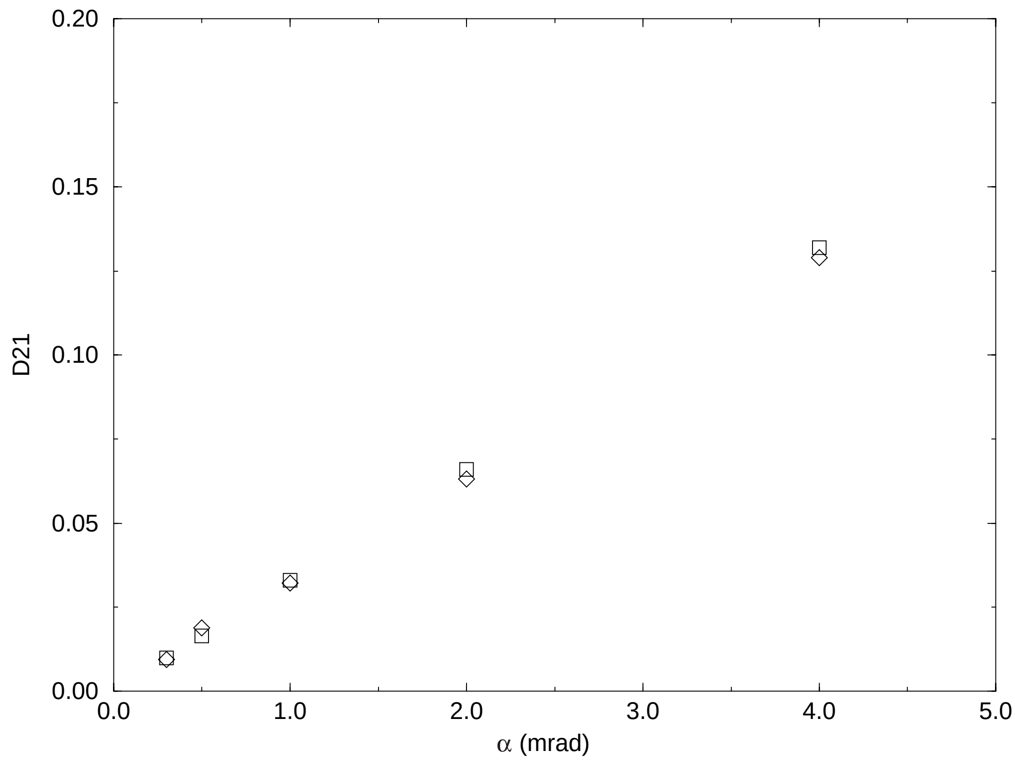

Fig. 2. – $D21$: ratio between double- and single-scattering contributions to received power, plotted *vs.* FOV semiaperture. Beam divergence = 0. Fictitious case of an isotropic phase function. Diamonds: analytic procedure, eq. (3). Squares: Monte Carlo results.

Stokes vector (**I** index i), the scattered one (**I** index s), and the scattering matrix **M** has the form $\mathbf{I}^{\rm s}(r) = \mathbf{M}I^{\rm i}$, with

$$\mathbf{M} = \frac{1}{r^2}\begin{bmatrix} P_1 & 0 & 0 & 0 \\ 0 & P_2 & 0 & 0 \\ 0 & 0 & P_3 & P_4 \\ 0 & 0 & -P_4 & P_3 \end{bmatrix},$$

where r is the distance from the scatterer and the scattered field point, and all the matrix elements P are functions of the scattering angle θ.

Let us now indicate as $D21P$ and $D21N$ the ratios of double-scattering contributions to received power and single-scattering contribution, respectively, with parallel and cross-polarisation with respect to the emitted linearly polarised pulse. In the analytic procedure we have

(4) $$D21P(t) = \frac{\sigma}{p(\pi)}c^2t^2\int_0^\alpha \alpha'\mathrm{d}\alpha'\int_D^b F(x,t,\alpha')F1(P_1,P_2,P_3,P_4)\mathrm{d}x\,,$$

$$D21N(t) = \frac{\sigma}{p(\pi)}c^2t^2\int_0^\alpha \alpha'\mathrm{d}\alpha'\int_D^b F(x,t,\alpha')\;F\mathbf{2}(P_1,P_2,P_3,P_4)\mathrm{d}x$$

with

$$(5)\qquad F1(P_1,P_2,P_3,P_4) = \frac{\pi}{8}\{3[P_1(\theta_1)P_1(\theta_2) + P_2(\theta_1)P_2(\theta_2)] - \\ - 2[P_3(\theta_1)P_3(\theta_2) - P_4(\theta_1)P_4(\theta_2)]\}\,,$$

$$F2(P_1,P_2,P_3,P_4) = \frac{\pi}{8}\{[P_1(\theta_1)P_1(\theta_2) + P_2(\theta_1)P_2(\theta_2)] + \\ + 2[P_3(\theta_1)P_3(\theta_2) - P_4(\theta_1)P_4(\theta_2)]\}$$

and $F(x,t,\alpha')$ as in eqs. (1), (2).

The Monte Carlo procedure is based on the consideration of probability laws for the decision of the trajectories parameters, in particular for the angle in space of a trajectory deviation at a scattering point. In comparison with the case where only the phase function is used, one can see that, in principle, one should consider that the probability of a scattering direction also depends on the azimuthal angle ϕ defining the scattered-photon direction together with the scattering angle θ, with respect to the incidence direction on

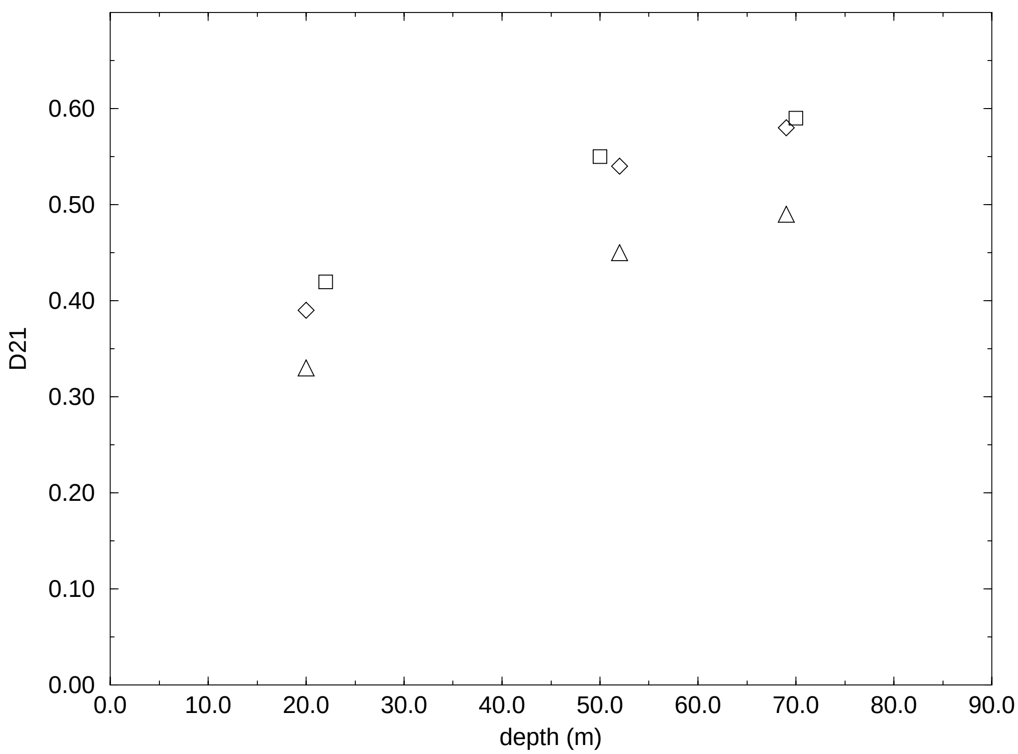

Fig. 3. – Comparison of ratio $D21$ obtained by Monte Carlo using Stokes vectors (squares), analytic formulas (eqs. (4), (5)) employing Stokes vectors (diamonds), and the formulas (eqs. (1), (2)) using a phase function (triangles). Case of Rayleigh scattering. FOV semiaperture 20 mrad. Beam divergence 0.5 mrad. Depth on the abscissa axis indicates the distance inside the cloud where single scattering occurs.

a scatterer. This is in principle true, because the P_1 and P_2 elements of the scattering matrices are in general different, apart from their values at $\theta = 0, \pi$. The dependence on the couple of angles θ and ϕ cannot be factorized in the form of a product of a θ- and a ϕ-dependence.

We have examined this point, since it is generally overlooked by Monte Carlo users, and solved the question by first assuming a probability of a scattering angle θ independent of ϕ, and then applying to the scattered photon a weight $W(\phi)$:

$$W(\phi) = 2\frac{[I_{1\mathrm{R}}P_1(\theta) + I_{2\mathrm{R}}P_2(\theta)]}{(P_1(\theta)+P_2(\theta))(I_{1\mathrm{R}}+I_{2\mathrm{R}})}\,, \tag{6}$$

where $I_{1\mathrm{R}}$ and $I_{2\mathrm{R}}$ are the first two elements of the incident Stokes vector after a rotation is applied in order to make them normal and parallel to the scattering plane, respectively. The integral of $W(\phi)$ over 2π is 1, and one can see that, given the scattering angle θ, $W(\phi)$ corresponds to the probability that the photon is scattered with the azimuthal angle ϕ.

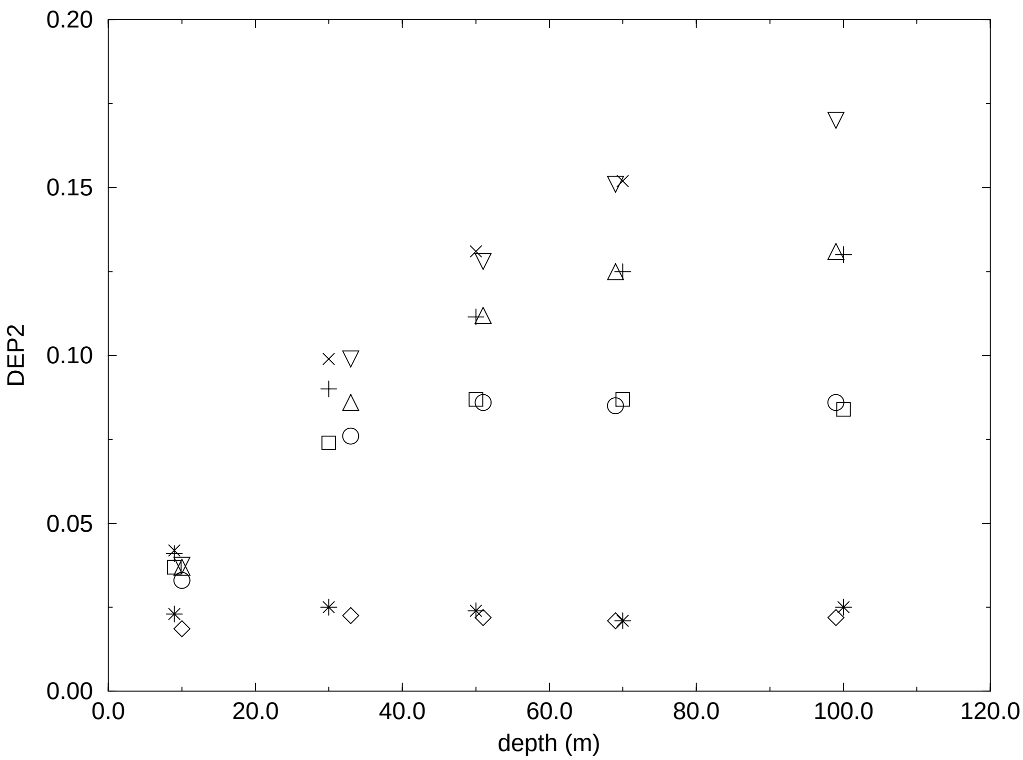

Fig. 4. – $DEP2$ ratio (eqs. (4), (5), (7)) between cross-polarised received power (double scattering) and parallel polarised received power = sum of single and second scattering contributions. Deirmendjian cloud model C1. Volume extinction (pure scattering) coefficient : $\sigma = 0.02\ \mathrm{m}^{-1}$. With reference to fig. 1: $D = 1000$ m, $\alpha = 0.5,\ 5,\ 10,\ 20$ mrad (respectively, diamonds, squares, triangles, $\times$ marks for analytic calculations (eqs. (4), (5)), and asterisks, circles, crosses, reverse triangles for Monte Carlo calculations). Beam divergence = 0.5 mrad.

By comparing a series of results, we found that in most instances of realistic cases of lidar sounding a cloud, the simplifying assumption that the scattering probability is independent of ϕ is a very good approximation. This is caused by the fact that small scattering angles greatly prevail, for a real lidar geometry and real characteristics of clouds, with P_1 and P_2 correspondingly differing very little. However, fig. 3 presents a case where even for received power (sum of parallel and cross-polarised components) a difference comes out. It refers to the case of a layer of very small particles, for which Rayleigh scattering has been assumed. The ratio $D21$ is calculated by the Monte Carlo code using the $W(\phi)$ correction factor, or by assuming independence from ϕ, which for power corresponds to using eqs. (1), (2), with the phase function taken equal to $\frac{(P_1+P_2)}{2}$. The figure shows the ratio $D21$ calculated by the procedure using the weight correction and that assuming azimuthal independence. One can see the slight, but not negligible difference. The assumption of azimuthal independence underestimates the contribution of second order of scattering. Although a situation like that of fig. 3 could only be of interest in some limit cases, such as that of a lidar sounding a thin haze, for the sake of completeness the consideration of the weight $W(\phi)$ is maintained in our code, since it does not imply a substantial increase for calculation time.

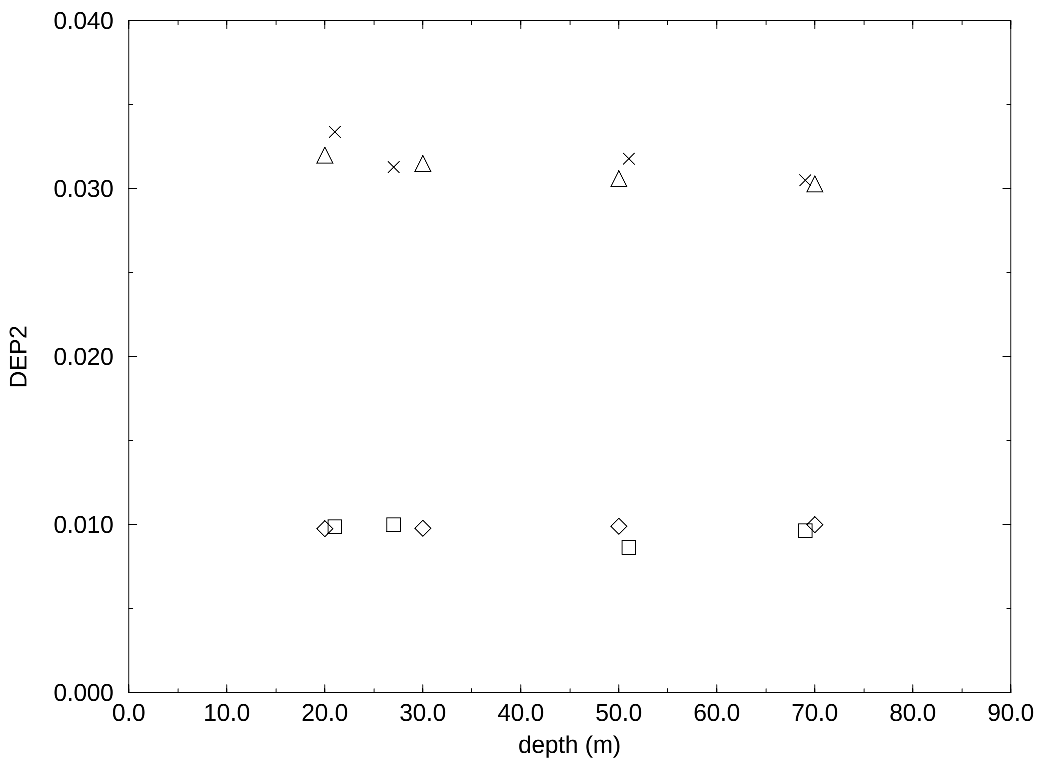

Fig. 5. – Same as fig. 4. Layer of Rayleigh particles. $D = 1000$ m, $\sigma = 0.02$ m^{-1}. α = 5, 20 mrad (respectively squares, crosses for Monte Carlo results and diamonds, triangles for analytic calculations).

Coming to the comparison of results obtained by means of analytic and Monte Carlo procedures, fig. 4 refers to calculated depolarisation due to second scattering.

The quantity $DEP2$ is defined as

$$DEP2 = \frac{W2n}{(W1 + W2p)} = \frac{D21N}{(1 + D21P)} \tag{7}$$

and indicates the ratio between second-scattering received power with cross-polarisation, $W2n$, and the sum of single-scattering power, $W1$, plus second-scattering power $W2p$ with parallel polarisation with respect to that of the emitted pulse.

The figure shows a comparison between results calculated by the Monte Carlo code and the double-scattering analytic formulas, and refers to the case of a wavelength of 1.06 μm, a distance D of 1 km, an extinction coefficient of 0.02 m^{-1} (pure scattering), and a C1 model cloud [11] which is a water cloud with mode radius 4 μm of the particle size distribution. The larger fields of view of the figure, though non-realistic for the lidar practice, have been considered since it can be seen (see, for instance, ref. [2]) that multiple-scattering effects for a large FOV and a near cloud (ground lidar) are almost equivalent to those for a much smaller FOV and a far cloud (satellite case), since the side width of the sounded part of the cloud, intercepted by the lidar FOV, is the essential geometrical parameter.

The figure shows the perfect agreement between the results of the Monte Carlo code and the integral formulas.

A confirmation comes from fig. 5, where calculated values of $DEP2$ refer to the case of a layer of spherical Rayleigh particles. For this case, depolarisation is much lower since the contribution of second (and higher orders) scattering to received power is much smaller than for the C1 model cloud, as the phase function is such that much more of the scattered power exits from the receiver FOV.

However, even in this case, where $DEP2$ is small, the agreement between Monte Carlo and analytic results is very good.

4. – Conclusion

This paper presented a series of checks for a Monte Carlo code for calculating lidar returns from clouds in regime of multiple scattering. A comparison was made of results of the code relevant to second order of scattering with those obtained by analytical formulas derived in a complete independent way. The consideration of second scattering is required and sufficient, as all the essential parts of the Monte Carlo procedure are employed for the calculations. Received power was considered, as well as its two components with parallel and cross-polarisation with respect to that of an assumed linearly polarised lidar pulse.

In all the considered cases the agreement between the results of the two procedures was very good. This series of checks, which come after others of different type (1,4,5,6), confirms the validity of our Monte Carlo code.

The paper also contains a brief discussion about a simplifying assumption on the possibility of neglecting the dependence of scattering probability on the azimuthal angle about the incident-photon direction.

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