Electron collection by a charged satellite in the ionospheric plasma (*)

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Summary. — The space charge region surrounding a highly charged, electron collecting, spacecraft moving in the ionospheric plasma, can be divided into an inner zone (close to the spacecraft), where electron collection is isotropic with respect to the magnetic-field direction, and an outer zone where the electrons are mainly collected along magnetic field lines. In this paper we outline a theory to obtain the current voltage characteristic of such a positive satellite. It is shown that the theoretical results compare very favorably with the experimental data obtained by the TSS-1R mission.

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1. – Introduction

In this paper we consider the problem of current-voltage characteristics for a positively charged satellite moving in the ionospheric plasma. The problem is important in relation to tethered satellite systems, like the TSS-1 flown in 1992 [1, 2] and reflown again in 1996 under the name TSS-1R. The theory presented aims in fact at explaining the data obtained from the TSS-1R mission [3].

In a previous paper [4] we considered several aspects of the problem of charged-particle collection by a spherical satellite moving in the Earth's ionosphere. One of the findings was that, for potentials Φ_s of the satellite above a certain value Φ_ε,

$$\Phi_s > \Phi_\varepsilon \,, \tag{1}$$

the charged-particle collection was becoming isotropic with respect to the magnetic-field direction. By introducing dimensionless potentials $\widetilde{\Phi}$ with

$$\widetilde{\Phi} = \frac{e\Phi}{kT_e}$$

(*) The authors of this paper have agreed to not receive the proofs for correction.

the limiting value Φ_ε was found to be given by

$$\widetilde{\Phi}_\varepsilon = \varepsilon^{-2} = \frac{r_s^2}{a_e^2}\,, \tag{2}$$

with r_s the satellite radius and a_e the electron Larmor radius. Notice that, in the above formula, the electron thermal velocity is defined as $v_{the} = (kT_e/m_e)^{1/2}$. For typical parameters relevant to the TSS missions, Φ_ε is about 40 V.

Condition (1) is precisely the condition under which the inertia terms, in the electron momentum equations, dominate over the magnetic force (proportional to $\mathbf{v} \times \mathbf{B}$). On the other hand, the same condition coincides with the violation of adiabaticity in a theory of electron drifts in crossed electric and magnetic fields [5].

Considering the behaviour of the self-consistent potential around the charged body, and for potentials Φ_s satisfying eq. (1), we therefore have a region of spherical symmetry surrounding the body, extending to a certain radial distance r_ε. Within that region, the potential decreases from Φ_s to Φ_ε. On the other hand, for $r > r_\varepsilon$ the magnetic force on the electrons becomes important so that electron channeling along magnetic field lines takes place. This outer region has therefore cylindrical symmetry with respect to the magnetic-field direction. This is true, at least, for $\Phi > \Phi_v$, where $\Phi_v = 5.3$ V is the voltage equivalent of the ion kinetic energy due to the relative motion between the spacecraft and the ionosphere. For $\Phi > \Phi_v$ we have in fact only electrons and, therefore, space charge effects. On the other hand, for $\Phi < \Phi_v$, ions will be present and will tend to make the plasma quasi-neutral.

The overall picture of the perturbed region surrounding the charged body, for $\Phi > \Phi_v$, is qualitatively shown in fig. 1. If $\Phi_s < \Phi_\varepsilon$, there is no isotropic region and the potential outside the satellite exhibits only cylindrical symmetry with respect to $\mathbf{B}$.

In the case of high potentials (eq. (1)), and in the quoted paper [4], we envisaged a way to calculate the collected current by essentially solving the problem only for the internal isotropic region. The work presented here is organized as follows. In sect. **2** we analyze the inner isotropic region (see fig. 1) and reformulate the derivation analyzed in ref. [4] in a different and clearer way; in so doing, we also correct an algebraic mistake that was present in that analysis. In sect. **3** we deal with the outer region (see fig. 1) characterized by electron channeling along magnetic lines. In sect. **4** we first show the data on *I-V* characteristics obtained from TSS-1R [3]. These data, as already

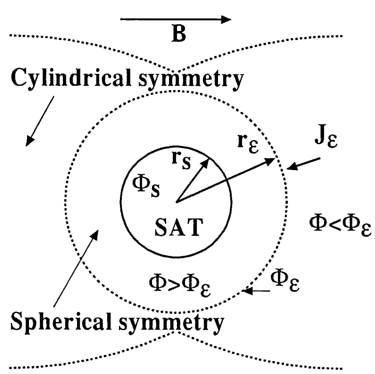

Fig. 1. – Schematic illustration of the perturbed region surrounding the charged body.

mentioned in ref. [3], suggest very strongly that the current, normalized to the thermal current, depends on the potential Φ_s of the satellite through the dimensionless parameter

$$\Phi_s^* = \widetilde{\Phi}_s \left(\frac{\lambda_d}{r_s} \right)^{4/3}, \tag{3}$$

which is characteristic of space charge theory. In eq. (3) λ_d is the electron Debye length defined by

$$\lambda_d = \left(\frac{kT_e}{4\pi e^2 n_0} \right)^{1/2}.$$

In the rest of sect. 4 we compare the experimental characteristics with those obtained from our theory which in fact predicts a dependence of the current on the dimensionless potential Φ_s^*. The agreement, for what concerns the functional dependence of currents from potentials, is indeed quite good. A correction factor (of about 2.7) is however needed to bring the theoretical curve to match the experimental points. We argue that the above factor can be due to effects of the relative velocity flow between satellite and plasma, as envisaged by several authors [6, 7].

2. – Current collection by the high-potential region

In spherical symmetry, the current I is a constant

$$I = 4\pi e n_0 v_e r^2 = \text{const}\,. \tag{4}$$

Referring to the radius r_ε, we write

$$I = 4\pi r_\varepsilon^2 j_\varepsilon\,, \tag{5}$$

with j_ε the current density at the radius r_ε.

In a problem of complete spherical symmetry (no magnetic field and no plasma flow relative to the spacecraft), r_ε would be the sheath radius, representing the boundary between the non-neutral and the neutral plasma, and j_ε would be the thermal current density

$$j_0 = \frac{1}{4} n_0 e v_{\text{the}} \tag{6}$$

with

$$v_{\text{the}} = \sqrt{\frac{8kT_e}{\pi m_e}}\,. \tag{7}$$

We do not have an overall spherical symmetry in our case. As $\widetilde{\Phi}_\varepsilon \gg 1$ (for TSS, $\widetilde{\Phi}_\varepsilon \sim 200$), outside r_ε we have a further region of suprathermal potentials which is controlled by the magnetic field for $\Phi > \Phi_v$ and, when $\Phi < \Phi_v$, by the velocity flow. Hence j_ε will be different and, plausibly, greater than j_0.

To evaluate (5), we must first evaluate r_ε which is the distance at which the potential has dropped to the value Φ_ε. We therefore need to solve for the radial distribution of

the self-consistent potential. The problem that we consider is that of a spherical diode with an external (electron) emitter at radius r_ε and an internal collector (the satellite) at radius r_s. We will see that the equation governing the potential distribution is, under certain assumptions, identical to that derived by Langmuir and Blodgett (LB) [8] but with only one different boundary condition. More precisely, whereas $\Phi(r_\varepsilon) = 0$ in the LB problem, in our case it is $\Phi(r_\varepsilon) = \Phi_\varepsilon$.
Let us start from Poisson's equation

$$\frac{d^2\Phi}{dr^2} + \frac{2}{r}\frac{d\Phi}{dr} = 4\pi e n_e . \tag{8}$$

Because of $I = 4\pi r^2 n_e e v_r = \text{const}$, we can write

$$\frac{d^2\Phi}{dr^2} + \frac{2}{r}\frac{d\Phi}{dr} = \frac{I}{r^2 v_r} . \tag{9}$$

From conservation of energy, in the spherically symmetric region, we have

$$m_e v_r^2 = m_e v_{r\varepsilon}^2 + 2e(\Phi - \Phi_\varepsilon) , \tag{10}$$

where $v_{r\varepsilon} = v_r(r = r_\varepsilon)$. The determination of $v_{r\varepsilon}$ requires now a matching of the spherically symmetric region with the region outside r_ε. In this outside region the electron motion is dominantly parallel to $\mathbf{B}$. For the velocity v_z of this motion, starting from ∞ with $\Phi = 0$, we obtain

$$m_e v_z^2 = 2e\Phi .$$

or, if we want to take into account an initial velocity of the order of the electron thermal velocity v_{the} of this motion, starting from ∞ with $\Phi = 0$, we obtain

$$m_e v_z^2 = m_e v_{the}^2 + 2e\Phi .$$

At $r = r_\varepsilon$, we obtain

$$m_e v_{z\varepsilon}^2 = m_e v_{the}^2 + 2e\Phi_\varepsilon \sim 2e\Phi_\varepsilon , \tag{11}$$

where the last approximation is based on $\widetilde{\Phi}_\varepsilon \gg 1$. Now, the transition between the motion parallel to $\mathbf{B}$ outside r_ε and the inward radial motion for $r < r_\varepsilon$ will not be of course discontinuous. In the impossibility of describing the real transition, we will assume that

$$v_{r\varepsilon} \sim v_{z\varepsilon} , \tag{12}$$

i.e. the parallel velocity is discontinuously translated, ar $r = r_\varepsilon$, into a radial velocity (in reality, eq. (12) will be true only when $v_{z\varepsilon}$ is already radial, otherwise, $v_{r\varepsilon}$ will tend to be smaller than $v_{z\varepsilon}$). Notice also that eq. (12) is crucial to obtain exactly the LB equation.

With (11) and (12), eq. (10) becomes

$$m_e v_r^2 = 2e\Phi , \tag{13}$$

Substituting in (9) we obtain

$$\frac{d^2\Phi}{dr^2}+\frac{2}{r}\frac{d\Phi}{dr}=\frac{I}{r^2}\left(\frac{m_e}{2e\Phi}\right)^{1/2}. \tag{14}$$

Now let us first introduce a dimensionless radius

$$\hat{r}=\frac{r}{r_\varepsilon}, \tag{15}$$

and then use

$$\gamma=\ln\hat{r}, \tag{16}$$

as the independent variable. Equation (14) then becomes

$$\frac{d^2\Phi}{d\gamma^2}+\frac{d\Phi}{d\gamma}=I\left(\frac{m_e}{2e\Phi}\right)^{1/2} \tag{17}$$

and Φ has to satisfy the following boundary conditions:

$$\Phi(\gamma=0)=\Phi_\varepsilon, \qquad \Phi(\gamma_s)=\Phi_s \tag{18}$$

with γ_s given, according to the definition (16), by

$$\gamma_s=\ln\frac{r_s}{r_\varepsilon}. \tag{19}$$

Notice that, as already anticipated, the first of conditions (18) is different from the corresponding one of the old LB problem (in that case $\Phi(\gamma=0)=0$). Our condition now involves the magnetic field.

The next step, following the LB formulation, is that of defining a new dependent variable $\alpha(\hat{r})$, in place of $\Phi(\hat{r})$, through

$$I=\frac{4}{9}\left(\frac{2e}{m_e}\right)^{1/2}\frac{\Phi^{3/2}}{\alpha^2}. \tag{20}$$

Using (20) in (17), we transform it in the following equation in α:

$$\left(\frac{d\alpha}{d\gamma}\right)^2+3\alpha\left[\frac{d^2\alpha}{d\gamma^2}+\frac{d\alpha}{d\gamma}\right]=1, \tag{21}$$

which is exactly the equation of Langmuir and Blodgett. The boundary conditions for α are now, from (18) and (20)

$$\alpha^2(\gamma=0)=\frac{4}{9}\left(\frac{2e}{m_e}\right)^{1/2}\frac{\Phi_\varepsilon^{3/2}}{I}, \qquad \alpha^2(\gamma_s)=\frac{4}{9}\left(\frac{2e}{m_e}\right)^{1/2}\frac{\Phi_s^{3/2}}{I}. \tag{22}$$

For zero magnetic field, we would obtain $\Phi_\varepsilon=0$ and, hence, $\alpha^2(\gamma=0)=0$ as in LB spherical theory. Because of the different boundary conditions, we can write for the solution of eq. (21)

$$\alpha^2(\gamma) = \alpha^2(\gamma = 0) + \alpha^2_{\mathrm{LB}}(\gamma), \tag{23}$$

with $\alpha^2(\gamma = 0)$ given by the first of equations (22) and $\alpha^2_{\mathrm{LB}}(\gamma)$ the LB solution.

Let us now come back to the problem of determining the I-V characteristic. As $I =$ const, writing (20) at $r = r_s$, we have

$$I = \frac{4}{9}\left(\frac{2e}{m_e}\right)^{1/2}\frac{\Phi_s^{3/2}}{\alpha^2(\gamma_s)}\,. \tag{24}$$

On the other hand, we can also use eq. (5) for the current so that

$$9\pi r_\varepsilon^2 j_\varepsilon \alpha^2(\gamma_s) = \left(\frac{2e}{m_e}\right)^{1/2}\Phi_s^{3/2}\,. \tag{25}$$

Substituting the solution (23), calculated in γ_s, and using (22) for $\alpha^2(\gamma = 0)$, we further obtain

$$9\pi r_\varepsilon^2 j_\varepsilon \alpha^2_{\mathrm{LB}}(\gamma_s) + \left(\frac{2e}{m_e}\right)^{1/2}\Phi_\varepsilon^{3/2} = \left(\frac{2e}{m_e}\right)^{1/2}\Phi_s^{3/2}\,. \tag{26}$$

If we now introduce the potential Φ_s^*, as defined in eq. (3), we can rewrite (26) as

$$\frac{9}{8\sqrt{\pi}}\,\frac{r_\varepsilon^2}{r_s^2}\,\hat{j}_\varepsilon \alpha^2_{\mathrm{LB}}(\gamma_s) + (\Phi_\varepsilon^*)^{3/2} = (\Phi_s^*)^{3/2}, \tag{27}$$

with

$$\Phi_\varepsilon^* = \widetilde{\Phi}_\varepsilon\left(\frac{\lambda_{\mathrm{d}}}{r_s}\right)^{4/3}$$

and having defined a normalized current density

$$\hat{j}_\varepsilon = \frac{j_\varepsilon}{j_0} \tag{28}$$

with j_0, the true current density at ∞, defined by eqs. (6) and (7).

Equation (27) must be solved for r_ε / r_s which is then obtained as a function of Φ_s^*. The result has to be substituted in eq. (5), which, normalized to I_0

$$I_0 = 4\pi r_s^2 j_0, \tag{29}$$

gives

$$\frac{I}{I_0} = \left(\frac{r_\varepsilon}{r_s}\right)^2 \hat{j}_\varepsilon\,. \tag{30}$$

Notice that eq. (27) depends on the background magnetic field through the term Φ_ε and the normalized current density $\hat{j}_\varepsilon$.

3. – Current collection by the low-potential region

The quantity $\hat{j}_\varepsilon$ must be determined by a theory of the exterior region ($r < r_\varepsilon$) where the potential should have a dominant axial symmetry with respect to the magnetic field (at least as long as $\Phi > \Phi_{\rm v}$).

The problem, therefore, is that of the current collected in the presence of a magnetic field by a sphere of radius r_ε charged at potential Φ_ε.

In the outer region we will assume the current density conservation

$$j_z \sim \text{const}$$

as the motion of the charged particles is dominantly parallel to **B** ($v_\perp < v_z$ in the drift approximation). This current density can therefore be equated to the value of the quasi-neutral region. Thus, neglecting, for the time being, effects connected to the relative flow between the satellite and the ionospheric plasma, we have

$$j_z \sim j_0 \tag{31}$$

with j_0 given by eq. (6).

The total current collected by the sphere of radius r_ε can be written as

$$I = I_z + I_\perp . \tag{32}$$

I_z is the contribution to the current from electrons moving parallel to the magnetic field and $I_\perp$ is the contribution coming from perpendicular electron drifts. For the parallel contribution we have, because of the axial symmetry of the problem,

$$I_z \sim 2\pi r_\varepsilon^2 j_0 . \tag{33}$$

As for the perpendicular contribution, we have that, in the drift approximation, an electron experiences, besides the motion parallel to **B** with velocity v_z, a radial drift $v_{\rm r}$ normal to the axis of the system given by [9]

$$v_{\rm r} = -\frac{ev_z}{m\omega^2}\,\frac{\partial^2\Phi}{\partial r\partial z} , \tag{34}$$

ω being the electron cyclotron frequency. The corresponding current density is

$$j_{\rm r} = -\frac{ej_z}{m\omega^2}\,\frac{\partial^2\Phi}{\partial r\partial z} .$$

The perpendicular current is then calculated from

$$I_\perp = I_{\rm r} \sim \frac{ej_0}{m\omega^2}\int\!\!\int r\,\mathrm{d}\phi\,\mathrm{d}z\,\frac{\partial^2\Phi}{\partial r\partial z} ,$$

where we have used eq. (31) for j_z. The integral in z goes from $-\infty$ to $-r_\varepsilon$ and then from $+r_\varepsilon$ to $+\infty$ if we denote by ∞ the unperturbed plasma (or the quasi-neutral region). The two pieces are identical and, integrating by parts with respect to z, we obtain

$$I_\perp \sim \frac{2ej_0}{m\omega^2}\int r\,\mathrm{d}\phi\,\frac{\partial\Phi}{\partial r} .$$

The integration in ϕ goes from 0 to 2π. As Φ, in axial symmetry, is independent of the azimuthal coordinate ϕ, the final result is, in order of magnitude,

$$I_\perp \sim 4\pi\Phi_\varepsilon \frac{ej_0}{m\omega^2} . \tag{35}$$

Substituting (33) and (35) in (32), using the definition (2) for Φ_ε, and then normalizing in the usual way, we obtain

$$\frac{I}{I_0} = \frac{1}{2}\left(\frac{r_\varepsilon}{r_\mathrm{s}}\right)^2 f(r_\varepsilon) \qquad \text{for } \Phi_\mathrm{s} > \Phi_\varepsilon , \tag{36}$$

with

$$f(r_\varepsilon) = 1 + 2\left(\frac{r_\mathrm{s}}{r_\varepsilon}\right)^2 . \tag{37}$$

The correspondence of (36) with our previous formulation (eq. (30)) is therefore obtained with

$$\hat{j}_\varepsilon = \frac{1}{2} f(r_\varepsilon) .$$

Finally, because of the fact that the quasi-neutral region extends to the potential Φ_v, it is likely that the current density there will be higher than the thermal current density, as supposed in eq. (31). Effects of the velocity flow leading to increased currents have been considered by several authors [6, 7]. We will take such effects into account by enhancing the current density above j_0 by a factor $\beta > 1$. With that, the equation for the current will be

$$\frac{I}{I_0} = \frac{1}{2}\left(\frac{r_\varepsilon}{r_\mathrm{s}}\right)^2 \beta f(r_\varepsilon) \qquad \text{for } \Phi_\mathrm{s} > \Phi_\varepsilon , \tag{38}$$

with $f(r_\varepsilon)$ given by eq. (37) and the radius r_ε obtained from the solution of

$$\frac{9}{16\sqrt{\pi}} \frac{r_\varepsilon^2}{r_\mathrm{s}^2} \beta f(r_\varepsilon)\, \alpha_\mathrm{LB}^2(\gamma_\mathrm{s}) + (\Phi_\varepsilon^*)^{3/2} = (\Phi_\mathrm{s}^*)^{3/2} . \tag{39}$$

4. – Comparison with experimental data

The first analysis of data from various experiments on TSS-1R, leading to current-voltage characteristics of the satellite, was reported in ref. [3]. Figure 2 shows the results obtained. On the top panel the normalized current I/I_0 is plotted as a function of Φ_s/Φ_0. Φ_0 corresponds to the Parker and Murphy (PM) potential [10], so that, according to the PM theory, I/I_0 should follow a law dependent on $(\Phi_\mathrm{s}/\Phi_0)^{1/2}$. On

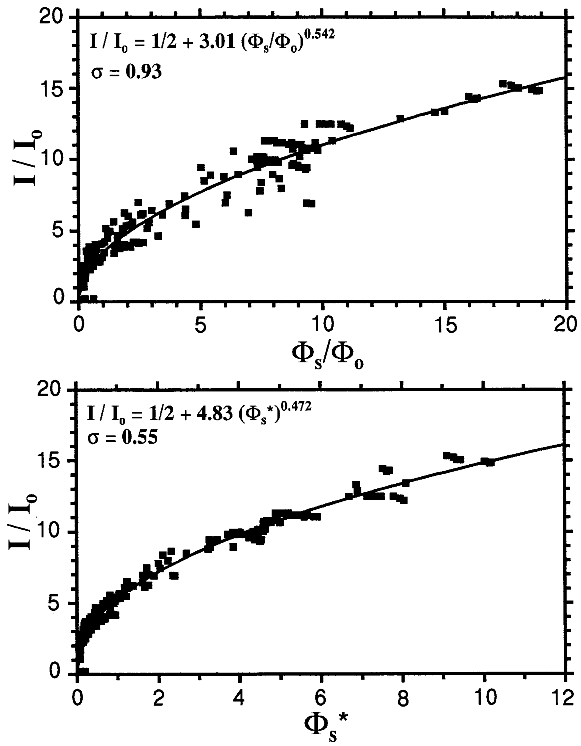

Fig. 2. – Current-voltage characteristic of the TSS-1R satellite. The normalized current is plotted as a function of Φ_s/Φ_0 in the top panel and Φ_s^* in the bottom panel. The solid lines represent the best fit of data with power functions. The relevant analytical expressions are shown in the figure.

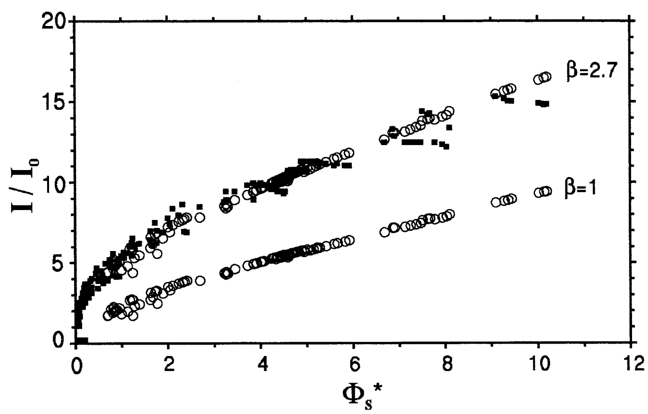

Fig. 3. – Comparison between the experimental and theoretical characteristics. The squares represent the TSS-1R data. The open circles are the theoretical points obtained from eqs. (38) and (39) for $\beta = 1$ and $\beta = 2.7$.

the other hand, in the bottom panel I/I_0 is plotted *vs.* the dimensionless potential Φ_s^*. It is immediately apparent, and it was already remarked in ref. [3], that the values of I/I_0 appear to be better organized as a function of Φ_s^* rather than as a function of Φ_s/Φ_0. The scatter of the experimental points is in fact much smaller for the plot on the bottom panel than for the one on the top.

Some statistical analysis supports that more quantitatively. In the figures, along with the experimental points, we have also reported the results of the best fit of the curves with power laws in the variables Φ_s/Φ_0, and Φ_s^*, respectively (solid lines). The power functions have been chosen with the additional constrain that the extrapolation at zero potential must reproduce a normalized current equal to 1/2 as expected for the channeling of the electrons along the magnetic field lines. The analitical functions obtained by the best fit are shown in the two panels of fig. 2. In both cases, as we see, the exponent of the power law is close to 0.5. However, the standard deviation is quite high ($\sigma = 0.93$) for the best fit in the plot of I/I_0 *vs.* Φ_s/Φ_0, whereas it is much smaller ($\sigma = 0.55$), when I/I_0 is plotted *vs.* Φ_s^*.

As Φ_s^* is the characteristic potential appearing in space charge theories, we think that the experimental data provide a strong indication for space charge effects taking place.

It is then worthy to compare the experimental results with the theory presented in this paper, which is in fact based on space charge.

Such a comparison between the experimental and theoretical current-voltage characteristics is reported in fig. 3. The squares in the figure are the experimental points obtained from TSS-1R data. The open circles are the theoretical points obtained from a computation of eqs. (38) and (39) corresponding to the values $\beta = 1$ and $\beta = 2.7$ of the parameter β. The value of $\beta = 2.7$ gives the best matching with the experimental points. If we fit the theoretical curve corresponding to $\beta = 2.7$ with a power law (and impose $I/I_0 = 1/2$ at $\Phi_s = 0$), we obtain

$$\frac{I}{I_0} = \frac{1}{2} + 4.31(\Phi_s^*)^{0.551}\,. \tag{40}$$

The exponent of this fit is reasonably close to the exponent obtained for the fit of the experimental points (see the bottom panel of fig. 2) so that we can say that theoretical and experimental characteristics agree well in the dependence of the normalized current I/I_0 on the potential Φ_s^*.

As already mentioned, several authors [6, 7], with different arguments, have pointed out that effects of the relative velocity flow between satellite and plasma can bring an increase in current. Even though the value of this increase varies in the different treatments (so that we still have an open problem here), it seems entirely reasonable to associate with this effect the value that we need for our scaling parameter β ($\beta \sim 2.7$), to obtain the best agreement with the data.

Notice, finally, that our theoretical points in fig. 3 refer to the region $\Phi_s > \Phi_\varepsilon$. We did not attempt any comparison of our model theory with the data below Φ_ε ($\Phi_\varepsilon \sim 40$ V), because we think that the significant errors on the data at low potentials (due to an uncertainty of ~ 10 V in the measurement of the Orbiter potential), would render such comparison highly uncertain.

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