Effects of reflectivity gradients on rainfall estimates based on specific differential phase measurements (*)

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Summary. — The rainfall estimates R_{DP} based on the specific differential phase shift K_{DP} are unaffected by errors in radar calibration and attenuation along the path. However, due to the signal fluctuations the estimates R_{DP} can be very noisy at low and moderate rain rates. In order to improve the accuracy of the rainfall estimates K_{DP} is to be estimated over a long path. In this way an error due to the reflectivity gradients, which could occur along a long path, can be introduced. In this paper we have analyzed two cases of reflectivity gradients along the path filled with rain; the first one, where the reflectivity varies linearly on dB scale that can be used to approximate regions with a steady increase or decrease of dBZ, and the other corresponding to sharp reflectivity gradient within the measurement cell, where an intense rainshaft is located adjacent in range to weak-echo regions. In both cases the error structure is discussed and the sensitivity of the normalized bias in K_{DP}-based rainfall estimates is evaluated from a theoretical viewpoint and by simulation.

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1. – Introduction

A coherent linear dual polarization weather radar can measure the propagation differential phase shift Φ_{DP} between horizontal and vertical polarization states. Seliga and Bringi [1], Sachidananda and Zrnic [2], and Chandrasekar *et al.* [3] have demonstrated that the rainfall rate R can be estimated utilizing the specific differential phase shift K_{DP} along the path between the measurement cell and the radar. Unlike the methods involving radar reflectivity measurements, the rainfall estimates based on K_{DP} (R_{DP}) are unaffected by errors in radar calibration and attenuation along the path. However, because of signal fluctuations the estimates R_{DP} can be very noisy at low and moderate rain rates. For a fixed number of sample pairs the accuracy of K_{DP} can be improved by estimating K_{DP} over a long path. This average determines a trade-off between resolution and accuracy in the estimates of rainfall rate. On the other hand, an error due to the reflectivity gradients, which might occur along a long path, can be

(*) The authors of this paper have agreed to not receive the proofs for correction.

introduced. This paper analyzes the effect of reflectivity variation on the radar observables Φ_{DP} and K_{DP} and more generally on the error structure of K_{DP}-based rainfall estimates.

2. – Rainfall model

Cloud models and measurements of raindrop size distribution (RSD) at the surface and aloft show that a gamma distribution model adequately describes many of the natural variations in the RSD [4]:

$$N(D) = N_0 D^{\mu} e^{-(3.67+\mu) D/D_0} \qquad (\mathrm{m}^{-3}\,\mathrm{mm}^{-1}), \tag{1}$$

where $N(D)$ is the number of raindrops per unit volume per unit size interval (D to $D + \Delta D$) and N_0, D_0, μ are the parameters of the gamma distribution.

The rainfall rate R and the radar observables of the rain medium, namely ($Z_{H,V}$, K_{DP}), can be expressed in moments of the RDS as

$$R = 0.6\pi \cdot 10^{-3} \int D^3 N(D)\, v(D)\, \mathrm{d}D \qquad (\mathrm{mmh}^{-1}), \tag{2}$$

where $v(D)$ is the fall speed of raindrop, which can be approximated as $v(D) = C_v D^{0.67}$. The radar reflectivity factor can be expressed as

$$Z_{H,V} = \frac{\lambda^4}{\pi^5 |K|^2} \int \sigma_{H,V}(D)\, N(D)\, \mathrm{d}D \qquad (\mathrm{mm}^6\,\mathrm{m}^{-3}), \tag{3}$$

where $Z_{H,V}$ and $\sigma_{H,V}$ represent the reflectivity factors and radar cross-sections at horizontal H and vertical V polarizations, λ the wavelength and $K = (\varepsilon_r - 1)/(\varepsilon_r + 2)$, where ε_r is the dielectric constant of water. Similarly, the specific differential phase can be obtained as

$$K_{DP} = \frac{180\lambda}{\pi} \mathrm{Re} \int [f_H(D) - f_V(D)]\, N(D)\, \mathrm{d}D \qquad (\mathrm{deg\,km}^{-1}), \tag{4}$$

where f_H and f_V are the forward scatter amplitudes at H, V polarization, respectively. The range cumulative differential phase shift Φ_{DP} at the range r_0 can be expressed as

$$\Phi_{DP} = \int_0^{r_0} K_{DP}(r)\, \mathrm{d}r \qquad (\mathrm{deg}). \tag{5}$$

We want to note here that the measurement Φ_{DP} is composed of two contributions, the first due to propagation and the second to backscatter differential phase, which is negligible at S-band. Assuming that raindrops are gamma distributed and their shape can be approximated by oblate spheroids, a linear estimate R_{DP} based on the differential phase shift at S-band is given by Gorgucci *et al.* [5]:

$$R_{DP} = 39.8\, K_{DP}. \tag{6}$$

Other nonlinear parameterizations to estimate R were performed by Sachidananda and Zrnic [2] and by Chandrasekar *et al.* [3]. We can note that the linear estimation has the advantage that it can be used to estimate directly the average rainfall rate over a nonhomogeneous path; however, the coefficients in the power law relationships are quite close to unity. In order to verify the accuracy of the rainfall radar estimates the mean-square error $\varepsilon_{\mathrm{SE}}$ or the error ε_{A} normalized to the average value of rainfall rate R are considered, respectively,

$$\varepsilon_{\mathrm{SE}} = \frac{\left[\frac{1}{M}\sum_{i=1}^{M}(R_{\mathrm{DP}} - R)_i^2\right]^{1/2}}{\frac{1}{M}\sum_{i=1}^{M} R_i}, \tag{7}$$

$$\varepsilon_{\mathrm{A}} = 1 - \frac{\sum_{i=1}^{M}(R_{\mathrm{DP}})_i}{\sum_{i=1}^{M} R_i}, \tag{8}$$

where $(R_{\mathrm{DP}})_i$ represents the i-th K_{DP}-based estimate of rainfall and the average is obtained over M estimates. It is easy to demonstrate that $\varepsilon_{\mathrm{SE}}$ and ε_{A} are related by

$$\varepsilon_{\mathrm{SE}} = \left\{\frac{\langle[(R - \overline{R}) - (R_{\mathrm{DP}} - \overline{R}_{\mathrm{DP}})]^2\rangle}{\overline{R}^2} + \varepsilon_{\mathrm{A}}^2\right\}^{1/2}, \tag{9}$$

where $\langle\,\rangle$ indicates expectation, $\overline{R}$ and $\overline{R}_{\mathrm{DP}}$ are the average values of R and R_{DP}, respectively. Conventionally the parameters (7) or (8) are shown as a function of rainfall rate; however, in this paper we are interested in the effects of reflectivity gradients so that $\varepsilon_{\mathrm{SE}}$ and ε_{A} are here computed as a function of reflectivity factor. Figure 1a shows the bias ε_{A} due to the parameterization of rainfall rate in terms of the observable K_{DP} as a function of horizontal reflectivity factor. It can be noted that the

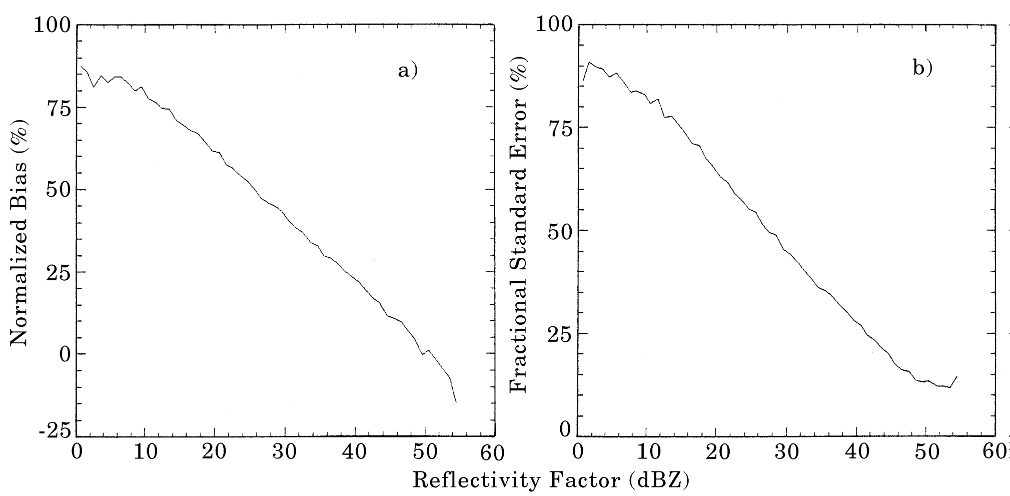

Fig. 1. – a) Normalized bias ε_{A} in K_{DP}-based rainfall rate estimate R_{DP} as a function of the horizontal reflectivity factor Z_{H} expressed in dBZ. b) Fractional Standard Error (FSE) of K_{DP}-based rainfall rate estimate R_{DP} as a function of the horizontal reflectivity factor Z_{H} expressed in dBZ.

high bias for $Z_H \leqslant 25$ dBZ is not a cause for concern due to the fact that the corresponding value of rainfall rate is less than $3\,\mathrm{mm\cdot h^{-1}}$. It can be seen from fig. 1a that R_{DP} underestimates R for $Z_H \leqslant 50$ dBZ and overestimates it for $Z_H \geqslant 50$ dBZ. Figure 1b shows the fractional standard error ε_{SE} of the estimate *vs.* the reflectivity factor. Note that, for the range of reflectivity values here considered, the contribution to the mean-square error due to the bias in the R_{DP} estimate is much higher than the contribution due to the variability of R and R_{DP} around their average values. We can note that ε_{SE} decreases by increasing the reflectivity factor (or the rainfall rate) and there is a minimum in correspondence with the unbiased estimate of rainfall rate ($\varepsilon_A = 0$) in agreement with (9).

3. – Reflectivity gradient effects

Improving the accuracy of the K_{DP}-based rainfall estimates requires that the specific differential phase shift is estimated as the slope of differential phase measurement over long paths. This procedure reduces the range resolution of rainfall rate; on the other hand, as the path length increases the rain medium ceases to be homogeneous. We study the sensitivity of the parameters (7) and (8) with reflectivity gradients along the rain-filled path for two cases, namely a) the one where reflectivity varies linearly on dB scale and b) the other corresponding to sharp reflectivity gradient within the measurement cell. Linear variation of reflectivity in dB scale (dBZ) can be used to approximate regions where there is a steady increase or decrease of dBZ. Step variation in dBZ can be used to describe regions in convective cells where an intense rainshaft is located adjacent in range to weak-echo regions.

3·1. *Constant reflectivity gradient.* – Linear variation of reflectivity in dB scale (dBZ) is commonly encountered [6]. Let us assume that the variation of Z_H occurs along the range r and in the other directions the reflectivity parameters are assumed uniform. It can be seen from (1), (3) and (4) that the exponential variation in linear scale of the radar observables can be easily obtained (in either way) assuming that the parameters D_0 and μ are constant and N_0 varies exponentially along the path of L length as

$$N_0 = N_0^* \exp[0.23\, G_{N_0} r]\,, \tag{9}$$

where N_0^* represents the value of N_0 at the first range gate and G_{N_0} is the corresponding gradient in dB/km. Under those assumptions it can be noted from (3) and (4) that G_{N_0} coincides both with the gradient of reflectivity Z_H and with the gradient of the parameter K_{DP}. The average value of K_{DP} along the path can be obtained by integration of (9) as

$$\langle K_{DP} \rangle = \frac{K_{DP}^*}{0.23\, G_{N_0} L}[\exp[0.23\, G_{N_0} L] - 1]\,, \tag{10}$$

where $\langle\ \rangle$ indicates average or expectation and K_{DP}^* represents the value of K_{DP} at the starting range bin. When the reflectivity factor is uniform along the rainfall path we can estimate the average value of the specific differential phase shift K_{DP} through the least-squares procedure as

$$K_{DP}^{S} = \frac{\sum_{i=1}^{N} [(\Phi_{DP})_i - \overline{\Phi}_{DP}](r_i - \overline{r})}{2 \sum_{i=1}^{N} (r_i - \overline{r})^2}, \tag{11}$$

where K_{DP}^{S} is the estimate of $\langle K_{DP} \rangle$ obtained as the slope of differential phase profile over a path, $(\Phi_{DP})_i$ is the two-way cumulative differential phase shift at the range bin i, Φ_{DP} is the average value of Φ_{DP} along the path, r_i the distance of the range bin i from the radar, $\overline{r}$ the average value of the path length and N the total number of range bins. It is easy to observe that for uniform reflectivity the cumulative differential phase Φ_{DP} is on average linear along the path so that from (11) we can obtain $K_{DP}^{S} = \langle K_{DP} \rangle$ without any bias in the retrieval procedure, as is expected. However, in the presence of exponential variation of reflectivity as described by (9), the parameter Φ_{DP} varies also exponentially as

$$\Phi_{DP} = \frac{K_{DP}^{*}}{0.23\, G_{N_0}} [\exp[0.23\, G_{N_0} r] - 1]. \tag{12}$$

Substituting summation with integral and developing (11) gives

$$K_{DP}^{S} = \frac{12\, K_{DP}^{*}}{(0.23\, \Delta Z)^2} \left[\frac{1}{2} + \frac{1}{0.23\, \Delta Z} + \left(\frac{1}{2} - \frac{1}{0.23\, \Delta Z} \right) \exp[0.23\, \Delta Z] \right], \tag{13}$$

where $\Delta Z = G_{N_0} L$ is the variation of reflectivity along the path. The bias BS_G, introduced by the least-squares procedure and due to the reflectivity gradients, is defined as

$$BS_G = 1 - \frac{K_{DP}^{S}}{\langle K_{DP} \rangle}. \tag{14}$$

Taking account of (13) and (10) BS_G can be written:

$$BS_G = 1 - \frac{\frac{12}{0.23\, \Delta Z} \left[\frac{1}{2} + \frac{1}{0.23\, \Delta Z} + \left(\frac{1}{2} - \frac{1}{0.23\, \Delta Z} \right) \exp[0.23\, \Delta Z] \right]}{\exp[0.23\, \Delta Z] - 1}. \tag{15}$$

Figure 2 shows the bias BS_G, computed from (15), as a function of the reflectivity variation ΔZ expressed in dB. We can note that K_{DP}^{S} underestimates $\langle K_{DP} \rangle$; moreover, the bias (14) depends only on the total variation of reflectivity ΔZ along the rain path and increases by increasing ΔZ up to approximately 40% for $\Delta Z = 30$ dB. We want to point out that BS_G is independent of the mean value of the reflectivity because, under the above assumptions, the radar observables Z_H and K_{DP} have the same variation along the path. Generally, this is not true and BS_G depends on the average value of reflectivity too. Moreover, it can be observed from (15) that the bias is independent of the sign of the reflectivity gradient. This interesting feature implies that for a fixed mean value of reflectivity a linear increase of dBZ introduces the same bias as a linear

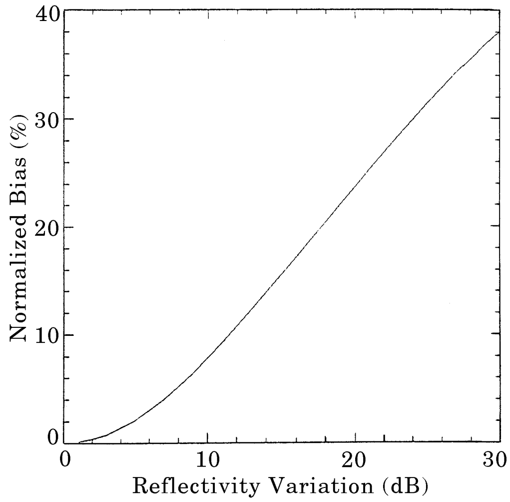

Fig. 2. – Normalized bias BS_G, analytically computed from (15), in the estimate of K_{DP} due to the least-squares procedure, as a function of the reflectivity variation ΔZ along the path where the reflectivity (dBZ) is varied linearly.

decrease of dBZ. Finally, the bias on K_{DP}-based rainfall estimates, which is introduced by the least-squares procedure, is still described by (15), due to the linearity between K_{DP} and the rainfall estimate R_{DP}.

3˙2. *Reflectivity variation due to beam filling.* – Let us consider a sharp reflectivity variation along the path due to beam filling. Assuming that f is the fraction of path where the reflectivity can be considered low and $1-f$ the portion of the path filled with intense rainshaft, it can be easily demonstrated that the bias BS_G in the estimate of K_{DP}, utilizing the slope of differential phase measurements over a path, is given by

$$BS_G = 1 - \frac{f(4f^2-9f+6)-\tau(4f^3-9f^2+6f-1)}{f+\tau(1-f)}, \tag{16}$$

where $\tau = 10^{\Delta Z/10}$ is the reflectivity step expressed on linear scale. It can be observed from (16), that the bias BS_G is equal to zero for uniform reflectivity ($f=0$ or $f=1$) and for $f=0.5$ due to symmetry reasons. Figures 3a and b show the bias BS_G, computed from (16) as a function of the fraction f of path with low reflectivity for the reflectivity jumps $\tau = 10$ and 100, which correspond to 10 and 20 dB on logarithmic scale, respectively. It is interesting to note that the algorithm K_{DP}^S overestimates the mean value of specific differential phase for $f \leqslant 0.5$; the overestimation is maximum at $f=0.25$. On the other hand, for $f \geqslant 0.5$ the bias is positive and K_{DP}^S underestimates the average value of K_{DP}. From the comparison of fig. 3a with fig. 3b one can observe that for $f \leqslant 0.5$ the bias is quite independent of the reflectivity jump τ; for $f \geqslant 0.5$ both the maximum of underestimation and the corresponding value of the fraction path f increase by increasing τ.

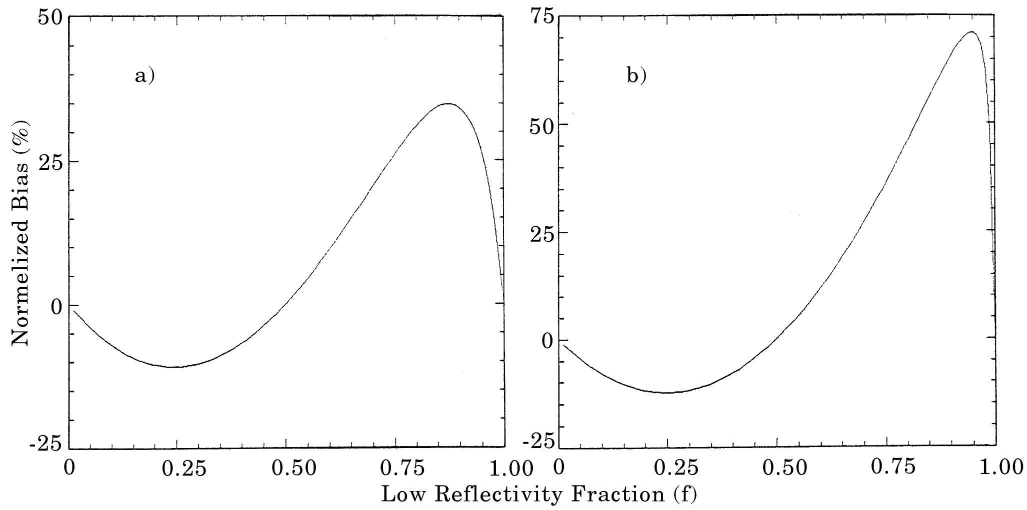

Fig. 3. – a) Normalized bias BS_G, analytically computed from (16), in the estimate of K_{DP} due to the least-squares procedure, as a function of the low reflectivity fraction f for a reflectivity step of 10 dB. b) Normalized bias BS_G, analytically computed from (16), in the estimate of K_{DP} due to the least-squares procedure, as a function of the low reflectivity fraction f for a reflectivity step of 20 dB.

4. – Simulation analysis

The theoretical results described in sect. **3** are obtained assuming that the variation of reflectivity is due to the parameter N_0 only, while D_0 and μ are kept constant. The general treatment is mathematically difficult. In this section we show the results obtained by simulation varying the three parameters of the drop size distribution.

We have studied a standard 1 km path with a 50 m resolution, where the reflectivity variation ranges between 0 to 30 dB and the minimum and maximum reflectivity along the path are equal to 25 and 55 dBZ, respectively. We note here that the resolution path length and the reflectivity gradient can be easily scaled to suit any experimental situation. The reflectivity profile is assumed linear on dB scale. Once the reflectivity is fixed at a resolution cell, the parameters of the RSD, namely N_0, D_0, and μ, are chosen randomly within the limits suggested by Ulbrich [4] and under the constraint that the RSD yields the current reflectivity value in the range bin, whereas the drop axis ratio is described by the Pruppacher and Pitter [6]. Subsequently, the values of K_{DP}, Φ_{DP} and the other radar observables are computed for each range location. As we are interested to study the influence of reflectivity variation on the measured K_{DP} and the derived rainfall rate estimates R_{DP}, we have to separate the effects due to the reflectivity averages from the effects due to the reflectivity variations. In this light the comparison between different reflectivity fields has been performed with the same mean value expressed in linear scale.

In this frame the error on the estimate K_{DP}^S of $\langle K_{DP} \rangle$, obtained as the slope of differential phase profile over a path and in the presence of nonuniform reflectivity, can

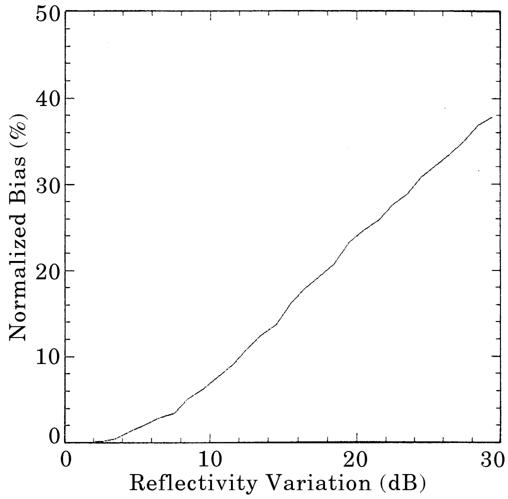

Fig. 4. – Simulated normalized bias BS_{G} in the estimate of K_{DP} due to the least-squares procedure, as a function of the reflectivity variation ΔZ along the path where the reflectivity (dBZ) is varied linearly with a mean reflectivity of 25 dBZ.

be expressed as

$$BS_{\mathrm{G}}^{*} = 1 - \frac{K_{\mathrm{DP}}^{\mathrm{S}}}{(K_{\mathrm{DP}}^{\mathrm{S}})_{Z\,=\,\mathrm{unif}}}\,. \tag{17}$$

Note that for Z uniform along the path it is easy to demonstrate that on average $\langle K_{\mathrm{DP}} \rangle_{Z\,=\,\mathrm{unif}} = (K_{\mathrm{DP}}^{\mathrm{S}})_{Z\,=\,\mathrm{unif}}$, so that the bias BS_{G} described by (14) coincides with the bias BS_{G}^{*} in eq. (17). Figure 4 shows the bias BS_{G} as a function of the reflectivity variation at

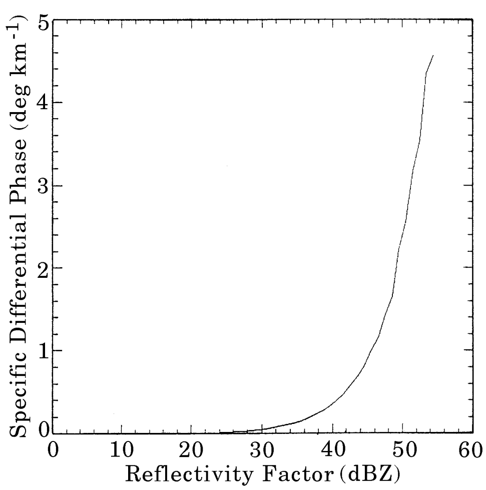

Fig. 5. – Average specific differential phase K_{DP} as a function of the average reflectivity factor Z_{H} in dBZ.

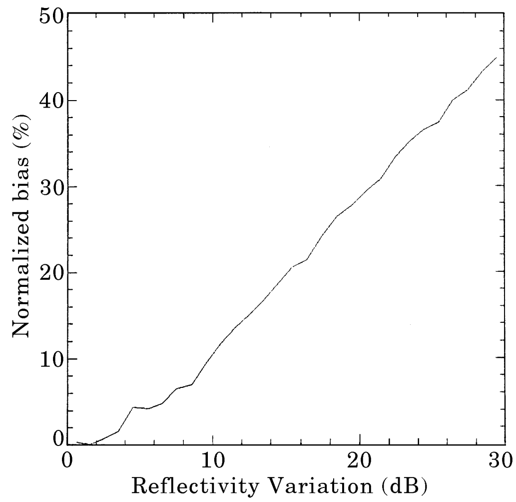

Fig. 6. – Simulated normalized bias BS_G in the estimate of K_{DP} due to the least-square procedure, as a function of the reflectivity variation ΔZ along the path where the reflectivity (dBZ) is varied linearly with a mean reflectivity of 40 dBZ.

1 dB step for mean reflectivity equal to 25 dBZ. In this case the reflectivity spans between 10 and 40 dBZ, which corresponds approximately to light-moderate (less than 50 $\mathrm{mm\,h^{-1}}$) rain rate. It can be noted that the theoretical curve of fig. 2 fits fairly well with the curve of fig. 4 obtained by simulation; moreover, the bias due to variation up to 5 dB can be considered negligible and for variation greater than 10 dB the bias BS_G increases quite linearly. We have also analyzed the effect on K_{DP} estimates due to the variation of the mean reflectivity for the same reflectivity variation. Figure 5 shows the average relationship between the specific differential phase K_{DP} and the reflectivity factor on dB scale obtained by varying the raindrop size distribution within the limits suggested by Ulbrich [4]. It can be observed from fig. 5 that the lower the mean reflectivity, the lower the variation of K_{DP} for the same reflectivity variation; this consideration suggests that the bias BS_G should decrease by decreasing the mean reflectivity. Figure 6 shows the bias BS_G as a function of the reflectivity variation for mean reflectivity equal to 40 dBZ; in that case the reflectivity ranges between 25 and 55 dBZ, which corresponds to moderate-heavy rain rate. Comparison of fig. 4 with fig. 6 shows the agreement with the theoretical results; moreover, it can be seen that the variation of the bias BS_G due to the mean reflectivity ranges from 0 at uniform reflectivity to 10% at reflectivity variation of 30 dB.

5. – Error on K_{DP}-based rainfall estimates over a path

The total mean normalized bias ε_T in K_{DP}-based estimates of rainfall over a nonhomogeneous path can be generally written as

$$\varepsilon_T = 1 - \frac{\langle R^S_{DP} \rangle}{\langle R \rangle} = 1 - \frac{\langle R^S_{DP} \rangle}{\langle R^S_{DP} \rangle_{Z=\mathrm{unif}}} \, \frac{\langle R^S_{DP} \rangle_{Z=\mathrm{unif}}}{\langle R \rangle_{Z=\mathrm{unif}}} \, \frac{\langle R \rangle_{Z=\mathrm{unif}}}{\langle R \rangle}\,, \tag{18}$$

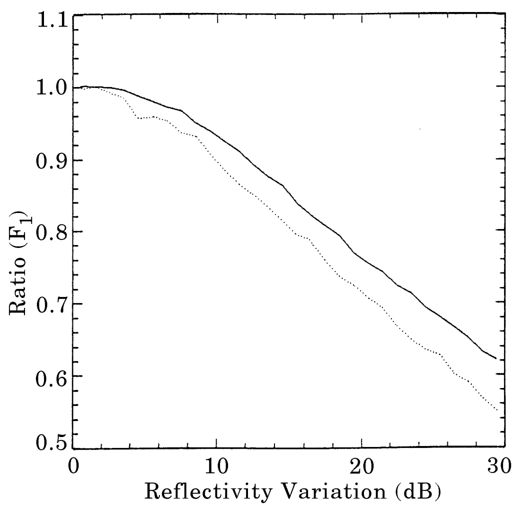

Fig. 7. – Ratio F_1 between the averages of the rainfall estimate R_{DP} for nonuniform and uniform reflectivity as a function of the reflectivity variation ΔZ along the path with a mean reflectivity of 25 dBZ (solid line) and 40 dBZ (dotted line), respectively.

where $\langle R^{\mathrm{S}}_{\mathrm{DP}} \rangle$ and $\langle R^{\mathrm{S}}_{\mathrm{DP}} \rangle_{Z=\mathrm{unif}}$ represent the mean values of the rainfall estimate $R^{\mathrm{S}}_{\mathrm{DP}}$ in the presence of reflectivity gradient and for uniform reflectivity along the path, respectively; $\langle R \rangle$ and $\langle R \rangle_{Z=\mathrm{unif}}$ are the corresponding mean values of the true rainfall rate R. It should be noted that the estimate $R^{\mathrm{S}}_{\mathrm{DP}}$ is an average rainfall over a path, whereas R represents a pointwise measurement; in this light the accuracy of these estimates can be quite different.

The ratio $F_1 = \langle R^{\mathrm{S}}_{\mathrm{DP}} \rangle / \langle R^{\mathrm{S}}_{\mathrm{DP}} \rangle_{Z=\mathrm{unif}}$ between the averages of the estimate $R^{\mathrm{S}}_{\mathrm{DP}}$ for nonuniform and uniform reflectivity can be easily obtained from the bias (17) because the estimate R_{DP} is linearly related to the radar observable K_{DP} and is given by

$$F_1 = 1 - BS^*_{\mathrm{G}} . \tag{19}$$

Figure 7 shows the ratio F_1 as a function of the reflectivity variation for two different mean reflectivities, respectively 40 and 25 dBZ. As discussed in sect. **3**, it can be seen from fig. 7 that the ratio F_1 is equal to 1 for uniform reflectivity and decreases down to approximately 0.62 and 0.55, respectively, at the reflectivity variation of 30 dB. $F_2 = \langle R^{\mathrm{S}}_{\mathrm{DP}} \rangle_{Z=\mathrm{unif}} / \langle R \rangle_{Z=\mathrm{unif}}$ represents the ratio between the mean values of rainfall estimate $R^{\mathrm{S}}_{\mathrm{DP}}$ and the true rainfall R in the case of uniform reflectivity; this factor takes account of the effects due to the drop size distribution which determines the parameterization (6). For the two values of mean reflectivity here considered, that is 40 and 25 dBZ, it can be seen from fig. 1 that the estimate $R^{\mathrm{S}}_{\mathrm{DP}}$ on average underestimates the rainfall rate and the ratio F_2 is equal to 0.77 and 0.48, respectively. $F_3 = \langle R \rangle_{Z=\mathrm{unif}} / \langle R \rangle$ represents the ratio between the mean values of rainfall rate R for uniform reflectivity and with reflectivity gradients, respectively. Of course, if the relationship between rainfall and reflectivity were linear the ratio F_3 should be 1. However, the R-Z relationships are nonlinear with a relation of the form $R = aZ^b$. Because the exponent b in the power law relation is less than 1, then it is easy to demonstrate that the mean rainfall rate for uniform reflectivity is greater than the mean rainfall rate for

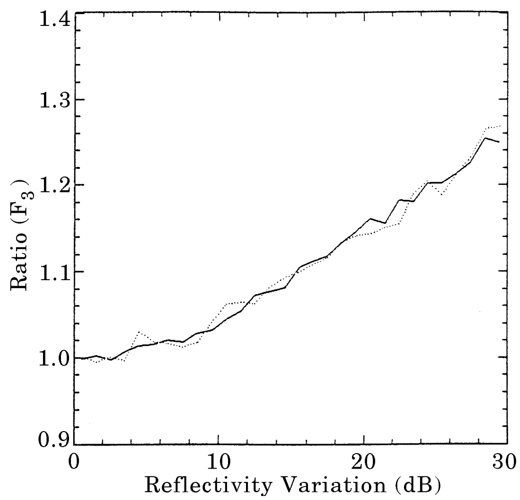

Fig. 8. – Ratio F_3 between the mean values of the rainfall rate R for uniform and nonuniform reflectivity as a function of the reflectivity variation ΔZ along the path with a mean reflectivity of 25 dBZ (solid line) and 40 dBZ (dotted line), respectively.

nonuniform reflectivity. This effect is slightly more evident by increasing the mean reflectivity along the path. As is shown in fig. 8, the ratio F_3 is 1 for uniform reflectivity; at the reflectivity variation of 30 dB F_3 increases up to 1.23 for mean reflectivity of 25 dBZ and 1.28 for mean reflectivity of 40 dBZ.

In conclusion, we can say that for uniform reflectivity F_1 and F_3 are approximately equal to 1 and the total normalized error in the estimate of mean rainfall is due to the

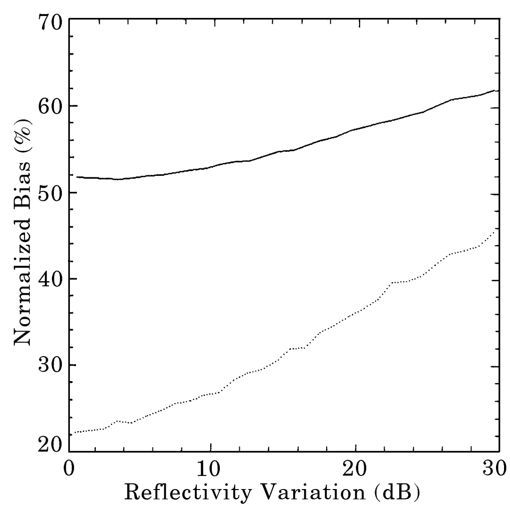

Fig. 9. – Normalized total bias ε_T in the rainfall estimate R_{DP} as a function of the reflectivity variation ΔZ along the path with a mean reflectivity of 25 dBZ (solid line) and 40 dBZ (dotted line), respectively.

parameterization given by (6) and decreases by increasing the mean reflectivity along the path, as seen in fig. 1. By increasing the reflectivity variation F_1 and F_3 have opposite behavior; however, the contribution to error due to F_1 is greater than F_3, and it increases by increasing the mean reflectivity along the path. Figure 9 shows the normalized total bias in the K_{DP}-based rainfall estimate as a function of reflectivity variation for mean reflectivity values of 25 and 40 dBZ. That bias is approximately 52% and 22%, respectively, for uniform reflectivity and it increases up to 62% and 46% for reflectivity variation of 30 dB.

6. – Summary and conclusions

In this paper we have analyzed the error introduced on K_{DP} estimates for two cases, namely a) when reflectivity varies linearly on dB scale and b) corresponding to sharp reflectivity gradient within the measurement cell. In the first case the results show that the estimate K_{DP}^{S} obtained as the slope of differential phase profile over a path underestimates the average value of K_{DP}; the corresponding bias BS_G increases by increasing the reflectivity variation ΔZ up to approximately 40% for $\Delta Z = 30$ dB. Our analysis demonstrates that BS_G is slightly dependent on the average value of reflectivity and is independent of the sign of the reflectivity gradient. For the case where a sharp reflectivity gradient is present, the bias is positive when the fraction f of path with low reflectivity is less than 0.5; for f greater than 0.5 the bias is negative and increases by increasing the reflectivity jump τ up to 70% for a sharp reflectivity variation of 20 dB. The same results are obtained for the bias on K_{DP}-based rainfall estimates due to the reflectivity gradients because of the linearity of the relationship between K_{DP} and rainfall rate.

Finally, we have analyzed the total mean normalized bias ε_T in K_{DP}-based estimates of rainfall in the presence of reflectivity variation for two different mean reflectivity values equal to 25 and 40 dBZ, respectively. The bias can be due to different reasons, namely a) bias BS_G due to the nonuniform path over which K_{DP} is estimated, b) bias B_P due to the parameterization of rainfall rate in terms of the radar observable K_{SP} and c) bias B_{NL} due to the non linearity between the reflectivity factor and the rainfall rate. We have found that the bias due to the parameterization gives the most important contribution to the error ε_T mostly at reflectivity values less than 40 dBZ; this result suggests to evaluate the parameterization error of a power law equation of the form $R(K_{DP}) = \alpha K_{DP}^{\beta}$ taking account of the error introduced by nonlinearity to the estimate rainfall rate over a path. As discussed above, the value and the sign of bias BS_G are closely related to the reflectivity field within the nonhomogeneous path; from our analysis we have found that the worst situation happens for very sharp reflectivity gradient, when an intense rainshaft is located adjacent in range to weak-echo regions. The bias B_{NL} is due to the nonlinearity of the relationship Z-R, because the mean rainfall rate for uniform reflectivity is greater than the mean rainfall rate for nonuniform reflectivity. This error ranges between 0 at uniform reflectivity and 25% for reflectivity variation up to 30 dB. Finally, we have found that for uniform reflectivity the total bias ε_T is due to the parameterization of the relation Z-R. By increasing the reflectivity variation the bias BS_G and the bias B_{NL} have opposite behaviour; however, ε_T increases because the contribution due to BS_G is greater than the one due to the bias B_{NL}.

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