IL NUOVO CIMENTO Vol. 24 C, N. 4-5 Luglio-Ottobre 2001

Particle density distributions of inclined air showers(*)

M. T. Dova, L. Epele and A. Mariazzi(**)

Departamento de Física, Universidad Nacional de La Plata - La Plata, Argentina

(ricevuto il 15 Ottobre 2000; approvato il 12 Febbraio 2001)

Summary. — The Nishimura-Kamata-Greisen cascade theory is re-analyzed in order to consider inclined showers. A new parameterization of the lateral distribution function including azimuth angle dependence is presented. Monte Carlo studies for 10^{19} eV proton-induced air showers indicate that the proposed lateral distribution function fits the data very well.

PACS 96.40 – Cosmic rays.
PACS 96.40.Pq – Extensive air showers.
PACS 01.30.Cc – Conference proceedings.

1. – Introduction

There are a number of reasons why EAS lateral distributions are of importance for the air shower phenomenon. The most important one is that from the number and distribution of ground particles the energy and mass of the primary can be deduced. The energies of primary cosmic rays are estimated by multiplying an observable such as the total number of electrons N_e, muons N_μ or the particle density at 600 m ρ_{600} by a conversion factor [1]. The quantities N_e, N_μ and ρ_{600} are determined by using the lateral distribution of electrons and muons. The lateral distribution functions also carry information on the related particle physics and astrophysics. Different hadronic models predict different lateral shapes. On the other hand, the expected lateral shape also depends on the mass of the primary cosmic rays.

Analytical and empirical parameterizations of the LDF for near vertical showers already exist [2-5]. For large zenith angles the axial symmetry is no longer valid, as reported by C. Pryke [6]. The geometrical effect can be corrected by projecting into the shower plane, but there are still deviations from this behavior due to the different stage of development at different azimuthal angles [7]. In order to improve inclined shower LDF

(*) Paper presented at the Chacaltaya Meeting on Cosmic Ray Physics, La Paz, Bolivia, July 23-27, 2000.

(**) E-mail: mariazzi@venus.fisica.unlp.edu.ar

parameterizations, we chose an analytical parameterization of a vertical shower derived by Nishimura, Kamata and Greisen [4] for pure electromagnetic cascades and studied its dependence on depth. The aim of the present work is to extend this parameterization to include inclined air showers.

2. – Electromagnetic cascade theory

Diffusion equations for electromagnetic cascade showers are

$$\frac{\partial N_\mathrm{e}}{\partial t} = -AN_\mathrm{e} + BN_\gamma + \epsilon\frac{\partial N_\mathrm{e}}{\partial E} + \sigma_1 N_\mathrm{e}\,, \tag{1}$$

$$\frac{\partial N_\gamma}{\partial t} = -CN_\mathrm{e} - \sigma_0 N_\gamma\,, \tag{2}$$

$N_\mathrm{e}(E_0, E, r, \theta, t)$ is the number of electrons at a depth t with energy $(E, E{+}\mathrm{d}E)$, traveling at an angle θ with the shower axis, at the position r. $N_\gamma(E_0, E, r, \theta, t)$ is the respective number of photons. σ_0 is the total cross-section of pair creation. $A(E, E_0, t)$, $B(E, E_0, t)$, $C(E, E_0, t)$ are integral operators corresponding to radiation and pair creation processes and $\sigma_1 N_\mathrm{e}$ is the variation in the number of electrons in an interval $(\theta, \mathrm{d}\theta)$ caused by scattering while traveling the thickness $\mathrm{d}t$.

Extensive showers develop in the atmosphere; hence variations in the density of air with height should be taken into account in the calculation. The quantity that naturally describes the varying atmospheric medium is the vertical atmospheric depth t measured in cascade units:

$$t = \frac{\int_z^\infty \rho_\mathrm{atm}(z)\mathrm{d}z}{X_0}\,. \tag{3}$$

The lateral structure function derived from the previous equations by Nishimura, Kamata and Greisen is

$$\rho_\mathrm{NKG}(r, s) = \frac{N_\mathrm{e}}{r_\mathrm{m}^2} f(s, r/r_\mathrm{m}) = \frac{N_\mathrm{e}}{r_\mathrm{m}^2} C(s)\left(\frac{r}{r_\mathrm{m}}\right)^{s-2}\left(1+\frac{r}{r_\mathrm{m}}\right)^{s-4.5}\,, \tag{4}$$

$s = s(t)$ is called age parameter because it is related with the shower stage of development, r_m is the Moliere radius and X_0 is the radiation length in air ($36.7\,\mathrm{g/cm}^2$).

The formula of Nishimura, Kamata and Greisen (NKG) has been much used both in theoretical developments (3D extensive air shower simulation) and in the description of experimental results [8].

2·1. *The lateral age parameter*. – The radial dependence of the age parameter was studied by Akeno experiment [9] and J. N. Capdevielle [10, 11].
The age parameter radial dependence in the NKG function is given by eq. (5):

$$s = \frac{3}{1 + \frac{2\left(\ln(\frac{E_0}{\epsilon_0} + \ln(r/r_\mathrm{m})\right)}{t}}\,. \tag{5}$$

In a first attempt to the problem we consider the age parameter only as a function of the vertical depth t:

$$s = \frac{3}{1 + \frac{2\,\ln\left(\frac{E_0}{\epsilon_0}\right)}{t}}\,, \tag{6}$$

ϵ_0 is the critical energy (84 MeV) and E_0 is the incident energy.

2.2. *The Moliere radius.* – The Moliere radius r_{m} is a characteristic unit of length in the scattering theory, the geometrical value of r_{m} is inversely proportional to the density of the medium (eq. (7)). For an isothermal atmosphere it will be inversely proportional to the vertical depth t (eq. (8)).

$$r_{\mathrm{m}} = \frac{4\pi.m.c^2}{\alpha}\frac{X_0}{\epsilon_0} \sim \frac{1}{\rho_{\mathrm{atm}}}\,, \tag{7}$$

$$r_{\mathrm{m}}(t) = r_{\mathrm{m}}(t_0)\frac{\rho_{\mathrm{atm}}(t_0)}{\rho_{\mathrm{atm}}(t)} \approx r_{\mathrm{m}}(t_0)\frac{t_0}{t}\,. \tag{8}$$

3. – Inclined showers

In this section we present a first attempt to extend the NKG solution to inclined showers. In the case of a homogeneous atmosphere, one has rotational invariance and the solution is the same as the vertical induced shower but evaluated in the coordinates of the rotated system:

$$x' = x\cos\theta + z\sin\theta = x_a\cos\theta\,, \tag{9}$$

$$y' = y = y_a\,, \tag{10}$$

$$z' = z\cos\theta - x\sin\theta = \frac{z}{\cos\theta} - x_a\sin\theta\,. \tag{11}$$

The (x_a, y_a) coordinates are the Cartesian coordinates of the particles measured from the intersection between the shower axis and a plane parallel to the ground. The inclined shower structure function is given by eq. (12):

$$f(r'/r'_{\mathrm{m}}, s') = C(s')\left(\frac{r'}{r'_{\mathrm{m}}}\right)^{s'-2}\left(1 + \frac{r'}{r'_{\mathrm{m}}}\right)^{s'-4.5}\,, \tag{12}$$

$$r'_{\mathrm{m}} = r_{\mathrm{m}}\frac{t}{t'}\,, \tag{13}$$

$$s' = \frac{3}{1 + \frac{2\,\ln(\frac{E_0}{\epsilon_0})}{t'}}\,, \tag{14}$$

$$t' = \frac{\int_{z'}^{\infty}\rho_{\mathrm{atm}}(z)\mathrm{d}z'}{X_0}\,. \tag{15}$$

Actually, the structure of an air shower is quite complex but, as we conclude in a previous work [7], an inverted cone seems to be a reasonable first approximation (eq. (16)). If the cone angle $\tan\alpha$ is considered nearly independent of z, then the particle density

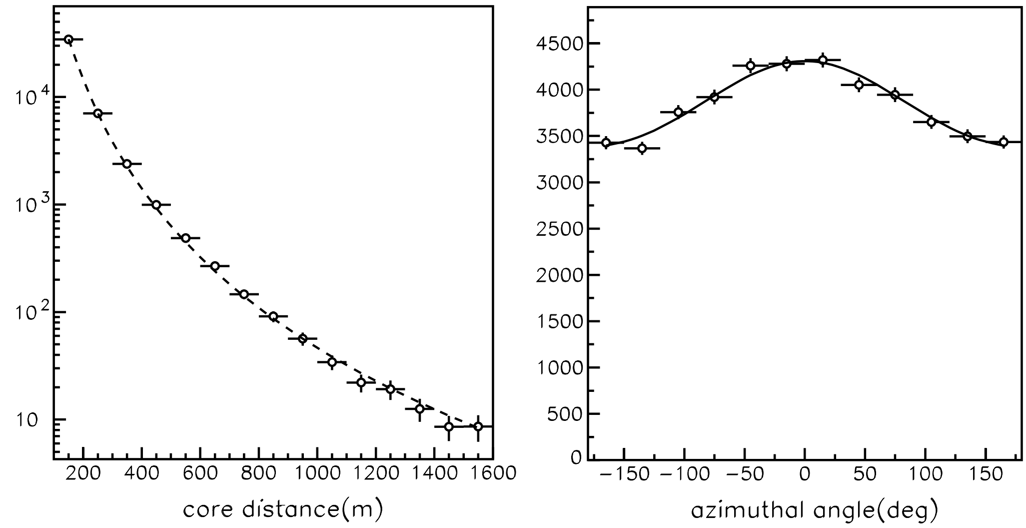

Fig. 1. – Density projections at $875\,\mathrm{g/cm}^2$ for a $30°$ inclined shower.

distribution for an inclined shower can be expressed as in eq. (12) with the slant depth t' which includes an azimuthal angle dependence.Then,

$$\tan\alpha = \frac{r'}{(L - z')}\,, \tag{16}$$

$$\frac{z}{\cos\theta} - L = (z' - L)\left(1 - \frac{r'}{(z' - L)}\tan\theta\cos\phi\right)\,, \tag{17}$$

$$\frac{\mathrm{d}z'}{\mathrm{d}z} = \frac{1}{\cos\theta\ (1 - \tan\alpha\ \tan\theta\ \cos\phi')} \tag{18}$$

and finally

$$t' = \frac{t}{\cos\theta\ (1 - \tan\alpha\ \tan\theta\ \cos\phi')}\,. \tag{19}$$

4. – Comparison with AIRES simulation data

Air showers were generated with the program AIRES with QGSJET hadronic model. 10^{19} eV proton-induced showers with first interaction on top of the atmosphere were simulated at zenith angles varying from 0° up to 70°. For high energy extensive air showers ground level is normally beyond the point of maximum development. The shower lateral distributions were evaluated at different observing levels. The new parameterization of the NKG lateral distribution function for an inclined shower (eq. (12)) is a function of 3 parameters: $N_{\mathrm{e}}, r_{\mathrm{m}}, \tan\alpha$. This parameterization was fitted to the previous shower simulation data at different zenith angles. Figures 1, 2, and 3 show particle density along the core distance and azimuthal angle projections of bi-dimensional fits to the data for 30°, 50° and 60° zenith angles.

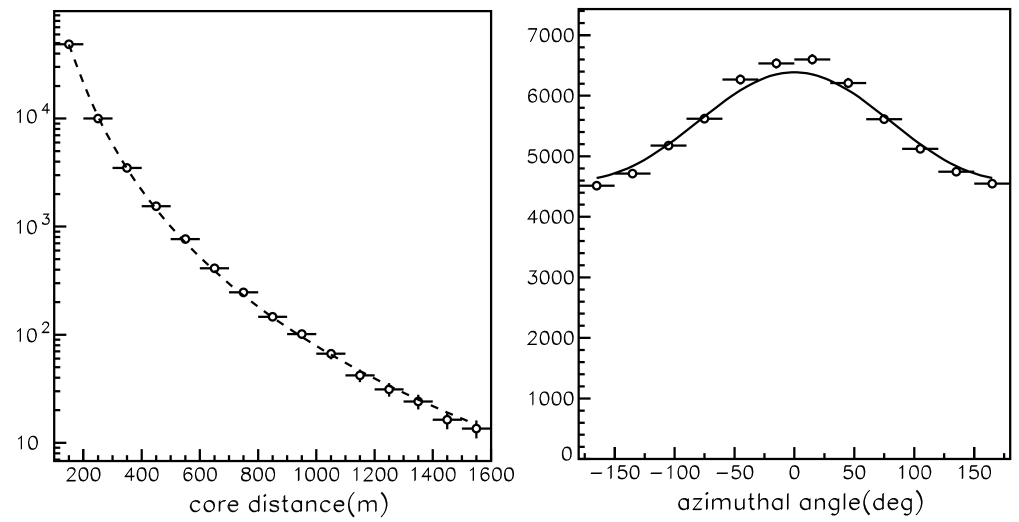

Fig. 2. – Density projections at $595\,\mathrm{g/cm^2}$ for a 50° inclined shower.

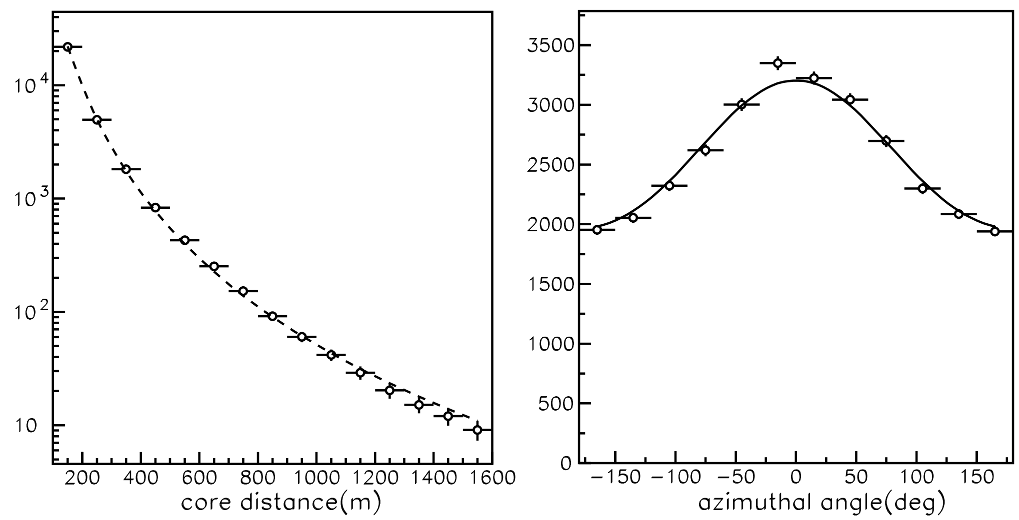

Fig. 3. – Density projections at $595\,\mathrm{g/cm^2}$ for a 60° inclined shower.

5. – Conclusions

We have studied the structure of vertical and inclined showers comparing analytical lateral distribution functions with shower simulations data. Starting from the Nishimura Kamata results we analyzed the depth dependence of the LDF for vertical showers. Then we extend this result to inclined showers using a conic approximation to the shower structure. The parameterization obtained fits well the simulation data at different zenith angles.

REFERENCES

[1] Hillas M. *et al.*, *Proceedings of the 12th ICRC (Hobart)*, vol. **3** (1971) p. 1001.
[2] Yoshida S. *et al.*, *Astropart. Phys.*, **3** (1995) 105.
[3] Kamata K. and Nishimura J., *Prog. Theor. Phys.*, **6** (1958) 93.
[4] Nishimura J., *Hand. Phys.*, **46** (1967) 1.
[5] Greisen K., *Ann. Rev. Nucl. Sci.*, **10** (1960) 63.
[6] Pryke C., Auger technical note GAP-98-034.
[7] Dova M. *et al.*, *Proceedings of the 26th ICRC (Salt Lake City)*, vol. **1** (1999) p. 478.
[8] KASCADE Collaboration, preprint astro-ph/00042333.
[9] Nagano M. *et al.*, *J. Phys. Soc. Jpn.*, **53** (1984) 1667.
[10] Capdevielle J. *et al.*, *J. Phys. G*, **4** (1978) 303.
[11] Capdevielle J. *et al.*, *J. Phys. G*, **6** (1980) 901.