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Nuclear interactions and the highest energy air showers(*)

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Summary. — Based on the energy distribution of particles produced in multiple particle production, which is formulated phenomenologically by the data of direct observation, we calculate the air shower development at $E_0 = 10^{20}$ eV. The calculation shows that the formula, extrapolated into the higher energy region, does not describe the highest energy air showers. We also argue that the energy estimation of the highest energy air showers may have an ambiguity of a factor ~ 2, due to our incomplete knowledge on high energy interactions.

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1. – Introduction

At present the energy spectrum of the primary cosmic rays, which is obtained by observing the highest energy air showers, seems to extend beyond the GZK cut-off energy. And there are several experimental efforts to confirm this discovery and many ambitious proposals to describe it, because it is one of the most interesting puzzles to be solved. One should, however, keep in mind that high energy nuclear interactions, which we cannot say to be established well above 10^{16} eV, are assumed to obtain the energy spectrum from the observed data of air shower size.

In this report we discuss whether the energy distribution of particles produced in multiple particle production, formulated by us [1], describes the highest energy ($\geq 10^{18}$ eV)

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air showers or not. The formulation is made phenomenologically on the basis of experimental data of direct observation by accelerator and cosmic ray experiments, assuming that the Feynman scaling law is valid at low energies but is violated at high energies. It is a merit of discussing the highest energy air showers that the energy dependence of the nuclear interaction characteristics shows itself in the most distinct way. Discussion enables us to examine how the nuclear interactions affect the air shower size at the highest energy region of 10^{18}–10^{20} eV, too.

2. – Air Showers

2˙1. *Elementary processes for air showers.*
1) Inelastic collision mean free path of hadrons in the air:

$$\lambda_N(E_0) = \lambda_N(E_0/B)^{-\beta} \qquad \lambda_\pi(E_0) = (1/\xi)\lambda_N(E_0)\,,$$

where $\beta = 0.056$, $B = 10^3$ GeV and $\xi \equiv \sigma_{\pi N}/\sigma_{NN} = 0.71$.
2) Energy distribution of the surviving particle:

$$(1-b)\,\delta(E-(1-K)E_0)\,\mathrm{d}E\,,$$

where K is the total inelasticity to be discussed below. The charge exchange probability of the surviving pion b is 0 and 0.3 for nucleon and pion collisions, respectively. That is, the charge exchange of the surviving pion, *i.e.* $\pi^\pm \to \pi^0$, is an important process to be taken into account, because the inelasticity is 1.0 in the process.
3) Energy distribution of *charged* produced particles:

$$(1) \quad \varphi(E_0,E)\mathrm{d}E = aD(1-a'x)^d/x\,\mathrm{d}x \qquad (x \equiv E/E_0,\ a=(E_0/A)^\alpha,\ a'=(E_0/A)^{\alpha'})\,,$$

where $D = 2(d+1)/3$, $d = 4.0$, $A = 200$ GeV, $\alpha = 0.105$ and $\alpha' = 0.210$. The formula is obtained on the basis of experimental data of direct observation by accelerator experiments ($\sqrt{s} = 53$, 200, 546, 630 and 900 GeV of CERN SPS $\bar{\mathrm{p}}\mathrm{p}$ collider) [2-4] and by cosmic ray experiment ($\langle\sqrt{s}\rangle = 500$ GeV of emulsion chamber experiment at Mt. Chacaltaya) [5, 1].

The formula with $a = a' = 1.0$, which is attained at low energy of $E_0 \sim 200$ GeV, is one of the empirical distributions to follow the Feynman scaling law [6]. Validity of the law is verified experimentally in the energy region $\sqrt{s} \leq 63$ GeV [7]. One can see in the formula that the law is violated strongly at high energies and consequently the average inelasticity decreases appreciably at high energies. (See fig. 1.)

The formula leads to the energy dependences of charged multiplicity and average total inelasticity (see fig. 1) defined as

$$m(E_0) \equiv \int \varphi(E_0,E)\mathrm{d}E\,, \qquad \langle K\rangle \equiv \frac{3}{2}\int \varphi(E_0,E)E\mathrm{d}E\,,$$

where the factor 3/2 is due to the charge independence of produced pions under the assumption that all the produced particles be pions.
4) Inelasticity is assumed to be distributed uniformly between 0 and $2\langle K\rangle$.

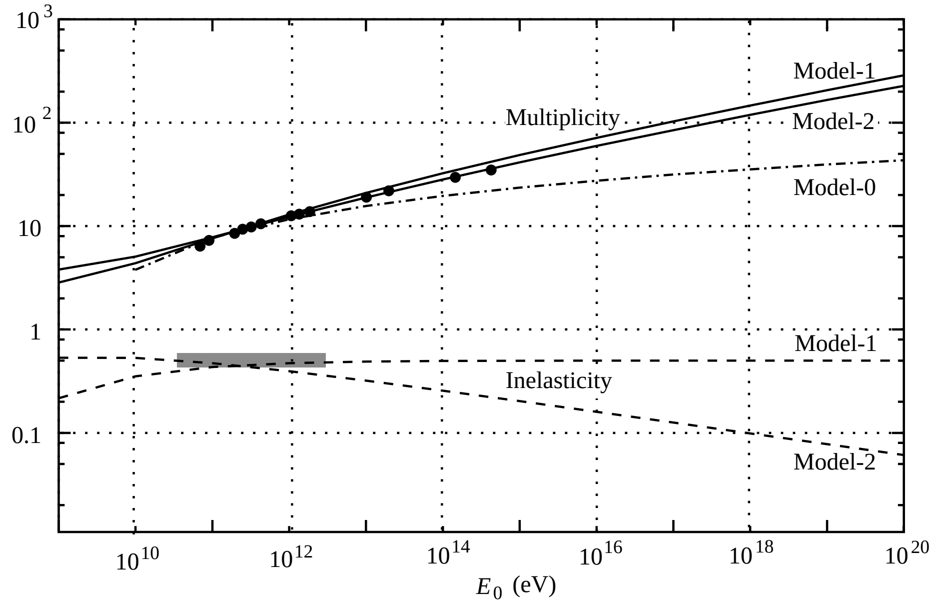

Fig. 1. – Energy dependences of charged multiplicity and average total inelasticity. Model-0, Model-1 and Model-2 are explained in table I. The hatched area is the energy region where the Feynman scaling law is verified experimentally. Full circles are the experimental data of charged multiplicity.

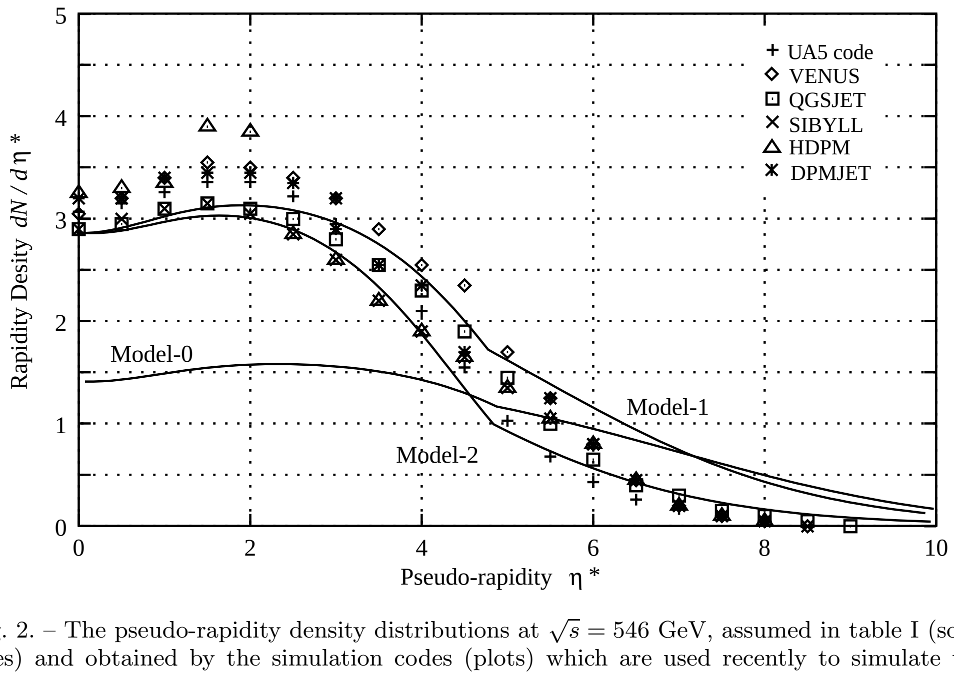

Fig. 2. – The pseudo-rapidity density distributions at $\sqrt{s} = 546$ GeV, assumed in table I (solid lines) and obtained by the simulation codes (plots) which are used recently to simulate the diffusion of cosmic rays in the atmosphere.

TABLE I. – *Scaling violation parameters assumed in the models.*

	α	α'	Feynman scaling law	Remark
Model-0	0	0	valid	$\langle K \rangle = 0.5$
Model-1	0.105	0.105	violated	$\langle K \rangle = 0.5$
Model-2	0.105	0.210	violated	the best-fit to the data

2·2. *Comparison with the models used in simulations.* – We assume three types of energy distributions for the discussion made below. (See table I.)

It is interesting to see whether the formula of eq. (1) is consistent with nuclear interaction models, such as VENUS, QGSJET, DPMJET, SYBILL, HDPM and UA5 code [8], which are incorporated in the simulations to follow the atmospheric diffusion of cosmic rays. Figure 2 shows the pseudo-rapidity density distributions at $\sqrt{s} = 546$ GeV by the formulated models in table I and by the simulation codes. In fig. 2, we can see the following(1):

1) The difference of the densities, predicted by the simulation codes, is not negligible.
2) In the central region all simulation codes predict rapidity densities consistent with the experimental data except HDPM.
3) In the middle rapidity region, *i.e.* $2.0 \leq \eta^* \leq 6.0$, which is the most important for the atmospheric diffusion of cosmic-ray particles, predictions by VENUS, QGSJET and DPMJET are higher than that of Model-2, while those by other codes are consistent with that of Model-2.
4) In the forward region all the codes predict consistent densities with that of Model-2.
5) The density distribution by UA5 code is almost consistent with that of Model-2 over all rapidity regions.

3. – Air showers of $E_0 = 10^{20}$ eV

Our plan is to solve the diffusion of cosmic ray particles in the atmosphere analytically on the basis of elementary processes, mentioned in sect. **2**, for the three models listed in table I(2). The diffusion equations can be solved for the cases listed in table II, although it is not easy to solve them in a general way.

Then we obtain the size of the air shower, which is initiated by an incident proton with fixed energy E_0, based on the solutions of diffusion equations. The size of the air shower is defined as the number of charged particles which pass the horizontal plane at the observation level. The electron component is dominant among the charged particles in the air shower, and hence we refer only to the electron number as the air shower size.
i) Figure 3 shows the transition curve of the air shower size for the primary proton with

(1) The distributions by simulations are for NSD events, while those by the formulation are for all inelastic events. The definitions of them are $\sigma_{NSD} = \sigma_{ND} + \sigma_{DD}$ and $\sigma_{\rm inel} = \sigma_{NSD} + \sigma_{SD}$, where NSD, ND, DD and SD stand for non-single-diffractive, non-diffractive, double-diffractive and single-diffractive, respectively. According to the data by UA5 collaboration [2] the pseudo-rapidity density of NSD events is higher by 10 % than that of all inelastic events in the range $0 \geq \eta^* \geq 3.5$ and is almost equal in the region $\eta^* \geq 3.5$ at $\sqrt{s} = 546$ GeV.
(2) $\pi \to \mu$ decay is neglected.

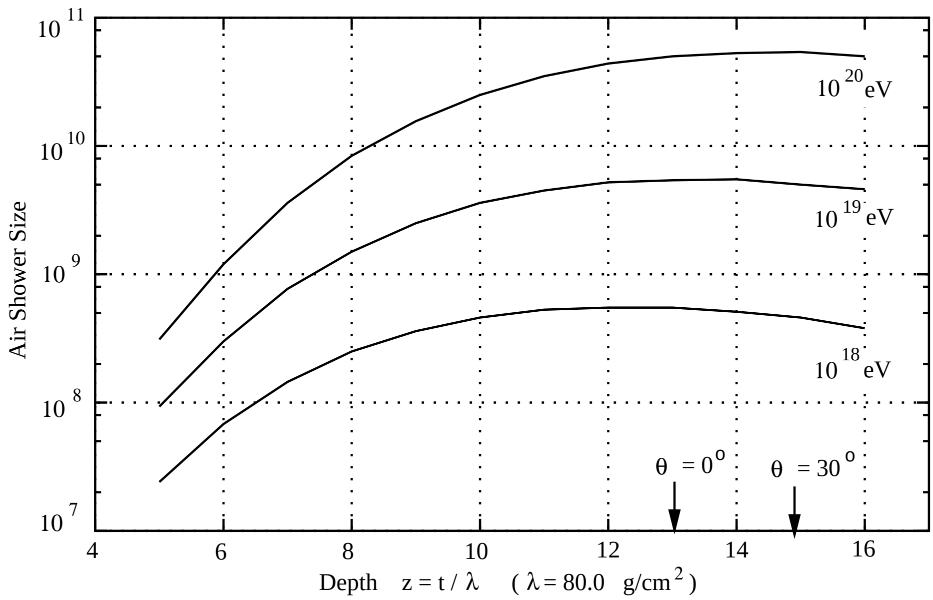

Fig. 3. – Transition curve of the air shower size for the primary proton with the energy $E_0 = 10^{18}$, 10^{19}, and 10^{20} eV, for case A (Model-0, constant cross-section and $b = 0$). The arrows indicate the depth of the sea level (1030 g/cm^2) for the air showers with the inclination $\theta = 0°$ and 30°.

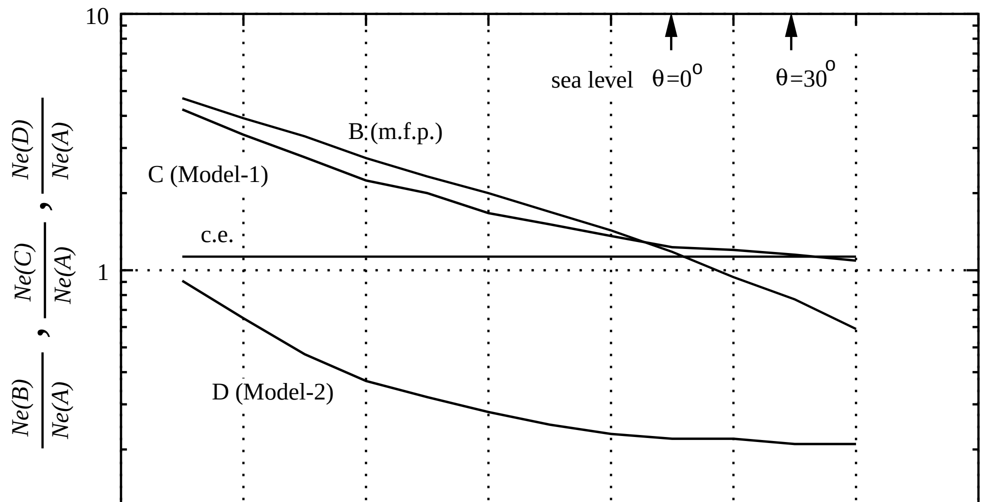

Fig. 4. – Ratio of air shower size, $N_e(B)/N_e(A)$, $N_e(C)/N_e(A)$ and $N_e(D)/N_e(A)$, along the depth. Cases A, B, C and D are tabulated in table II. The primary energy of a proton is 10^{20} eV.

TABLE II. – *The cases possible to be solved.*

	α	α'	β	b	Remark		
					Model	σ	$\langle K\rangle$
Case A	0	0	0	0, 0.3	Model-0	const	0.5
Case B	0	0	0.056	0	Model-0	increasing	0.5
Case C	0.105	0.105	0	0	Model-1	const	0.5
Case D	0.105	0.210	0	0	Model-2	const	decreasing

TABLE III. – *Air shower size at sea level by the models for the incident proton of* $E_0 = 10^{20}$ eV.

	Model-0	Model-1	CORSIKA	Model-2
size[a]	5.0×10^{10}	6.2×10^{10}		1.1×10^{10}
ratio to Model-0	$(\times 1.0)$	$(\times 1.23)$		$(\times 0.22)$
charge exchange	$\times 1.13$	$\times 1.13$		$\times 1.13$
increasing cross-section	$\times 1.18$	$\times 1.18$		$\times 1.18$
size (expected)[b]	6.7×10^{10}	8.3×10^{10}	5.5×10^{10}	1.5×10^{10}

([a]) Without the processes of increasing cross-section and the charge exchange.
([b]) With the processes of increasing cross-section and the charge exchange.

energies $E_0 = 10^{18},\ 10^{19},\ 10^{20}$ eV for Case A (with $b = 0$), where the exact analytical solutions are possible. One can see in the figure that the air showers are at the maximum development at sea level and that the relation $E_0/N_\mathrm{e} \simeq 2.0$ (GeV) holds approximately.
ii) Figure 4 shows the ratio of the air shower size between cases B, C, D and case A for the primary energy $E_0 = 10^{20}$ eV. In fig. 4 one can see the following:
1) The effect of the charge exchange process of the surviving pion is almost constant over the atmospheric depth, amounting to 13%([3]).
2) The effect of increasing cross-section is large (100–200%) at high altitude, but is small (~ 18 %) at sea level([3]).
3) The effects of scaling violation, in Model-1 and in Model-2, have similar depth dependence, but their absolute values differ by a factor five.
4) Model-2 gives smaller air shower size, and the attenuation of the air shower size after the shower maximum is very slow due to the small value of inelasticity([3]).
5) At sea level the air shower size is dependent most strongly on the energy distribution of the produced particles, but less strongly on the increasing cross-section and on the charge exchange.
iii) The air shower size at sea level, expected by the present calculation, is tabulated in table III for an incident proton energy of $E_0 = 10^{20}$ eV. In the table the effects of the charge exchange process and the increasing cross-section are obtained by fig. 4. To calculate the expected air shower size, to which the effects of charge exchange probability and increasing cross-section are included, we multiplied all the factors because the factors are near 1.0.
iv) M. Nagano *et al.* examined the method of energy determination of extremely high

([3]) This tendency can be explained by the analytic expression of the air shower size.

energy air showers, employed by AGASA experiment, by the simulation code of CORSIKA [8](with QGSJET code). And they reached the conclusion that the method works well for the highest energy air showers [9]. The simulation gives $N_e = 5.5 \times 10^{10}$ for the proton-induced air showers of $E_0 = 10^{20}$ eV([4]). We can see the following points by comparing the value with the expected sizes in table III.

1) The value by the simulation is between those of Model-1 and Model-2. In this sense our calculation and the simulation are consistent with each other, because we saw in sect. **2** that the pseudo-rapidity density distribution by the QGSJET code is between those by Model-1 and Model-2.

2) If we take Model-1, the energy spectrum of highest energy air showers shifts to the left (toward lower energy) by a factor 1.5.

3) If we take Model-2, which is the best-fitted to the experimental data, the energy spectrum shifts to the right (toward higher energy) by a factor 3.7.

Among the major factors which govern the cosmic ray diffusion in the atmosphere —the energy distribution of produced particles, the charge exchange probability of the surviving pion and the increasing cross-section of hadron-air collisions—, the first one has the largest effect on the size of extremely high energy air showers. Hence *we have to specify the energy distribution of produced particles in multiple particle production in more detail, in order to confirm the extremely high energy cosmic rays exceeding the GZK cut-off energy.*

v) The item (3) in the above paragraph iv) makes the puzzle of extremely high-energy cosmic rays more serious. Probably it is not irrelevant to conclude that the energy dependences of the scaling violation parameters in Model-2 are not valid in the extremely high energy region. In other words Model-2 does not describe the extremely high-energy air showers, although the model is formulated based on the experimental data of direct observation. This is due to the fact that Model-2 predicts quite small inelasticity at high energies. For example, the value is as small as 0.2 even at $E_0 = 10^{16}$ eV (see fig. 1). According to our previous analysis of the attenuation mean free paths of the hadron and (e, γ) components [10], the inelasticity $\langle K \rangle = 0.5$ is compatible but a smaller inelasticity is not compatible with the experimental data in the energy region 10^{14}–10^{16} eV.

REFERENCES

[1] Ohsawa A., *Proceedings of the XI International Symposium on Very High Energy Cosmic Ray Interactions, Campinas (2000)*, *Nucl. Phys. B (Proc. Suppl.)*, **97** (2001) 16.

[2] Alner G. L. *et al.*(UA5 Collaboration), *Z. Phys. C*, **33** (1986) 1; *Nucl. Phys. B*, **291** (1987) 445; *Phys. Rep.*, Nos. 5 and 6 (1987) 247.

[3] Pare E. *et al.*, *Phys Lett. B*, **242** (1990) 531.

[4] Haar R. *et al.*, *Phys. Lett. B*, **401** (1997) 176.

[5] Arata N., *Nucl. Phys. B*, **211** (1983) 189.

[6] Feynman R., *Phys. Rev. Lett.*, **23** (1969) 1415.

[7] Taylor F. E. *et al.*, *Phys. Rev. D*, **14** (1976) 1217.

[8] References of the simulation codes are found in the paper by Knapp J. *et al.*, Preprint of Forchungszentrum Karlsruhe, FZKA 5828 (1996).

[9] Nagano M. *et al.*, Preprint of Forschungszentrum Karlsruhe, FZKA 6191 (1998).

[10] Ohsawa A. *et al.*, *Phys. Rev. D*, **49** (1992) 3128-3133.

([4]) We obtained $N_e = 5.5 \times 10^{10}$ for the proton-induced air showers of $E_0 = 10^{19}$ eV, from the figure in ref. [9], and multiplied it by 10.