Sensitivity of plume descriptors of a Gaussian plume model to deposition and source elevation (*)

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Summary. — Plume descriptors, constituting the first four moments of the concentration distribution, have been obtained for a Gaussian deposition plume model. Analytical expressions for the plume descriptors (centroid, variance, skewness and kurtosis) for point source ground-level and elevated releases are derived from a solution to the advection-diffusion equation accounting for deposition at the ground surface. A sensitivity analysis has been carried out to investigate the effect of deposition velocity and source height on the plume descriptors. For the particular case of a fully reflected Gaussian plume from a ground-level source, the centroid and the standard deviation have been found to vary as square root of downwind distance whereas skewness and kurtosis are independent of downwind distance, the approximate values being 0.995 for skewness and 3.87 for kurtosis.

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1. – Introduction

The statistical description of the concentration distribution involves determination of plume descriptors such as centroid, variance, skewness and kurtosis. Plume descriptors provide useful information regarding the plume behaviour which, in turn, helps decision-making regarding the siting of new industrial units, stack height, operational timing etc. Gaussian plume models have been known to be extensively used in regulatory applications all over the world because of their obvious advantages (Hanna *et al.*, 1982; Seinfeld, 1986; Zannetti, 1990; Turner, 1994). Therefore, plume descriptors for Gaussian plume models are often found in the literature (Pasquill and Smith, 1984). At the same time, these models have certain limitations. The assumption of constant wind and diffusivity fields in Gaussian plume models is an inherent limitation in describing near-source dispersion from near-surface sources. However,

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this is no longer a limitation in simulating dispersion in the far field. Non-Gaussian models have been known to perform better, particularly in the near-source region. In a recent study, Brown *et al.* (1997) have obtained the plume descriptors from a non-Gaussian model with vertically varying mean wind and diffusivity.

The removal of pollutants by dry deposition is an important mechanism to be modelled and studied properly. In fact, accurate measurement and modelling of dry deposition is an important issue for understanding the transport, transformation and the ultimate state of atmospheric pollutants. The deposition process can have vital impact on plume behaviour and concentration distribution. Recently, Xu *et al.* (1997) have studied the effect of winds on dry deposition at the coastline of Long Island Sound.

In this study, an attempt is made to examine the effect of deposition on plume behaviour through plume descriptors. A simple Gaussian plume model incorporating deposition at the ground surface has been considered to study the sensitivity of plume descriptors with the deposition velocity and the source height. Two particular cases have been discussed: one for a perfectly reflecting surface and the other for a ground level source.

2. – Model formulation and solution

Based on the gradient-transfer theory, the steady-state advection-diffusion equation governing the dispersion of a passive pollutant emitted from a point source located at $(0, 0, z_s)$ can be written as

$$U\frac{\partial \overline{C}}{\partial x} = \frac{\partial}{\partial y}\left(K_y \frac{\partial \overline{C}}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial \overline{C}}{\partial z}\right) + Q\delta(y)\,\delta(z - z_s)\,, \tag{1}$$

where x, y, and z are the alongwind, crosswind and vertical directions, respectively, U is the mean wind speed along the x-direction, $\overline{C}$ is the mean concentration, K_y and K_z are the crosswind and vertical eddy diffusivity coefficients, respectively, Q is the source strength (mass/time), δ is the Dirac-delta function, and z_s is the source height.

Assuming constant diffusivities, eq. (1) is a three-dimensional linear elliptic partial differential equation which can be solved analytically subject to the boundary conditions

$$\overline{C} \to 0\,; \qquad x,\ |y|\,,\ z \to \infty \tag{2}$$

and

$$K_z \frac{\partial \overline{C}}{\partial z} = v_d \overline{C} \qquad \text{at } z = 0 \tag{3}$$

to yield the solution in the form (Lin and Hildemann, 1997; Rao, 1986)

$$\overline{C}(x, y, z) = \frac{Q}{4\pi x\sqrt{K_y K_z}}\, e^{-\frac{Uy^2}{4xK_y}} \left\{ \exp\left[-\frac{U(z+z_s)^2}{4xK_z}\right] + \exp\left[-\frac{U(z-z_s)^2}{4xK_z}\right] \right\} - \\ - \frac{v_d}{UK_z} \exp\left[\frac{v_d(z+z_s)}{K_z} + \frac{v_d^2 x}{UK_z}\right] \operatorname{erfc}\left[\frac{z+z_s+2v_d x/U}{2\sqrt{K_z x/U}}\right], \tag{4}$$

where erfc is the complementary error function (see appendix A for definition and properties). The boundary condition (3) describes the mass flux at the ground surface ($z = 0$) in terms of the deposition velocity v_d. The crosswind integrated concentration (CWIC) can be obtained by integrating the above equation with respect to y from $-\infty$ to ∞. Assuming the source strength to be unity, CWIC is given by

$$\overline{C}_y(x, z) = \frac{1}{\sqrt{4\pi x U K_z}} \left\{ \exp\left[-\frac{U(z+z_s)^2}{4xK_z} \right] + \exp\left[-\frac{U(z-z_s)^2}{4xK_z} \right] \right\} - \\ - \frac{v_d}{UK_z} \exp\left[\frac{v_d(z+z_s)}{K_z} + \frac{v_d^2 x}{UK_z} \right] \operatorname{erfc}\left[\frac{z+z_s+2v_d x/U}{2\sqrt{K_z x/U}} \right]. \tag{5}$$

In the limiting case $v_d \to 0$, solution (5) reduces to the simple well-known result

$$\overline{C}_y(x, y) = \frac{1}{\sqrt{4\pi x U K_z}} \left\{ \exp\left[-\frac{U(z+z_s)^2}{4xK_z} \right] + \exp\left[-\frac{U(z-z_s)^2}{4xK_z} \right] \right\}. \tag{6}$$

This can also be obtained directly by considering no-flux condition at the ground surface and ignoring crosswind diffusion in the transport equation.

3. – Plume descriptors

Plume descriptors are essentially the first four moments of the concentration distribution. They are defined as follows (Brown *et al.*, 1997):

1) Centroid

$$\bar{z}(x) = \int_0^\infty z\overline{C}_y(x, z)\,\mathrm{d}z \Big/ \int_0^\infty \overline{C}_y(x, z)\,\mathrm{d}z\,; \tag{7}$$

2) Variance

$$\sigma_z^2(x) = \int_0^\infty (z-\bar{z})^2\,\overline{C}_y(x, z)\,\mathrm{d}z \Big/ \int_0^\infty \overline{C}_y(x, z)\,\mathrm{d}z\,; \tag{8}$$

3) Skewness

$$Sk(x) = \int_0^\infty (z-\bar{z})^3\,\overline{C}_y(x, z)\,\mathrm{d}z/\sigma_z^3 \int_0^\infty \overline{C}_y(x, z)\,\mathrm{d}z\,; \tag{9}$$

4) Kurtosis

$$Ku(x) = \int_0^\infty (z-\bar{z})^4\,\overline{C}_y(x, z)\,\mathrm{d}z/\sigma_z^4 \int_0^\infty \overline{C}_y(x, z)\,\mathrm{d}z\,. \tag{10}$$

For the sake of computational ease, the expressions (8)-(10) can be written in the form

$$\sigma_z^2(x) = \frac{\int_0^\infty z^2 \overline{C}_y(x, z)\,\mathrm{d}z}{\int_0^\infty \overline{C}_y(x, z)\,\mathrm{d}z} - (\overline{z})^2, \tag{11}$$

$$Sk(x) = \frac{\int_0^\infty z^3 \overline{C}_y(x, z)\,\mathrm{d}z}{\sigma_z^3 \int_0^\infty \overline{C}_y(x, z)\,\mathrm{d}z} - 3\left(\frac{\overline{z}}{\sigma_z}\right) - \left(\frac{\overline{z}}{\sigma_z}\right)^3, \tag{12}$$

$$Ku(x) = \frac{\int_0^\infty z^4 \overline{C}_y(x, z)\,\mathrm{d}z}{\sigma_z^4 \int_0^\infty \overline{C}_y(x, z)\,\mathrm{d}z} - 4Sk\left(\frac{\overline{z}}{\sigma_z}\right) - 6\left(\frac{\overline{z}}{\sigma_z}\right)^2 - \left(\frac{\overline{z}}{\sigma_z}\right)^4. \tag{13}$$

With solution (5), the integrals appearing in the expressions (7), (11)-(13) have been evaluated analytically. These are given by

$$\int_0^\infty \overline{C}_y(x, z)\,\mathrm{d}z = \tag{14}$$

$$= \frac{1}{U} - \frac{1}{U}\,\mathrm{erfc}\left(\frac{z_\mathrm{s}}{2}\sqrt{\frac{U}{K_z x}}\right) + \frac{1}{U}\exp\left[\frac{v_\mathrm{d}^2 x}{UK_z} + \frac{v_\mathrm{d} z_\mathrm{s}}{K_z}\right]\mathrm{erfc}\left[\frac{z_\mathrm{s} + 2v_\mathrm{d} x/U}{2\sqrt{K_z x/U}}\right],$$

$$\int_0^\infty z\overline{C}_y(x, z)\,\mathrm{d}z = \tag{15}$$

$$= \frac{z_\mathrm{s}}{U} + \left(\frac{K_z}{v_\mathrm{d}}\right)\frac{1}{U}\,\mathrm{erfc}\left(\frac{z_\mathrm{s}}{2}\sqrt{\frac{U}{K_z x}}\right) - \left(\frac{K_z}{v_\mathrm{d}}\right)\frac{1}{U}\exp\left[\frac{v_\mathrm{d}^2 x}{UK_z} + \frac{v_\mathrm{d} z_\mathrm{s}}{K_z}\right]\mathrm{erfc}\left[\frac{z_\mathrm{s} + 2v_\mathrm{d} x/U}{2\sqrt{K_z x/U}}\right],$$

$$\int_0^\infty z^2 \overline{C}_y(x, z)\,\mathrm{d}z = \tag{16}$$

$$= \frac{z_\mathrm{s}^2}{U} - \left(2\frac{K_z^2}{v_\mathrm{d}^2} + 2\frac{K_z}{v_\mathrm{d}} z_\mathrm{s} + z_\mathrm{s}^2\right)\frac{1}{U}\,\mathrm{erfc}\left(\frac{z_\mathrm{s}}{2}\sqrt{\frac{U}{K_z x}}\right) + \frac{1}{\sqrt{\pi U K_z x}}\int_0^{z_\mathrm{s}} t^2 \exp\left[\frac{-Ut^2}{4K_z x}\right]\mathrm{d}t +$$

$$+ \frac{4}{U}\sqrt{\frac{K_z x}{\pi U}}\left(\frac{K_z}{v_\mathrm{d}} + z_\mathrm{s}\right)\exp\left[\frac{-Uz_\mathrm{s}^2}{4K_z x}\right] + \left(\frac{2K_z^2}{v_\mathrm{d}^2}\right)\frac{1}{U}\exp\left[\frac{v_\mathrm{d}^2 x}{UK_z} + \frac{v_\mathrm{d} z_\mathrm{s}}{K_z}\right]\mathrm{erfc}\left[\frac{z_\mathrm{s} + 2v_\mathrm{d} x/U}{2\sqrt{K_z x/U}}\right],$$

$$(17)\qquad \int_0^\infty z^3 \overline{C}_y(x,z)\,\mathrm{d}z = \frac{z_s^3}{U} + \left(\frac{6K_z^3}{v_d^3} + \frac{6K_z^2}{v_d^2} z_s + \frac{3K_z}{v_d} z_s^2\right)\frac{1}{U}\,\mathrm{erfc}\left(\frac{z_s}{2}\sqrt{\frac{U}{K_z x}}\right) +$$

$$+ \frac{3z_s}{\sqrt{\pi U K_z x}}\int_0^{z_s} t^2 \exp\left[\frac{-Ut^2}{4K_z x}\right]\mathrm{d}t - \frac{4}{U}\sqrt{\frac{K_z x}{\pi U}}\left(\frac{3K_z^2}{v_d^2} + \frac{3K_z}{v_d} z_s\right)\exp\left[\frac{-Uz_s^2}{4K_z x}\right] -$$

$$-\left(\frac{6K_z^3}{v_d^3}\right)\frac{1}{U}\exp\left[\frac{v_d^2 x}{UK_z} + \frac{v_d z_s}{K_z}\right]\mathrm{erfc}\left[\frac{z_s + 2v_d x/U}{2\sqrt{K_z x/U}}\right] + \frac{12K_z x}{U^2\sqrt{\pi}}\left(\frac{K_z}{v_d} + z_s\right)\Gamma\left(\frac{3}{2}, \frac{Uz_s^2}{4K_z x}\right),$$

$$(18)\qquad \int_0^\infty z^4 \overline{C}_y(x,z)\,\mathrm{d}z =$$

$$= \frac{z_s^4}{U} - \left(\frac{24K_z^4}{v_d^4} + \frac{24K_z^3}{v_d^3} z_s + \frac{12K_z^2}{v_d^2} z_s^2 + \frac{4K_z}{v_d} z_s^3 + z_s^4\right)\frac{1}{U}\,\mathrm{erfc}\left(\frac{z_s}{2}\sqrt{\frac{U}{K_z x}}\right) +$$

$$+ \frac{6z_s^2}{\sqrt{\pi U K_z x}}\int_0^{z_s} t^2 \exp\left[\frac{-Ut^2}{4K_z x}\right]\mathrm{d}t - \frac{1}{\sqrt{\pi U K_z x}}\int_0^{z_s} t^4 \exp\left[\frac{-Ut^2}{4K_z x}\right]\mathrm{d}t +$$

$$+ \frac{4}{U}\sqrt{\frac{K_z x}{\pi U}}\left[\frac{12K_z^3}{v_d^3} + \frac{8K_z^2 x}{Uv_d} + \left(\frac{12K_z}{v_d^2} + \frac{8x}{U}\right)K_z z_s + \frac{8K_z}{v_d} z_s^2 + 4z_s^3\right]\exp\left[\frac{-Uz_s^2}{4K_z x}\right] +$$

$$+\left(\frac{24K_z^4}{v_d^4}\right)\frac{1}{U}\exp\left[\frac{v_d^2 x}{UK_z} - \frac{v_d z_s}{K_z}\right]\mathrm{erfc}\left[\frac{z_s + 2v_d x/U}{2\sqrt{K_z x/U}}\right] -$$

$$-\left(\frac{4K_z}{v_d}\right)\frac{12K_z x}{U^2\sqrt{\pi}}\left(\frac{K_z}{v_d} + z_s\right)\Gamma\left(\frac{3}{2}, \frac{Uz_s^2}{4K_z x}\right).$$

In eqs. (17) and (18) Γ is the incomplete gamma-function (see appendix A for definition and properties). Expressions given by eqs. (14-18) can be used to compute plume descriptors for the general case of elevated point source emissions with deposition at the surface. These general expressions are lengthy and would involve large computations. However, they can be simplified in certain special cases such as surface release and deposition, elevated release and no deposition. Plume descriptors in such special cases are conceptually more appealing.

Particular Case: I. – Here, we consider a situation of a near surface release. The integrals (14)-(18) for emissions taking place at $z_s = 0$ reduce to

$$(19)\qquad \int_0^\infty \overline{C}_y(x,z)\,\mathrm{d}z = \frac{1}{U}\exp\left[\frac{v_d^2 x}{UK_z}\right]\mathrm{erfc}\left[v_d\sqrt{\frac{x}{K_z U}}\right],$$

$$(20)\qquad \int_0^\infty z\overline{C}_y(x,z)\,\mathrm{d}z = \frac{K_z}{Uv_d} - \frac{K_z}{Uv_d}\exp\left[\frac{v_d^2 x}{UK_z}\right]\mathrm{erfc}\left[v_d\sqrt{\frac{x}{K_z U}}\right],$$

(21) $$\int_0^\infty z^2 \overline{C}_y(x, z)\,\mathrm{d}z = -\frac{2K_z^2}{Uv_\mathrm{d}^2} + \frac{2K_z^2}{Uv_\mathrm{d}^2}\exp\left[\frac{v_\mathrm{d}^2 x}{UK_z}\right]\mathrm{erfc}\left[v_\mathrm{d}\sqrt{\frac{x}{UK_z}}\right] + \frac{4K_z}{Uv_\mathrm{d}}\sqrt{\frac{K_z x}{\pi U}}\,,$$

(22) $$\int_0^\infty z^3 \overline{C}_y(x, z)\,\mathrm{d}z =$$

$$= \frac{6K_z^3}{Uv_\mathrm{d}^3} - \frac{6K_z^3}{Uv_\mathrm{d}^3}\exp\left[\frac{v_\mathrm{d}^2 x}{UK_z}\right]\mathrm{erfc}\left[v_\mathrm{d}\sqrt{\frac{x}{UK_z}}\right] - \frac{12K_z^2}{Uv_\mathrm{d}^2}\sqrt{\frac{K_z x}{\pi U}} + \frac{6K_z^2 x}{U^2 v_\mathrm{d}}\,,$$

(23) $$\int_0^\infty z^4 \overline{C}_y(x, z)\,\mathrm{d}z = -\frac{24K_z^4}{Uv_\mathrm{d}^4} + \frac{24K_z^4}{Uv_\mathrm{d}^4}\exp\left[\frac{v_\mathrm{d}^2 x}{UK_z}\right]\mathrm{erfc}\left[v_\mathrm{d}\sqrt{\frac{x}{UK_z}}\right] +$$

$$+ \frac{4}{U}\sqrt{\frac{K_z x}{\pi U}}\left(\frac{12K_z^3}{v_\mathrm{d}^3} + \frac{8K_z^2 x}{Uv_\mathrm{d}}\right) - \frac{24K_z^3 x}{U^2 v_\mathrm{d}^2}\,.$$

Expressions given by (19)-(23) can be used to compute plume descriptors for a near surface release incorporating the effect of deposition. In the absence of deposition, *i.e.*, in the limit $v_\mathrm{d} \to 0$, the above integrals further simplify to give

(24) $$\int_0^\infty \overline{C}_y(x, z)\,\mathrm{d}z = \frac{1}{U}\,,$$

(25) $$\int_0^\infty z\overline{C}_y(x, z)\,\mathrm{d}z = \frac{2}{U}\sqrt{\frac{K_z x}{\pi U}}\,,$$

(26) $$\int_0^\infty z^2 \overline{C}_y(x, z)\,\mathrm{d}z = \frac{2K_z x}{U^2}\,,$$

(27) $$\int_0^\infty z^3 \overline{C}_y(x, z)\,\mathrm{d}z = \frac{8K_z x}{U^2}\sqrt{\frac{K_z x}{\pi U}}\,,$$

(28) $$\int_0^\infty z^4 \overline{C}_y(x, z)\,\mathrm{d}z = \frac{12K_z^2 x^2}{U^3}\,.$$

These equations can be used to obtain plume descriptors for a reflected (no deposition) Gaussian plume from a ground level source. Explicit expressions for plume descriptors in this case are given by

$$(29)\qquad \begin{cases} \bar{z} = 2\sqrt{\dfrac{K_z x}{\pi U}}\,, \qquad \sigma_z^2 = \dfrac{2K_z x}{U}\left(1 - \dfrac{2}{\pi}\right), \\ Sk = \dfrac{4-\pi}{\pi - 2}\sqrt{\dfrac{2}{\pi - 2}} \simeq 0.995\,, \qquad Ku = \dfrac{3\pi^2 - 4\pi - 12}{(\pi - 2)^2} \simeq 3.87\,. \end{cases}$$

It can be easily seen that the centroid and the standard deviation vary as square root of the downwind distance while the coefficients of skewness and kurtosis assume constant values. These results can also be obtained as a particular case by substituting $\alpha = 2$, $h_s = 0$ in eqs. (9)-(12) of Brown *et al.* (1997). Recall that for a non-reflected Gaussian distribution, $Sk = 0$ and $Ku = 3$ (Brown *et al.*, 1997).

Particular Case: II. – Here, we consider the case of a release from an elevated point source in the absence of deposition. In this case, the integrals for the plume descriptors can be obtained by taking the limit $v_d \to 0$ in the general expressions (14)-(18). The resulting expressions are given by

$$(30)\qquad \int_0^\infty \overline{C}_y(x, z)\,\mathrm{d}z = \frac{1}{U}\,,$$

$$(31)\qquad \int_0^\infty z\overline{C}_y(x, z)\,\mathrm{d}z = \frac{2}{U}\sqrt{\frac{K_z x}{\pi U}}\exp\left[\frac{-Uz_s^2}{4K_z x}\right] + \frac{z_s}{U}\operatorname{erf}\left(\frac{z_s}{2}\sqrt{\frac{U}{K_z x}}\right),$$

$$(32)\qquad \int_0^\infty z^2\overline{C}_y(x, z)\,\mathrm{d}z = \frac{2K_z x}{U^2} + \frac{z_s^2}{U}\,,$$

$$(33)\qquad \int_0^\infty z^3\overline{C}_y(x, z)\,\mathrm{d}z = \frac{8K_z x}{U^2}\sqrt{\frac{K_z x}{\pi U}}\left(1 + \frac{Uz_s^2}{4K_z x}\right)\exp\left[\frac{-Uz_s^2}{4K_z x}\right] + \\ + \frac{1}{U}\left(z_s^3 + \frac{6K_z x}{U}z_s\right)\operatorname{erf}\left(\frac{z_s}{2}\sqrt{\frac{U}{K_z x}}\right),$$

$$(34)\qquad \int_0^\infty z^4\overline{C}_y(x, z)\,\mathrm{d}z = \frac{12K_z^2 x^2}{U^3} + \frac{12K_z x}{U^2}z_s^2 + \frac{z_s^4}{U}\,.$$

In eqs. (31) and (33) erf is the mathematical error function (see appendix A). For a ground level source ($z_s = 0$), these expressions (30)-(34) match identically with those given by eqs. (24)-(28) in Case I.

4. – Sensitivity analysis

The analytical expressions leading to evaluation of the plume descriptors for a general case of a Gaussian deposition plume for an elevated point source are given by eqs. (14)-(18). These have been used to carry out two sensitivity analyses on the plume descriptors, one with the deposition velocity and the other with the source height. The case of total reflection and no absorption ($v_\mathrm{d} = 0$) has also been considered for comparison.

4·1. *Sensitivity of plume descriptors to deposition velocity.* – Figures 1(a-d) give the plots of the four descriptors of an elevated ($z_\mathrm{s} = 10$ m) Gaussian plume against the downwind distance for different values of deposition velocity. A special case ($v_\mathrm{d} = 0$, $z_\mathrm{s} = 0$) of emissions from a ground level source and perfectly reflecting surface has been included for comparative analysis. Theoretically, the cases $v_\mathrm{d} = 0$ and $v_\mathrm{d} = \infty$ (in practice, even $v_\mathrm{d} \approx 1\ \mathrm{ms}^{-1}$ may be deemed as infinite value) correspond to fully reflecting and fully absorbing boundary at $z = 0$, respectively. Figure 1a shows that the centroid (the position of the centre of distributed mass) moves upwards with the downwind distance and with the increase in the deposition velocity. This trend can be explained by more depletion of mass due to downward flux at the ground for higher values of v_d. The increase in standard deviation, the measure of dispersion about the mean, in the downwind direction, and with the increase in deposition velocity is induced by vigorous spreading of the plume due to vertical diffusion and depletion at the ground (fig. 1b).

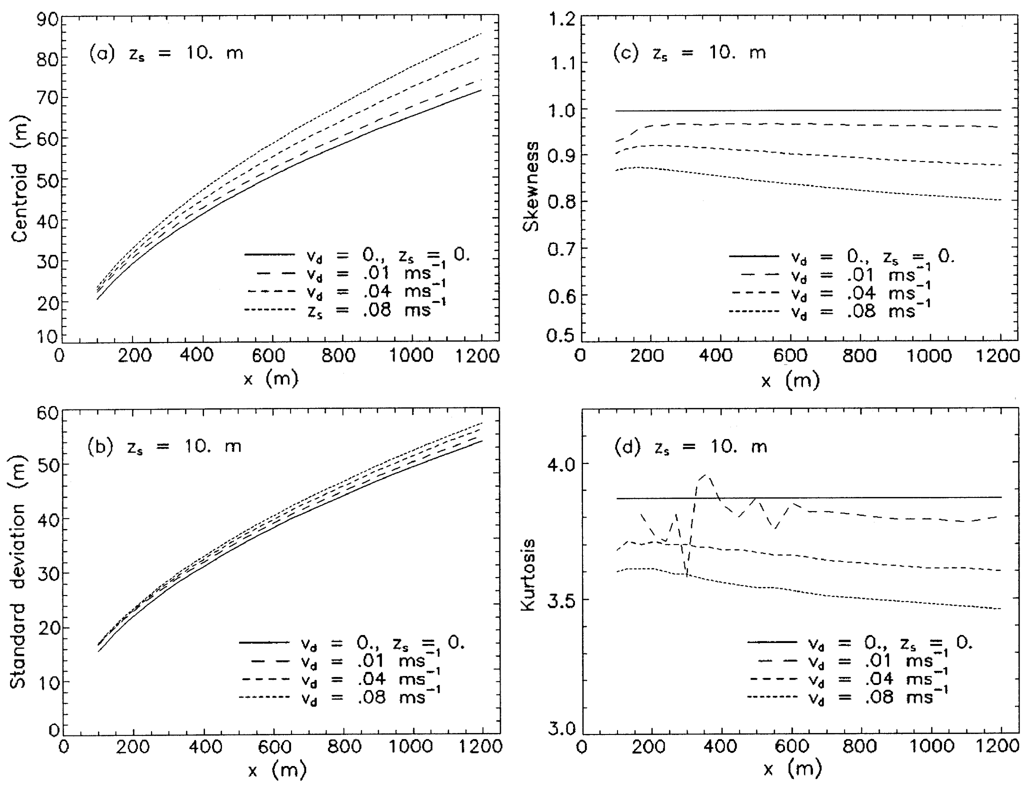

Fig. 1. – Variation of plume descriptors with downwind distance for different values of deposition velocity v_d for an elevated point source with $z_\mathrm{s} = 10$ m: (a) centroid, (b) standard deviation, (c) skewness, (d) kurtosis. Solid curves refer to the case $z_\mathrm{s} = 0$, $v_\mathrm{d} = 0$.

On the other hand, skewness and kurtosis—the statistical measures of symmetry and flatness/peakedness, respectively—of the concentration distribution decrease along the downwind direction except in the initial stages of the plume where the source height effects are perceptible (figs. 1c, d). The trend of all the four descriptors for a ground level source remains unchanged almost everywhere except near the source. In the near source region, the centroid comes down with the subsequent decrease in the standard deviation and the distribution becomes more skewed and peaked as the effects of reflection start right near the source. The extent of this region depends upon the source height and the rate of deposition. The peculiar behaviour, as seen in fig. 1d, near the source for small v_d may be due to computational noise arising out of the error function.

In the case of a fully reflecting boundary $z=0$ and ground level source, the coefficients of skewness and kurtosis are independent of the downwind distance and have the uniform values $Sk \approx 0.995$ and $Ku \approx 3.87$. In the absence of reflection at $z=0$ ($v_d = \infty$), the distribution is symmetric about the source height ($Sk=0$) while, on the other hand, total reflection at $z=0$ ($v_d=0$) implies a highly skewed distribution ($Sk \approx 0.995$ for $z_s=0$). It is observed from fig. 1d that kurtosis is maximum in the case of total reflection at $z=0$ ($v_d=0$); its minimum value which is attained when $v_d=\infty$ (complete absorption at $z=0$) is 3.

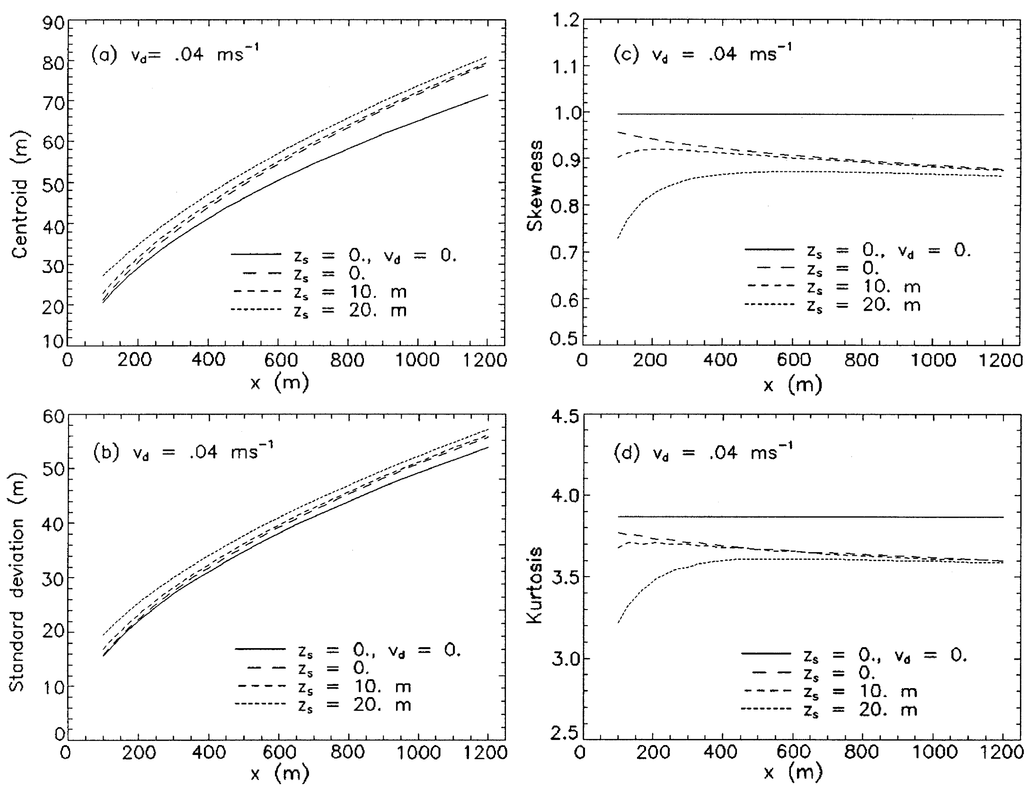

Fig. 2. – Variation of plume descriptors with downwind distance for different values of source height z_s in the case of a partially absorbing surface with $v_d = 0.04\ \mathrm{ms}^{-1}$: (a) centroid, (b) standard deviation, (c) skewness, (d) kurtosis. Solid curves refer to the case $z_s=0$, $v_d=0$.

4.2. *Sensitivity of plume descriptors to source height.* – Figures 2(a-d) display the response of plume descriptors of a partially reflected ($v_d = 0.04\ \mathrm{ms}^{-1}$) Gaussian plume for different values of source height. Solid curves correspond to ground-based source ($z_s = 0$) and a completely reflecting boundary at $z = 0$. An increase in the stack height expectedly pushes the centroid upwards, accompanied by an increase in the standard deviation (fig. 2b). The values of these descriptors increase along the downwind direction. The plots (figs. 2c, d) of *Sk* and *Ku* indicate that the concentration distribution becomes more symmetric and flat with the increase in source height. In the near source region, the effects of source height on skewness and kurtosis are manifested in increased symmetry and reduced peakedness as compared to the far field. The values of the coefficients of *Sk* and *Ku* rapidly increase with downwind distance up to a stage (depending upon source height) where the effects of absorbing surface are still pronounced and thereafter the variation is very slow because the stack height will no longer have appreciable influence on dispersion.

On the other hand, for a fully reflecting surface, skewness and kurtosis clearly exhibit a boundary layer structure (rapid increase in value with variation in x) up to nearly 200 m downwind irrespective of the height of the source (see fig. 3). Beyond a distance of 1200 m downwind, skewness and kurtosis attain approximately uniform values close to 0.995 and 3.87, respectively.

It may also be pointed out that the results stated above correspond to uniform values of U and K_z which have a bearing on the atmospheric stability.

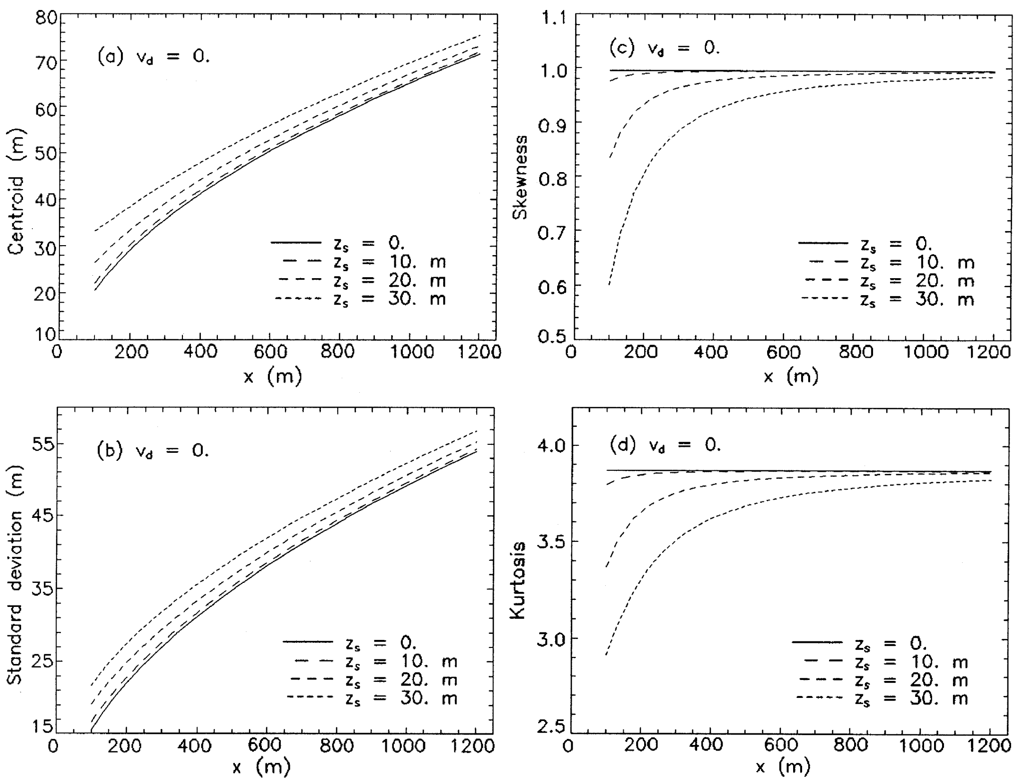

Fig. 3. – Same as fig. 2 but with a fully reflecting surface $v_d = 0$.

5. – Conclusions

The plume descriptors are useful for the statistical description of a plume behaviour. In this study, analytical expressions have been obtained for the evaluation of plume descriptors from a Gaussian deposition plume model for an elevated point source. Two special cases have been derived: one, ignoring deposition (total reflection) and taking an arbitrary source height and the other, partial absorption and a ground level source. A sensitivity study of plume descriptors with the deposition velocity and the source height has been performed in the present work. The results of sensitivity analysis indicate that the general behaviour of centroid and standard deviation resembles for various values, including zero, of deposition velocity and source height. While both centroid and standard deviation vary as square root of downwind distance in the case of zero source height and no deposition, skewness and kurtosis assume constant values, 0.995 and 3.87, respectively. It would be worthwhile and desirable to undertake the validation of the results of this study.

The inherent limitation (constant wind and diffusivity fields) of the Gaussian plume model would not have any significant impact on the results of this study in the far field. However, it may be noted that in our results (figs. 2 and 3) the changes in plume descriptors are relatively small for near-surface releases as compared to higher source elevations. The trend in these changes, particularly in the near-source region, may be different after the limitation of constant wind and diffusivity is removed.

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Appendix A

Here, we give the definition of the functions mentioned in this study. The error function and the complementary error function are defined as (Abramowitz and Stegun, 1972)

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2}\,\mathrm{d}u\,, \tag{A.1}$$

$$\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-u^2}\,\mathrm{d}u \tag{A.2}$$

with the following properties:

$$\mathrm{erf}(x) + \mathrm{erfc}(x) = 1\,; \qquad \mathrm{erf}(-x) = -\,\mathrm{erf}(x)\,. \tag{A.3}$$

The incomplete gamma-function can be represented as (Abramowitz and Stegun, 1972)

(A.4) $$\Gamma(\alpha, x) = \int_x^\infty e^{-t} t^{\alpha-1} \mathrm{d}t$$

with the following properties:

(A.5) $$\Gamma(\alpha+1, x) = \alpha\Gamma(\alpha, x) + x^{\alpha} e^{-x},$$

(A.6) $$\Gamma(\alpha, 0) = \int_0^\infty e^{-t} t^{\alpha-1} \mathrm{d}t = \Gamma(\alpha); \quad \Gamma(1/2) = \sqrt{\pi}; \quad \Gamma(\alpha+1) = \alpha\Gamma(\alpha).$$

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