# **Optimization of the configuration in a CAES-TES system**

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### Abstract

A compressor with heat recovery is thermodynamically analyzed from a second-law point of view, in terms of entropy generation minimization. The unit is optimized, accounting for heat exchange and pressure loss irreversibility, and the maximum output is found, in terms of combined mechanical and thermal energy, for a given input energy at the compressor shaft.

This analysis is intended as a part of the thermal optimization on a CAES TES (Compressed Air Energy Storage with Thermal Energy Storage) plant. The latter, if an artificial reservoir is used for compressed air, requires high operating pressure. Hence, this work focuses on a staged compression with intermediate refrigeration. The principle of equal distribution of irreversibility, which is one of the aspects of the "Constructal Law" is then demonstrated. From these findings, a good approximation can be found for the optimal configuration of a complex multi-stage system.

### Nomenclature

A	Area [m <sup>2</sup> ]	Greek	
С	Flow heat capacity [W K <sup>-1</sup> 1]	β	Volumetric expansion coefficient
С	Specific heat [J kg <sup>-1</sup> K <sup>-1</sup> ]	β	Compression ratio
D	Diameter [m]	γ	Specific heat ratio
E	Energy [J]	, ε	Heat exchanger efficiency
ex	Exergy [J]	η	Efficiency
h	Specific enthalpy [J kg <sup>-1</sup> ]	1	-
L	Length [m]		
ṁ	Mass flow rate [kg s <sup>1</sup> ]	subscripts	
N	Entropy generation index	0	Ambient
п	Polytropic exponent	С	Compressor
NTU	Number of heat transfer units	env	Environment
р	Pressure [Pa]	gen	Generated
Q	Heat [J]	i	Inlet
r	Temperature ratio	0	Outlet
R	Ideal gas constant [J kg <sup>-1</sup> K <sup>-1</sup> ]	pol	Polytropic
S	Entropy [J K <sup>-1</sup> ]	rev	Reversible
t	Time [s]		
Т	Temperature [K]		
v	Specific volume [m <sup>3</sup> kg <sup>-1</sup> ]	superscripts	
W	Work [J]	0	Total (thermal + kinetic + potential)
Z.	Heat capacity ratio		

### 1. Introduction

In the present energy situation Compressed Air Energy Storage (CAES) is receiving new interest, due to (hopefully) increasing share of renewable energy sources and market liberalization. Among various energy storage systems, CAES has been proposed for the high-energy, high-power end of the range, but practical realizations, at now, are just two [1,2]. Many authors have guessed that CAES could be useful in smaller sizes, as a complement, for example, to a wind farm or any other renewable power plant [3].

The two existing CAES plant include a combustion chamber where compressed air is pre-heated before expansion. Hence, they consume fossil fuels and emit pollutants. Moreover, the thermal energy released by the air after its compression is wasted, reducing the system efficiency.

These two problems have prompted other proposals [4], based on the integration between a CAES and a Thermal Energy Storage (TES). Such integration can solve both issues: thermal energy is drawn from compressed air during the energy storage phase and released to expanding air during the energy recovery phase. As a whole, such a plant is adiabatic, that is free of emissions and fuel consumption.



Figure 1 – Scheme of CAES TES plant

On the other hand, a CAES-TES system has many degrees of freedom and claims for a complex thermal optimization. Some aspects of this optimization have been presented elsewhere [5, 6]. For example, it has been shown that an artificial compressed-air storage has a decreasing cost with increasing storage pressure.

The variation of storage pressure during charge/discharge can be addressed by a variable configuration of the multi stage compression-expansion train, as shown for example in [6]. Here the focus is on heat transfer problems, accounting for pressure losses in the heat exchangers and trying to optimize the whole system configuration

#### 2. Background

As shown for example in [7], for a system exchanging heat with various sources at different temperatures  $T_i$  (including the environment at  $T_0$ ) and mass flows with different enthalpy, kinetic and potential energy through various inlet and outlet ports (including a port toward the environment), the energy and entropy balance are:

$$\frac{dE}{dt} = \sum_{i=0}^{n} \dot{Q}_{i} - \dot{W} + \sum_{in} \dot{m}h^{0} - \sum_{out} \dot{m}h^{0}$$

$$\dot{S}_{gen} = \frac{dS}{dt} - \sum_{i=0}^{n} \frac{\dot{Q}_{i}}{T_{i}} - \sum_{in} \dot{m}s + \sum_{out} \dot{m}s \ge 0$$
(1)

In these equations, E is the system internal, kinetic and potential energy, S is its entropy and  $h^0$  is the total enthalpy (including kinetic and potential terms) of the inlet/outlet flows. Solving for the heat and mass transfers towards the environment and summing the two balances, the maximum work can be calculated as:

$$\dot{W} = -\frac{d}{dt}(E - T_0 S) + \sum_{i=1}^{n} \left(1 - \frac{T_0}{T_i}\right) \dot{Q}_i + \sum_{in} \dot{m} \left(h^0 - T_0 S\right) - \sum_{out} \dot{m} \left(h^0 - T_0 S\right) - T_0 \dot{S}_{gen}$$
(2)

Maximum work is obtained through reversible process and by definition is:

$$\dot{W}_{rev} = -\frac{d}{dt}(E - T_0 S) + \sum_{i=1}^n \left(1 - \frac{T_0}{T_i}\right) \dot{Q}_i + \sum_{in} \dot{m} (h^0 - T_0 S) - \sum_{out} \dot{m} (h^0 - T_0 S)$$
(3)

The system optimization is then pursued by minimizing the term:

$$\dot{W} - \dot{W}_{rev} = T_0 \dot{S}_{gen} \tag{4}$$

In the case of a CAES-TES system, both mechanic and thermal energy are stored. The system objective is to reach the maximum energy recovery efficiency, that is the ratio between compression and expansion work.

The elemental system unit is a compressor coupled with a heat exchanger (or an exchanger coupled with a turbine stage). The best working conditions are sought for this unit and the relation between this elemental optimum and the complete system optimum is investigated.

By the way, such analysis may be useful beyond the specific CAES study, being the compressed air production a primary process in any industry and being the heat recovery from compressed air a significant opportunity, if low-temperature thermal loads are to be covered.

#### **3.** Heat Exchanger optimization

Heat exchanger optimization has been dealt with by many authors [8, 9]. For example, the following analysis is a generalization of the one presented in [10]. Given a heat exchanger between a hot fluid 1 and a cold fluid 2, neglecting any energy loss towards the environment, the entropy production is:

$$dS = \dot{m}_1 \, ds_1 + \dot{m}_2 \, ds_2 \tag{5}$$

Being the entropy variation:

$$ds = c_p \frac{dT}{T} - \left(\frac{\partial v}{\partial T}\right)_p dp \tag{6}$$

by integration from inlet to outlet, one has:

$$\Delta S = \dot{m}_1 \left\{ c_{p1} \ln \left( \frac{T_o}{T_i} \right)_1 - \int_i^o \left( \frac{\partial v}{\partial T} \right)_{1,p} dp_1 \right\} + \dot{m}_2 \left\{ c_{p2} \ln \left( \frac{T_o}{T_i} \right)_2 - \int_i^o \left( \frac{\partial v}{\partial T} \right)_{2,p} dp_2 \right\}$$
(7)

where specific heats are averaged between inlet and outlet temperatures.

For a liquid: 
$$\left(\frac{\partial v}{\partial T}\right)_p = \beta v = \cos t$$

$$\int_{i}^{o} \left(\frac{\partial v}{\partial T}\right)_{p} dp = \int_{i}^{o} \frac{R}{p} dp = R \ln \frac{p_{o}}{p_{i}}$$

For an ideal gas:

In the second expression, we can introduce the pressure loss  $\Delta p = p_i - p_o$  and consider that  $\Delta p \ll p_i$  in order to reduce the logarithm to its first order Taylor series:

$$\int_{i}^{o} \left(\frac{\partial v}{\partial T}\right)_{p} dp = R \ln\left(1 + \frac{\Delta p}{p_{i}}\right) \approx \frac{R}{p_{i}} \Delta p$$

Therefore, in both cases we have:

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$$\int_{l}^{o} \left(\frac{\partial v}{\partial T}\right)_{p} dp = l\Delta p \qquad \text{being} \quad l = \beta v \text{ for liquids and } l = R/p_{i} \text{ for ideal gasses}$$

Introducing Kays efficiency: 
$$\varepsilon = \frac{C_1(T_{i1} - T_{o1})}{C_1(T_{i1} - T_{i2})} = \frac{C_2(T_{o2} - T_{i2})}{C_1(T_{i1} - T_{i2})}$$

assuming  $c_{p1}m_1 = C_1 = C_{min}$  and  $c_{p2}m_2 = C_2 = C_{max}$ , and introducing the ratios  $r = T_2/T_1$  and  $z = C_1 / C_2$ we have:  $(T_o/T_i)_1 = 1 - (T_{i1} - T_{o1}) / T_{i1} = 1 - \varepsilon (T_{i1} - T_{i2}) / T_{i1} = 1 - \varepsilon (r - 1)/r$ and  $(T_o/T_i)_2 = 1 + (T_{o2} - T_{i2}) / T_{i2} = 1 + \varepsilon z (T_{i1} - T_{i2}) / T_{i2} = 1 + \varepsilon z (r - 1)$ 

The entropy variation therefore is:

$$\Delta S = C_1 \left[ ln \left( \frac{T_o}{T_i} \right)_1 - \frac{l_1 \Delta p_1}{c_{p_1}} \right] + C_2 \left[ ln \left( \frac{T_o}{T_i} \right)_2 - \frac{l_2 \Delta p_2}{c_{p_2}} \right] =$$

$$= C_1 \left\{ ln \left[ 1 - \varepsilon \frac{r - 1}{r} \right] + \frac{1}{z} ln \left[ 1 + \varepsilon z (r - 1) \right] - \frac{l_1 \Delta p_1}{c_{p_1}} - \frac{l_2 \Delta p_2}{z c_{p_2}} \right\}$$
(8)

This quantity has two contributions, accounting for heat exchange and pressure loss irreversibility. Entropy generation can be divided by the minimum heat capacity  $C_{min}$ , so introducing the non-dimensional entropy generation index:

$$N_S = \Delta S/C_{min} = N_{SP} + N_{S\varepsilon} \tag{9}$$

where:

$$N_{SP} = -\frac{l_1 \Delta p_1}{c_{p1}} - \frac{l_2 \Delta p_2}{c_{p2}}$$

$$N_{S\varepsilon} = ln \left[ 1 - \varepsilon \frac{r-1}{r} \right] + \frac{1}{z} ln \left[ 1 + \varepsilon z (r-1) \right]$$
(10)

As a rule, any improvement in terms of heat exchange gives increased pressure losses and viceversa. Therefore, we expect the two contributions  $N_{SP}$  and  $N_{s\varepsilon}$  to have opposite trend and their sum to have a minimum. Actually this may often be untrue, and so other entropy generation indexes have been introduced. For example, the entropy generation  $\Delta S$  can be related to its maximum value, which would be attained if heat were directly transferred between the farthest system temperatures  $T_{i1} \in T_{i2}$ . Such entropy generation would be:

$$\Delta S_Q = Q(1/T_{i1} - 1/T_{i2})$$

Another entropy generation index is hence defined [10]:

$$N_Q = \Delta S / \Delta S_Q = (N_S \varepsilon) [r/(r-1)^2]$$
(11)

In view of application to a CAES, we focus on a counterflow, shell and tube heat exchanger, with air flowing inside the tubes and refrigerant around them. Optimization is performed with fixed heat exchange area and length and variable number of tubes. Concentrated pressure losses are neglected.

The TES fluid is water, at suitable pressure in order to avoid evaporation. Water enters the exchanger at ambient temperature. Air and water properties are evaluated by interpolation from data found in [11].

Air conditions are those calculated at the exit of a compressor with pressure ratio  $\beta = 3$  and polytropic efficiency  $\eta_{pol} = 0.92$ .

Table 1 – Heat exchanger data								
$T_{\rm air}$	422	Κ	$T_{\rm water}$	288	Κ			
$p_{\rm air}$	304	kPa	$p_{\text{water}}$	101	kPa			
$m_{\rm air}$	1	kg/s	mwater	0.2432	kg/s			
L	10	m	Α	25	$m^2$			

Other variables are the heat capacity ratio  $z = C_1/C_2$  and the shell diameter. This latter, given the low pressure loss on the water side, is set to the minimum geometrical value for the given number of tubes. This minimum has been graphically evaluated and can be expressed in the form  $D_2 = y D_1 n^x$ , being  $y \approx 1.3$  and  $x \approx 0.47$ . In practice we assume y = 1.35, in order to avoid direct contact between tubes.



Figure 2 – Entropy generation index  $N_s N_{s\varepsilon}$  and  $N_{sp}$  for a water-air, shell and tube heat exchanger - data from table 1, z = 1

Even if water flow section has been set to a minimum value, the entropy contribution due to water pressure loss is always less than 1% of that due to air pressure loss.

The  $N_s$  index in figure 2 has a relative minimum, but it also shows an upward slope for NTU < 1. Even if this zone must be discarded for design purpose, it makes the optimization result less evident. Moreover, at certain values of the design parameters, the minimum disappears. Therefore, the  $N_0$  index is more convenient, having always a minimum as shown in figure 3.

The minimum position slightly changes with z between NTU = 3.5 and NTU = 4. Further data are given in table 2.

Table 2 – Optimum heat exchanger design for various $z$								
Z.	$N_Q$	NTU	n. of tubes	$D_{\text{Tube}} [\text{mm}]$	D <sub>Shell</sub> [mm]			
0.3	0.446	3.53	31	26	172			
0.5	0.362	3.77	33	24	166			
0.8	0.283	4.00	35	23	161			
1	0.263	3.89	34	23	163			



Figure 3 – Entropy generation index  $N_Q$  v/s NTU for various z values

The  $N_Q$  index decreases with z, even if curves become closer as this parameter approaches unity. Assuming z = 1 and optimum design, heat exchanger efficiency is  $\varepsilon = 0.795$ .

#### 4. Entropy generation in the compressor

The compressor can be modeled by a polytropic transformation  $pv^n = \text{cost.}$  For a perfect gas, the entropy generation along this transformation (which, by definition, is equal to that of the real compression) is given by:

$$ds = c_p \frac{dT}{T} - R \frac{dp}{p}$$

Coupling the polytropic and the ideal gas relation we have:  $\frac{dT}{T} = \frac{n-1}{n} \frac{dp}{p}$ . Substituting:

$$ds = c_p \left(\frac{n-1}{n} - \frac{\gamma - 1}{\gamma}\right) \frac{dp}{p}$$

where  $\gamma = c_p / c_v$ . Introducing the compression ratio  $\beta$  and integrating:

$$\Delta s = c_p \left( \frac{n-1}{n} - \frac{\gamma - 1}{\gamma} \right) ln\beta$$
(12)

The entropy increment can hence be related to the polytropic exponent n, which in turn is related to the polytropic efficiency:

$$\eta_{pol} = \frac{n}{n-1} \frac{\gamma-1}{\gamma}.$$

Therefore:

$$\Delta s = c_p \frac{\gamma - 1}{\gamma} \left( \frac{1 - \eta_{pol}}{\eta_{pol}} \right) ln \beta$$
(13)

Use of polytropic efficiency is correct for a compression, as it measures the "quality" of the transformation independently from  $\beta$ . Again, the entropy generation can be translated in a non-dimensional entropy generation index  $N_{s-comp} = \Delta s / c_p$ .

Figure 4 shows that  $N_{s-comp}$  increases with  $\beta$  and decreases with the compressor polytropic efficiency, as expected.



Figure 4 – Entropy generation index for a compressor with given  $\beta e \eta_{pol}$ 

#### 5. Compressor plus heat exchanger

When the compressor is coupled to a heat exchanger with a given pressure loss  $\Delta p$ , if the delivery pressure of the system is fixed, it must work at an increased compression ratio:

$$\beta^* = \frac{p_i}{p_{ic}} = \frac{p_o + \Delta p}{p_{ic}} = \beta + \frac{\Delta p}{p_{ic}}$$

being  $p_{ic}$  the compressor inlet pressure.

At this point we must decide what the real objective of the system is. If the air has to be stored in a reservoir at ambient temperature, any excess temperature at the compressor exit is actually lost. Therefore we must add this loss to the system irreversibility, in terms of entropy generation.

In the following, fluid 1 is air and fluid 2 is water. If  $T_{o1}$  is the air residual temperature at heat exchanger exit, the heat  $Q_{env} = C_1 (T_{o1} - T_0)$  is discharged to the environment. It should be reminded that:

 $p_{i1} = \beta^* \cdot p_{ic}$  and  $T_{i1} = T_{ic} (\beta^*)^{(n-1)/n}$ .

For a single stage we may assume  $T_{i2} = T_{ic} = T_0$  and  $p_{ic} = p_0$ . Hence  $r = (\beta^*)^{(n-1)/n}$ . Furthermore:

$$T_{o1} = T_{i1} - \varepsilon(T_{i1} - T_{i2}) = T_0 [r - \varepsilon(r - 1)].$$

Air enters the compressor at ambient conditions  $p_0$ ;  $T_0$  and is stored at  $p = \beta p_0$ ;  $T_0$ . Its entropy variation is:

$$\Delta S_{air} = -C_1 \frac{\gamma - 1}{\gamma} \ln \beta \tag{14}$$

Water enters the exchanger at  $p_0$ ,  $T_{i2} = T_0$  and, neglecting its pressure loss, exits at  $p_0$ ,  $T_{o2} = T_{i2} + z \varepsilon (T_{i1} - T_{i2})$ . Its entropy variation is:

$$\Delta S_{water} = \frac{C_1}{z} ln \left( \frac{T_o}{T_i} \right)_2 = \frac{C_1}{z} ln [1 + z \varepsilon (r - 1)]$$
(15)

The environment receives the heat  $Q_{env}$  while remaining at  $T_0$ .

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$$\Delta S_{env} = C_1 \left[ \frac{T_{o1}}{T_0} - 1 \right] = C_1 \left[ r - \varepsilon (r - 1) - 1 \right] = C_1 (1 - \varepsilon) (r - 1) = C_1 (1 - \varepsilon) \left[ (\beta^*)^{\frac{n-1}{n}} - 1 \right]$$
(16)

Finally:

$$\frac{\Delta S_{air} + \Delta S_{water} + \Delta S_{env}}{C_1} = -\frac{\gamma - 1}{\gamma} ln \beta + \frac{1}{z} ln [1 + z\varepsilon(r-1)] + (1 - \varepsilon)(r-1)$$
(17)

The same result could be obtained through an exergy balance. If air enters the system at  $T_0$  and is stored at the same temperature, the air non-dimensional exergy increase, per unit mass flow rate, is:

$$\frac{\Delta e x_{air}}{c_p T_0} = (h_{o1} - h_{ic}) - T_0(s_{o1} - s_{ic}) = \frac{\gamma - 1}{\gamma} \ln \beta$$
(18)

Meanwhile, the water flow increases its specific exergy (neglecting pressure losses), by:

$$\Delta ex_{water} = (h_{o2} - h_{i2}) - T_0(s_{o2} - s_{i2}) = c_{p,water} \left[ (T_{o2} - T_{i2}) - T_0 \ln \frac{T_{o2}}{T_{i2}} \right]$$
(19)

Using the heat exchanger efficiency:

 $\dot{m}_{water}c_{p,water}(T_{o2}-T_{i2}) = \varepsilon \cdot \dot{m}c_p(T_{i1}-T_{i2})$ 

Hence, being  $z = \dot{m}c_p / \dot{m}_{acqua}c_{p,acqua}$ 

$$T_{o2} = (1 - z\varepsilon)T_{i2} + z\varepsilon T_{i1}$$

Therefore, referring the exergy increase to the unit mass flow of air:

$$\frac{\Delta e x_{water}}{c_p T_0} = \varepsilon \frac{\left(T_{i1} - T_{i2}\right)}{T_0} - \frac{1}{z} ln \left(1 - z\varepsilon + z\varepsilon \frac{T_{i1}}{T_{i2}}\right)$$
(20)

The system exergy input is the compression work:

$$W = c_p T_{ic} \left[ \left( \boldsymbol{\beta}^* \right)^{\frac{n-1}{n}} - 1 \right]$$
(21)

Using again the assumptions  $T_{i2} = T_{ic} = T_0$  and  $p_{ic} = p_0$ , we may write the system exergy destruction as follows:

$$\frac{ex_{loss}}{c_{p}T_{0}} = \frac{W}{c_{p}T_{0}} - \frac{\Delta ex_{air}}{c_{p}T_{0}} - \frac{\Delta ex_{water}}{c_{p}T_{0}} = \left[ (\beta^{*})^{\frac{n-1}{n}} - 1 \right] - \frac{\gamma-1}{\gamma} ln \beta - \left\{ \varepsilon(r-1) - \frac{1}{z} ln [1 + z\varepsilon(r-1)] \right\} =$$

$$= (r-1)(1-\varepsilon) - \frac{\gamma-1}{\gamma} ln \beta + \frac{1}{z} ln [1 + z\varepsilon(r-1)]$$
(22)

Therefore the entropy generation index  $N_s = \Delta s/C$  calculated by equation (17) is equal to the nondimensional exergy loss calculated by equation (22) and will be used in the following to find the system optimum.

Figure 5 shows that optimum NTU for the heat exchangers increases with  $\beta$ . Selecting the optimum values, we obtain the results shown in Figure 6, where the increase of NTU is shown together with the increase of entropy generation.

For  $\beta = 3$  we find an optimum NTU around 3.75, as shown in the heat exchanger analysis. The increase of optimum NTU can be interpolated as a polynomial in  $\beta$  as follows:

NTU = 
$$-1.717 \cdot 10^{-3} \beta^4 + 4.466 \cdot 10^{-2} \beta^3 - 4.510 \cdot 10^{-1} \beta^2 + 3.060 \beta - 2.533$$



Figure 5 – Entropy generation index v/s exchanger NTU for various values of  $\beta$  ( $\eta_{pol} = 0.9, z = 1$ )



Figure 6 – Optimum heat exchanger design and exergy efficiency for a single stage v/s  $\beta$ ( $\eta_{pol} = 0.9, z = 1$ )

### 6. Two compressors/heat exchanger assemblies, arranged in series

If the compression ratio has a high value, as in a CAES system, it may be useful to divide the compression into several stages, each followed by a heat exchanger.

Starting from two stages, we may fix a global compression ratio  $\beta = 10$  and try to find the best values of the stage compression ratios  $\beta_1$  and  $\beta_2$ , being  $\beta_1 \cdot \beta_2 = \beta$ , in terms of minimum entropy production (or maximum exergy efficiency). For each couple of compression ratios, the two heat exchangers are optimized, in terms of NTU, through the beforehand discussed optimizing criterion. The exergy destruction is the sum of those registered in the two stages.

The second stage has a different behavior with respect to the first one. The residual temperature at first stage exit is not simply an energy loss, but it causes an increase in the compression work of the second stage. The entropy generation index of the second stage has therefore a different shape.

Again, the entropy generation increases with the compression ratio and its minimum occurs for higher values of NTU. This is summarized in figure 8.

Coupling the two stages we have the final results shown in figure 9, where the two entropy contributions are plotted with their sum. The compression ratio reported on the abscissa refers to the first stage. The second is  $\beta_2 = 10 / \beta_1$ . Therefore the entropy generation of the first stage increases and the second decreases.



Figure 7 - Entropy generation index of the second stage v/s heat exchanger NTU for various values of  $\beta$  ( $\eta_{pol} = 0.9, z = 1$ )



Figure 8 – Optimum heat exchanger design and exergy efficiency for the second stage v/s  $\beta$ ( $\eta_{pol} = 0.9, z = 1$ )

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The global entropy generation has a minimum which basically coincides with the intersection of the two curves, i.e. with the point where the entropy generation is equal between the two stages. This is in good agreement with all the literature on entropy generation minimization, as well as with one of the basic principles of Constructal Design [12], i.e. the equal distribution of irreversibility between the units of a complex system.



Figure 9 – Optimum design of a two stage system v/s  $\beta$  of the first stage

The global entropy generation can be interpolated by the polynomial:

 $N_s = 6.961 \cdot 10^{-6} x^6 - 2.070 \cdot 10^{-4} x^5 + 2.519 \cdot 10^{-3} x^4 - 1.616 \cdot 10^{-2} x^3 + 5.850 \cdot 10^{-2} x^2 - 1.139 \cdot 10^{-1} x + 0.230$ 

The minimum of this polynomial is at  $\beta_1 = 3.26$ , i.e.  $\beta_2 = 3.07$ . The intersection of the two curves is instead at  $\beta_1 = 3.05$  and  $\beta_2 = 3.28$ . Apparently, this increased load on the first stage is beneficial because it reduces the loss toward the environment at the exit of second stage.

Actually, the system design is quite robust, as shown in figure 8, being the minimum zone very flat from  $\beta_1 = 3$  to 3.5.

The optimum value of  $N_s$  is around 0.14, while for a single stage system with  $\beta = 10$  it was around 0.16 (see figure 6).

### 7. Higher number of stages

When the number of stages exceeds two, the single stage can be optimized independently, using the method above described. If the equal distribution of loss has given a close-to-optimal solution in the case of two stages, even if the last stage works in slightly different conditions, we may well say that it should work for the other stages, which work in the same condition.

For a more precise solution, one should use a complex multi-variable optimization method, but the equal distribution of losses is useful for a first design and may even suffice as a thermal analysis, if the precise system design is dictated by other constraints.

# 8. Closure

Compressed air energy storage is widely recognized as a valuable option for energy storage. Coupling it with a thermal energy storage may significantly improve its viability and attractiveness, but requires a careful thermal optimization. To this aim, well assessed results on heat exchangers optimization through entropy generation minimization can be used as a starting point. When the system becomes increasingly complex, as in the case of several stages connected in series, the principles of Constructal Law, i.e. the use of a single optimized building block and the assembly of several blocks having the same entropy generation, can be invoked.

In this paper the compression phase of the energy storage has been analyzed. The principle of uniform distribution of losses has been shown to be valid in a simple configuration and is proposed as a guideline for the design at higher degrees of complication.

Expansion phase can be dealt with in a similar fashion.

Further work will be done on the Constructal optimization of the complete system, including the storage size and pressure.

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