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On the lifetime of an intense localized barotropic vortex on the β-plane (*)(**)

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Summary. — We present an asymptotic theory for the long-time evolution of an intense barotropic vortex on the β-plane. Three stages are described: first, the near-field development of β-gyres; second, the intensification of the quadrupole and secondary axisymmetric components with vortex deceleration; third, the vortex decay. Our theory takes account of all three stages and provides estimates for the vortex lifetime.

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1. – Vortex evolution stages

In its simplest form the problem of the evolution of a localised vortex on the rotating Earth can be stated as follows:

$$\frac{\partial}{\partial t}(\nabla^2\Psi - R_\mathrm{d}^{-2}\Psi) + \beta\frac{\partial\Psi}{\partial x} + J(\Psi,\,\nabla^2\Psi) = 0\,, \qquad \Psi(x,\,y,\,0) = \Psi_0(r)\,. \tag{1}$$

Here the motion is assumed to be barotropic, and the β-plane and quasigeostrophic approximations are used, Ψ is the streamfunction, x, y denote eastward and northward coordinates and t time; ∇^2 and J are Laplacian and Jacobian operators, while R_d is the Rossby radius of deformation. Importantly, R_d is finite and non-zero in the present theory. $\Psi_0(r)$ is the initial axisymmetric vortex.

We suppose that the initial vortex scale is of the order of R_d and the typical orbital

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velocity is U_p. Then we can produce three different time scales using the parameters β, U_p, R_d; $T_\mathrm{a} = R_\mathrm{d}/U_p$, $T_\mathrm{w} = 1/\beta R_\mathrm{d}$ and $T_\mathrm{d} = U_p/\beta^2 R_\mathrm{d}^3$.

Only two of these scales are independent. The "advective" scale T_a is a typical time required for a fluid particle to move the distance of the order of the vortex size. The "wave" scale T_w is the typical time it takes for the vortex to move its own size; accordingly, the vortex translational speed scale is equal to $U_\mathrm{t} = \beta R_\mathrm{d}^2$ (see Reznik, 1992; Reznik and Dewar, 1994). The "distortion" time T_d is the time for the vortex relative vorticity change to be of the order of the relative vorticity itself. Conservation of potential vorticity $\Omega = Q + \beta y$, where $Q = \nabla^2 \Psi - R_\mathrm{d}^{-2}\Psi$ is the relative vorticity, implies that the vortex relative vorticity change is $\Delta Q = -\beta L_y$, where L_y is the meridional drift of the vortex. Since Q scales with U_p/R_d we see that ΔQ is of the order of Q when $L_y = U_p/\beta R_\mathrm{d}$. T_d is then determined as L_y/U_t.

It is now readily seen that

$$\frac{T_\mathrm{w}}{T_\mathrm{a}} = \frac{1}{\alpha}, \qquad \frac{T_\mathrm{d}}{T_\mathrm{a}} = \frac{1}{\alpha^2}, \qquad \text{where } \alpha = \frac{U_\mathrm{t}}{U_p} = \frac{\beta R_\mathrm{d}^2}{U_p}. \tag{2}$$

Since a real atmospheric or oceanic eddy is generally highly nonlinear, its orbital velocity greatly exceeds the translation velocity, and the parameter α is small, $\alpha \ll 1$. In this case the typical times are well separated, $T_\mathrm{a} \ll T_\mathrm{w} \ll T_\mathrm{d}$. Hence we are able to divide the vortex evolution into three different stages.

The first stage, $T < T_\mathrm{w}$, is characterised by the development of a secondary dipole circulation (the so-called β-gyres) in the vicinity of the vortex. To describe this mechanism we see that the monopole, for instance a cyclone, induces a northward (southward) motion to the east (west) of itself. In accordance with the conservation of potential vorticity the β-effect generates anticyclonic (cyclonic) vorticity to the east (west) of the initial vortex, *i.e.* a dipole (β-gyre) is generated. Due to nonlinearity, the β-gyres advect the vortex along the dipole axis. In turn the dipole is advected by the vortex resulting in a turning of the dipole axis in the sense coinciding with the sense of the vortex; that is, clockwise (counterclockwise) for an anticyclone (cyclone). Thus a cyclone (anticyclone) moves northwestward (southwestward) along some curved trajectory; the trajectory shape and the β-gyres structure are related to the strength and structure of the initial vortex (see, for instance, the laboratory experiments of Firing and Beardsley, 1976, the numerical experiments of McWilliams and Flierl, 1979, Mied and Lindemann, 1979, Fiorino and Elsberry, 1989, and the analytical theories of Reznik, 1992, Reznik and Dewar, 1994, and Sutyrin and Flierl, 1994).

In the second stage, $T_\mathrm{w} \leqslant t \leqslant T_\mathrm{d}$, the influence of the other azimuthal harmonics generated by wave radiation and nonlinearity has to be taken into account. This influence gradually reduces the vortex amplitude and in doing so decelerates the vortex motion (*e.g.*, Reznik and Dewar, 1994, Sutyrin *et al.*, 1994). At the same time changes to the relative vorticity of the vortex remain relatively small, so that the vortex amplitude exceeds the amplitude of the radiated field. In the final third stage, $t > T_\mathrm{d}$, the vortex distortion becomes strong and its amplitude decreases to the background level, *i.e.* the vortex ceases to exist as a coherent structure.

The second stage has been less well studied than the first stage. Numerically it was investigated by Sutyrin *et al.* (1994), while there have been a few attempts to describe this stage analytically for various types of eddies (Flierl, 1984, Korotaev and Fedotov, 1994, Flierl and Haines, 1994). But all these theories are based on the assumption that in the course of time the vortex tends to some quasistationary state when its zonal velocity greatly exceeds the meridional one and the Rossby wave wake can be considered to be resonantly excited. This means that the wake is calculated in a similar manner to the calculation of lee waves behind an obstacle in a uniform zonal flow on the β-plane.

Although these theories give qualitatively reasonable results, nevertheless there remain unanswered some important questions. First, there is no evidence (numerical or observational) that the wave field radiated by the vortex can be considered as quasiresonant. Second, the quasiresonant Rossby wave wake has infinite energy (see, for example, eq. (23) from Flierl, 1984). This is of no great importance when considering the flow around an obstacle, in which case the infinite energy is a consequence of making a long-time quasisteady approximation in the frame of reference of the obstacle. But a vortex does not behave like an obstacle in this respect, it loses energy when radiating Rossby waves and therefore cannot possess an infinite-energy wave wake. Third, *none* of the theories cited above (not only the last three ones) provide conservation of energy and enstrophy which are particularly important for the second stage of the vortex development. In this paper we report briefly on an asymptotic theory for a vortex which initially has piece-wise constant relative vorticity. Here we will focus on the issue of the vortex lifetime. A much fuller account which addresses all issues in greater detail is given by Reznik and Grimshaw (1998) (henceforth denoted as RG).

2. – Intense vortex with initially piece-wise constant relative vorticity

To focus in detail on the second stage, we consider the model of an intense initial vortex with piece-wise constant relative vorticity, suggested by Sutyrin and Flierl (1994). We proceed from the non-dimensional form of eq. (1):

$$\frac{\partial}{\partial t}(\nabla^2\Psi-\Psi)+J(\Psi+\alpha Uy-\alpha Vx,\,\nabla^2\Psi-\Psi)+\alpha\frac{\partial\Psi}{\partial x}=0,\qquad \Psi(x,\,y,\,0)=\Psi_0(r)\,,\tag{3}$$

which is written in moving coordinates attached to the vortex center; here U and V are non-dimensional zonal and meridional translation speeds, respectively, scaled with $U_{\mathrm{t}}=\beta R_{\mathrm{d}}^2$; the advective time $T_{\mathrm{a}}=R_{\mathrm{d}}/U_p$ is used as a time scale.

Let the initial potential vorticity be

$$\Omega=\Omega_0=H(1-r)+\alpha y\,,\tag{4}$$

where $H(z)$ is the usual Heaviside function. By the conservation of potential vorticity the discontinuity on the right-hand side of (4) is conserved in time, *i.e.* the potential vorticity can be represented as follows:

$$\Omega=Q+\alpha y\,,\qquad Q=\nabla^2\Psi-\Psi=H(r_0-r)+\alpha q(x,\,y,\,t)\,.\tag{5}$$

Here $q(x,\,y,\,t)$ is a continuous function and the patch boundary $r=r_0(\theta,\,t)$ depends on θ and t.

Substituting (5) into (3) and equating to zero the singular and regular parts in the resulting equation we obtain,

$$\frac{\partial q}{\partial t}+\frac{\partial\Psi}{\partial x}+J(\Psi^*,\,q)=0\,,\tag{6}$$

$$\frac{\partial r_0}{\partial t} + \frac{1}{r_0}\frac{\partial \Psi^*}{\partial r}\bigg|_0 \frac{\partial r_0}{\partial \theta} + \frac{1}{r_0}\frac{\partial \Psi^*}{\partial \theta}\bigg|_0 = 0\,, \tag{7}$$

where $\Psi^* = \Psi + Uy - Vx$ and the subscript "0" means evaluation at $r = r_0(\theta, t)$. The initial conditions are that

$$\Psi(r, \theta, 0) = \Psi_0(r), \qquad \nabla^2\Psi_0 - \Psi_0 = H(1-r), \qquad q(r, \theta, 0) = 0, \qquad r_0(\theta, 0) = 1\,. \tag{8}$$

Now we define a method to determine the translation speed $\mathbf{U} = (U, V)$, based on tracing the centroid of the vortex patch $\mathbf{S}$ bounded by the curve $r = r_0(\theta, t)$. In moving coordinates attached to the centroid $x_c = y_c = 0$, and so, on using the equations,

$$x_c = \frac{\int_S x\,dx\,dy}{\int_S dx\,dy}\,, \qquad y_c = \frac{\int_S y\,dx\,dy}{\int_S dx\,dy} \tag{9}$$

we have

$$\int_0^{2\pi} r_0^3(\theta, t)\cos\theta\,d\theta = 0\,, \qquad \int_0^{2\pi} r_0^3(\theta, t)\sin\theta\,d\theta = 0\,. \tag{10}$$

Equations (10) close the problem (6), (7) and (8).

In the limit $\alpha \ll 1$ of an intense vortex the solution is sought in the following asymptotic form:

$$\Psi = \Psi_0(r) + \alpha\Psi_1(r, \theta, t) + \ldots, \qquad q = q_1(r, \theta, t) + \alpha q_2(r, \theta, t) + \ldots, \tag{11}$$

$$r_0 = 1 + \alpha\bar{r}_1(\theta, t) + \alpha^2\bar{r}_2(\theta, t) + \ldots, \qquad (U, V) = (U_0, V_0) + \alpha(U_1, V_1) + \ldots\,. \tag{12}$$

3. – Asymptotic solution

Substitution of (11) and (12) into the governing equations gives at each order a system of linear equations for the coefficients in the expansions. To solve these systems we use decomposition into Fourier series in the angle variable θ.

The initial streamfunction $\Psi_0(r)$ has the form (fig. 1)

$$\Psi_0 = \begin{cases} -1 + K_1(1)\,I_0(r), & r \leqslant 1\,, \\ -I_1(1)\,K_0(r), & r > 1\,, \end{cases} \tag{13}$$

where $K_n(r)$, $I_n(r)$ are the modified Bessel functions of order n. For the first-order quantities we have

$$q_1 = q_{1s}\sin\theta + q_{1c}\cos\theta, \qquad Q_1 = q_{1s} + iq_{1c} = -\,r(1 - \exp[-i\bar{\Omega}t])\,, \tag{14}$$

$$\bar{r}_1(\theta, t) = 0\,, \qquad \Psi_1 = A_{s1}(r, t)\sin\theta + A_{c1}(r, t)\cos\theta\,, \tag{15}$$

$$U_0 + iV_0 = -A^*(1, t)\,. \tag{16}$$

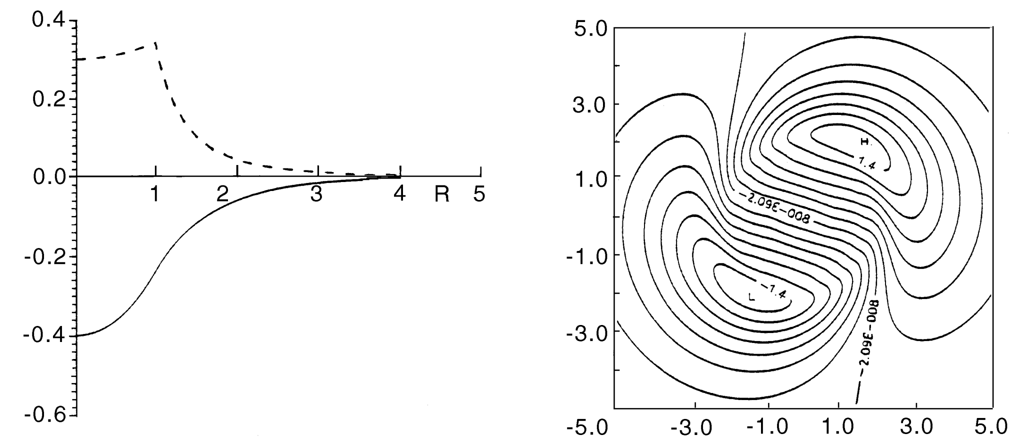

Fig. 1. – The lowest- and the first-order fields. Left: the initial streamfunction $\Psi_0(r)$ (solid) and the angular velocity $\overline{\Omega}(r)$ (dashed). Right: the streamfunction Ψ_1 (β-gyres) at $t = 100$.

Here

$$A = A_{s1} + iA_{c1} = -I_1(r)\int_r^\infty rQ_1 K_1(r)\,\mathrm{d}r - K_1(r)\int_0^r rQ_1 I_1(r)\,\mathrm{d}r\,, \tag{17}$$

and $\overline{\Omega}(r)$ is the angular velocity of the initial vortex, that is $\overline{\Omega}(r) = \partial\Psi_0/r\partial r$. This solution was derived by Sutyrin and Flierl (1994) (with a slightly different notation). Here we consider some important properties of this solution which were not analysed in their paper.

This lowest-order approximation describes the developing beta-gyres (the function Ψ_1 in (15)) gradually amplifying and expanding, with a broadening large-scale approximately rectilinear flow forming in the central region (fig. 1); the vortex patch shape remains unchanged. This flow advects the vortex along the dipole axis, *i.e.* northwestward (see figs. 1 and 4 below) with the translation speed $\mathbf{U} = (U_0, V_0)$ given by (16). The advecting flow appears to be practically uniform in some region near the vortex center since the residual flow $\Psi_{\mathrm{res}} = \Psi_1 + U_0 y - V_0 x$ practically vanishes in some region $r < r_{\mathrm{res}}(t)$ centred at the vortex center, *i.e.*

$$\Psi_{\mathrm{res}} = \Psi_1 + U_0 y - V_0 x \approx 0\,, \qquad r < r_{\mathrm{res}}(t)\,. \tag{18}$$

The size (somewhat arbitrary) $r_{\mathrm{res}}(t)$ of this region monotonically increases with increasing time: $r_{\mathrm{res}}(50) \cong 1$, $r_{\mathrm{res}}(100) \cong 2$. So, as time passes the region becomes larger than the vortex patch.

The smallness of the residual flow $\Psi_1 + U_0 y - V_0 x$ in the main vortex region has been seen in all numerical experiments with localised vortices (*e.g.* Fiorino and Elsberry, 1989) and is of great importance for understanding the long-term vortex evolution. For example, it greatly reduces the Jacobian term in the right-hand side of the equation

$$\frac{\partial q_2}{\partial t} + \overline{\Omega}(r)\,\frac{\partial q_2}{\partial \theta} = -\,J(\Psi_1 + U_0 y - V_0 x,\; q_1 + y) - V_0 \tag{19}$$

for the second-order vorticity correction q_2. According to (14), the spatial derivatives of q_1 grow proportionally to t with increasing t but due to the exponential behaviour of Ψ_0 as $r \to \infty$ the region of rapid growth is concentrated near the vortex center where the growth is compensated by the smallness of Ψ_{res} in the Jacobian. This effect results in a very slow increase of this Jacobian with increasing time. But the right-hand side of (19) multiplied by α^2 is nothing but the main part of the remainder arising when substituting the approximate solution $\Psi_0 + \alpha\Psi_1$ into the basic equation (3). The smallness of the remainder means that this solution can be a good approximation for many turnaround times exceeding the wave time T_{w}. This conclusion is confirmed by numerical experiments performed for various initial vortices in divergent (Sutyrin *et al.*, 1994) and non-divergent (Reznik and Dewar, 1994) models. Obviously, the behaviour of the right-hand side of (19) retards also the growth of the second vorticity correction q_2 which also contributed to the greater "longevity" of the expansions (11) and (12).

One can show that *locally* the approximate solution $\Psi_0 + \alpha\Psi_1$ tends to

$$\widetilde{\Psi}_\infty = \Psi_0\left(\sqrt{(x+\alpha t)^2 + y^2}\right) + \alpha y \qquad \text{as} \qquad t \to \infty\,. \tag{20}$$

The state $\widetilde{\Psi}_\infty$ is an exact solution to eq. (3) (with $U = V = 0$) for an *arbitrary* axisymmetric function $\Psi_0(r)$. One can say that the transformation (20) "kills" the β-effect, which is why an arbitrary axisymmetric vortex moving with the drift velocity on the background of a uniform zonal flow with the same velocity does not radiate any Rossby waves. The tendency of the solution to the state (20) can be considered as a fine example of a transient non-linear self-organisation when the non-linearity and *near-field radiated* Rossby waves (the β-gyres) create the tendency for the vortex to adopt a *non-radiating* state and in doing so retard the vortex decline. Of course, this tendency holds only when the vortex is sufficiently strong.

At the second approximation q_2 is determined from (19), followed by Ψ_2, $\bar{r}_2$ and (U_1, V_1). We shall not give details here (see RG), but note that at this stage the relative vorticity consists of an axisymmetric component $q_{20}(r, t)$ as well as quadrupole components proportional to $\sin 2\theta$ and $\cos 2\theta$. The streamfunction Ψ_2 has an analogous structure. Also it is found that $\bar{r}_2$ contains only quadrupole components, while $(U_1, V_1) = 0$. A plot of q_{20} and Ψ_2 is shown in fig. 2 at a late time in the evolution. In accordance with the potential vorticity conservation, q_{20} is negative in a region around the vortex center. As the vortex travels along the meridian the region becomes wider and "deeper" giving rise to a broadening annulus in the field $Q_0 + q_{20}$ with vorticity opposite in sign to the main vortex. The vorticity in the annulus has a clearly seen fine structure with scale decreasing with increasing time because of the relatively rapid differential rotation in the main vortex for the range $1 \leqslant r \leqslant 2$ (see fig. 1). One can expect that such a structure exists only in the absence of friction which, being included in the model, would result in homogenisation of the vorticity inside the annulus.

The occurrence of such an annulus was qualitatively predicted and demonstrated in some numerical experiments by Sutyrin *et al.* (1994) and Korotaev and Fedotov (1994). Note, however, that this annulus is not alone. It is followed by two broadening rings

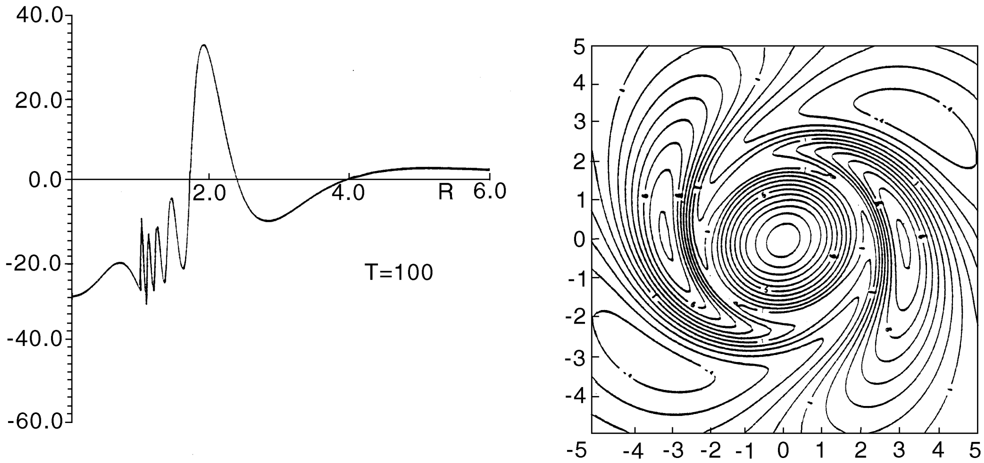

Fig. 2. – The second-order corrections. Left: radial profile of the relative vorticity q_{20}. Right: the streamfunction Ψ_2 at $t = 100$.

with alternating signs. We emphasize also that q_{20} is of the same sign as Q_0 at the vortex periphery, *i.e.* the angle-averaged relative vorticity *increases* with increasing time far from the vortex center. It means that the localized vortex weakens in the central region and intensifies at the periphery, *i.e.* it gradually broadens and "flattens". This behaviour is qualitatively the same as the behaviour of azimuthally averaged perturbation vorticity in the case of a *non-divergent* vortex examined numerically by Smith *et al.* (1995) (see fig. 2 of their paper). Therefore one might expect that this behaviour is typical for vortex evolution on the β-plane.

The second-order correction Ψ_2 is also shown in fig. 2. The developed structure consists of a strong axisymmetric central vortex surrounded by weaker azimuthally elongated four vortex-satellites with alternating signs. The system expands and intensifies with incrasing time, the satellites centres gradually migrating radially outward rather than rotating around the main vortex center. The evolution of vorticity q_2 is quite similar to that of Ψ_2, except that much stronger gradients develop in the vorticity field.

At the third order, there is the generation of secondary β-gyres (*i.e.* dipoles proportional to $\sin\theta$ and $\cos\theta$, but of $O(\alpha^3)$) as well as components proportional to $\sin 3\theta$ and $\cos 3\theta$. The secondary β-gyres are responsible for (U_2, V_2), the $O(\alpha^2)$ corrections to the vortex velocity. It is found that whereas $U_0 < 0$ and $V_0 > 0$, both U_2, $V_2 > 0$. Hence the secondary β-gyres tend to oppose the zonal motion to the west, but to enhance the meridional motion to the north (for more details, see RG).

4. – Energy and enstrophy decay

The energy conservation law for (3) is

$$\int [(\nabla\Psi)^2 + \Psi^2]\, \mathrm{d}s = -\int \Psi Q\, \mathrm{d}s = E_0 = \text{const}\,, \tag{21}$$

where E_0 is the initial energy. Substitution of the expansions (11) and (12) into (21)

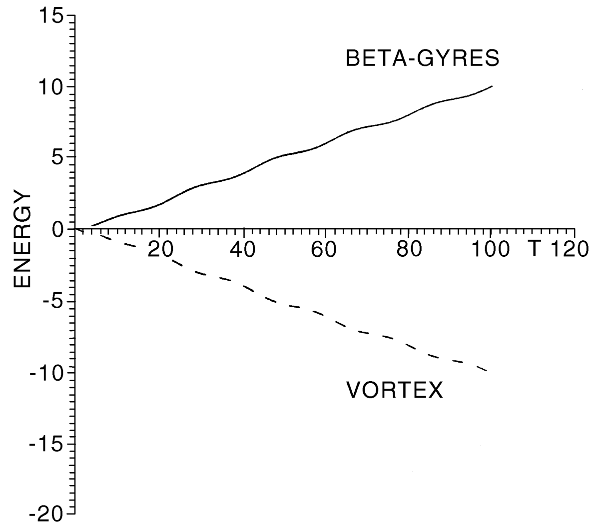

Fig. 3. – The energy $E_\beta(t)$ gained by the beta-gyres and lost by the axisymmetric component.

gives the equation

$$\int (\Psi_2 Q_0 + \Psi_0 q_2)\, \mathrm{d}s + \int \Psi_1 q_1\, \mathrm{d}s = 0 \tag{22}$$

relating the energy of the amplifying β-gyres to the decreasing energy of the axially symmetric vortex component (the second and first terms on the left-hand side of (22), respectively). We find that the dependence of the β-gyres energy $E_\beta(t)$ on the time t is almost linear (fig. 3) and with a good accuracy can be approximated as

$$E_\beta(t) = 2\pi\alpha^2(0.1t)\,. \tag{23}$$

Note that the vortex energy is then $E_0 - E_\beta(t)$.

A qualitative explanation for this linear dependence on t can be found by observing first that only the axisymmetric component Ψ_{20} of Ψ_2 and q_{20} of q_2 contribute to the first term in (22), and second that $Q_0 \equiv 0$ in $r > 1$, while we can approximately put Ψ_0 equal to zero in $r > r_{\mathrm{res}}(t) > 1$ (see fig. 1 and recall that $r_{\mathrm{res}}(t)$ increases with t). Hence, the time rate of decay of the β-gyres is approximately given by

$$\frac{\mathrm{d}E_\beta}{\mathrm{d}t} \approx 4\pi\alpha^2 \int_0^{r_{\mathrm{res}}} \Psi_0\, q_{20t}\, r\, \mathrm{d}r\,, \tag{24}$$

where we have used integration by parts on the first term in (22). But we recall from (19) that the Jacobian term on the right-hand side is approximately zero for $r < r_{\mathrm{res}}$, and so $q_{20t} \approx -V_0$. Hence, it follows that

$$\frac{\mathrm{d}E_\beta}{\mathrm{d}t} \approx 2\pi\alpha^2 I(t)\, V_0(t)\,, \tag{25}$$

where $I(t) = 1 - 2I_1(1)\, r_{\mathrm{res}} K_1(r_{\mathrm{res}})$. Here we have used (13) to evaluate the integral of

$r\Psi_0$. Although this expression is not constant in time, it is shown in (RG) that $r_{\mathrm{res}}(t)$ and $V_0(t)$ are both slowly varying functions of t. Indeed we estimate that at $t = 50$, $IV_0 \approx 0.08$ while at $t = 100$, $IV_0 \approx 0.14$. The agreement with the numerical result (23) is reasonable.

Using (23) one can find the typical lifetime t_{lf} of the vortex as the time when the energy of the axisymmetric component becomes equal to the energy of beta-gyres (*i.e.* $\alpha^2 E_\beta = E_0/2$),

$$t_{\mathrm{lf}} = 0.8/\alpha\beta R_{\mathrm{d}}. \tag{26}$$

According to (26) the lifetimes of mid-oceanic eddies and rings are approximately equal to 130 days and 650 days, respectively. Typical oceanic parameters in midlatitudes are $\beta = 2.10^{-13}\,\mathrm{m}^{-1}\,\mathrm{s}^{-1}$ and $R_{\mathrm{d}} = 45$ km; the typical swirl velocity in mid-oceanic eddies (rings) is assumed to be 20 m/s (1 m/s). Being rather crude (the energy losses caused by the higher azimuthal harmonics are not taken into account) these estimates are quite reasonable and suggest that the energy transfer from the axisymmetric component to the beta-gyres really plays a substantial role in the vortex decay.

Similar estimates for the vortex lifetime can be obtained from the enstrophy conservation law

$$\int Q^2\,\mathrm{d}s = N_0 = \mathrm{const}\,. \tag{27}$$

Substitution of the expansions (11) and (12) now gives

$$\int 2Q_0 q_2\,\mathrm{d}s + \int q_1^2\,\mathrm{d}s = 0\,. \tag{28}$$

We find that the enstrophy of the β-gyres $N_\beta(t)$ is also almost linear in time t, and given by (see (RG))

$$N_\beta(t) = \pi\alpha^2(0.45t)\,. \tag{29}$$

Note that the vortex enstrophy is then $N_0 - N_\beta(t)$. Lifetime estimates based on (29) are slightly larger by a fraction of 1.4 than those given by (26). Since $Q_0 = 0$ for $r > 1$ and $q_{20t} \approx -V_0$ when $r < 1$ for large times, we can obtain the following approximate expression for the rate of change of $N_\beta(t)$ analogously to (25) for $E_\beta(t)$:

$$\frac{\mathrm{d}N_\beta}{\mathrm{d}t} \approx 2\pi\alpha^2 V_0. \tag{30}$$

Using the results from (RG) we find that $V_0 \approx 0.25(0.2)$ for $t \approx 50(100)$, which is in very good agreement with (29).

We conclude that the vortex decay is mainly determined by the growth of the primary β-gyres, and most significantly by the generation of the quasi-steady state (20) in the vortex centre for large times. Although, as shown in (RG), this quasi-steady state essentially holds for t in the range α^{-1} to α^{-2}, and does not persist indefinitely, nevertheless its occurrence seems to be the major factor in determining the near-linear decay in the vortex energy and enstrophy.

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