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Self-Organized Criticality (SOC) model of solar flares(*)

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Summary. — To describe the distribution of the total number of flares per time unit $p(E)$, we bring forward a new self-organized critical model subject to uniform small-scale magnetic element and driving and dissipation. Due to diversity and complex interrelation of processes in the solar atmosphere, one needs to find the "main" process that "drives" the other ones. Magnetic-field reconnection in the sun atmosphere was usually treated as the main process by the SOC models. We, however, give the crucial role to the annihilation of oppositely charged magnetic elements on the sun surface, the elements being intersections of magnetic tubes with the sun surface.

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1. – Introduction

The mechanism of energy accumulation and release in the solar atmosphere is one of the key problems in astrophysics. The observational data and theoretical results available to date allow to reduce the possible variety of answers to the analysis of convective motions of plasma with frozen-in magnetic field. However, the mechanism of energy accumulating and release is not yet clear.

Observations of flare activity resulted in establishing the power law in the distribution of the flares peak of amplitude, *i.e.* the number of flares N with amplitude P is defined by the formula

$$N(P) \sim P^{-\alpha_P}\,, \tag{1}$$

where $\alpha_P = 1.75 \pm 0.15$, valid for several values of P. Similar formulae are relevant for a "spectrum" of flares of different energy output (fluency) E and continuances T

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with values amounting to $\alpha_E = 1.45 \pm 0.15$ and correspondingly $\alpha_T = 1.6 \pm 0.4$ [1-3]. Considerable ambiguity in the values is not due to the lack of statistics only, but also to the difficulties in the systematization of data in different spectral bands.

Power law distributions (1), with no characteristic values for certain parameters, are typical for systems in the critical state, where systems become holistic in spite of short-range interaction between their elements. However, the fine adjustment of a control parameter necessary to reach the critical state is possible only under laboratory environment. So the reason for the prevalence of power law in nature was understood only in the late 80th with the advent of the theory of self-organized criticality, with a pile of sand as a base model [4].

Lu and Hamilton and co-authors in [5,6] made the first attempt to put new a theory for solar processes modeling. The model (hereinafter LH model) was based on the assumption that, if the angle between the vectors of the magnetic field at the current sheet opposite sides is less than a certain threshold value, reconnection goes slowly due to the high conductivity of the solar corona plasma. However, if the angle exceeds the threshold, the reconnection can be explosive. Such a mechanism of reconnection allows the accumulation of energy in the helicity magnetic field, followed by the energy discharge during the avalanche evolution of the flare.

The LH model interprets the active zone as a 3D grid with elements of the field corresponding to its cells (hereby the notion of the field is treated differently by different authors). If the value of the field is considerably different from the average in the six closest neighboring cells, a reconnection occurs. It is a uniform distribution of the field excess in the given cell between this cell and its neighbors. In its turn, there connection can provoke reconnections in the neighboring cells, *i.e.* an avalanche develops. Since the field energy is in pro rata square, every reconnection is followed by emission of energy. Computer analysis of the LH model shows a power law distribution of amplitude, energy and duration of flares, which is in good agreement with the observational data.

In the frame of this LH model (and versions thereof), the areas are independent and occur at a constant rate; hence the distribution of intervals between flares (waiting time distribution—WTD) obeys Poisson statistics.

However, Boffeta *et al.* [7] and Lepreti *et al.* [8] showed that the WTD constructed on the base of the GOES catalog yields evidence of deviation from Poisson statistics, which fact is assumed to be inconsistent with the avalanche model.

In response, Wheatland [9,10] argued that

i) over several solar cycles included in the GOES data, the flare rate varies by more than an order of magnitude, so the flares cannot be assumed to occur at a constant rate (if the flaring process can be represented by a piecewise constant Poisson process, WTD was shown to qualitatively reproduce the observed power law tail);

ii) the deviation from Poisson statistics is due to the failure to detect flares occurring soon after large flares because of increased soft–X-ray flux associated with the large flare.

Recently Georgoulis, Vilmer and Crosby [11] compared the statistical properties of avalanche flare models with the respective properties of WATCH data base. They found no correlation of time intervals between successive bursts arising from the same active region with peak intensity of the flare.

Also, Norman *et al.* [12] built a modified LH model driven by a non-stationary random process. In the resulting models waiting times frequency distribution includes a power law tail.

All above-mentioned shows that neither real to model time transformation, nor real solar atmosphere to model plane transformation are trivial as usually. In this case there is no argument in favor of direct correspondence between SOC-simulated WTD for avalanches and real WTD for solar flares (*e.g.*, work by Norman *et al.*, when a minor change in the model results in a final distribution of another kind).

2. – Solution

When undertaking to build a model of processes in the solar chromosphere, we had for the key priority to make it as close as possible to the real processes in the Sun. Hereby, we followed the basic assumption of synergetics, that inside the multitude of different processes, there should be a dominant one, responsible for the discharge of the bulk of energy, by which all the other processes are regulated. The energy supplied to the atmosphere is encapsulated in the tubular magnetic field frozen into plasma. The analysis of the solar magnetograms with are solution of up to an angular minute shows [13] a number of small loops on the solar surface, which appear and disappear from time to time. The pattern formed by the loops looks like a carpet. The "carpet" pattern configuration can change very quickly, whereby the local increase in brightness coincides with disappearance of the loops, which means that they have all been reconnected and disappeared. This fact by itself changes the existing views of the magnetic activity in the solar atmosphere, since the existence of such a magnetic carpet increases the volume of the energy available to heat the solar corona, as the chromosphere, formerly magnetoneutral, appears to be composed of zones of alternate polarity.

Bearing in mind the above mentioned, as well as the ideas suggested by the other authors, we built a model based on the concept of magnetic elements being the points of entry into the chromosphere of small-scale tubes of magnetic field frozen into plasma, whereby these tubes are arranged along the Sun radius. Hereby, we assume that the processes affecting the magnetic field in the corona stem from the processes affecting the magnetic elements in the chromosphere, *i.e.* the dominant process takes place in the area of concentration of the bulk of matter and energy.

Our model is a simple cellular automaton representing the active region by means of a two-dimensional rectangular lattice with periodic boundary conditions. Each lattice cell can contain one or more magnetic elements or it can be empty. Model rules are as follows:

1) Two magnetic elements of opposite sign and equal absolute value appear simultaneously in two random cells. Their value is Poisson random deviated with mean Q.

2) Coming into a cell tests it for the presence of elements of opposite sign. If any opposite element is present, then one of them chosen at random annihilates with the incoming element. The absolute values of annihilating elements are decreased by one and the unit of energy is released. If the element's value becomes zero, the element disappears.

Any release of energy in a cell causes an outward disturbance wave. This wave carries out all elements from the cell to its neighbors picked out at random among eight adjacent cells. These elements can also cause annihilations there (step 2)) resulting in an avalanche of annihilations. Such avalanche is nothing else than flare. If the transfer of elements does not give rise to new annihilations, then the avalanche is over and the step 1) repeats.

An important feature of the model under consideration is the assumption that the

TABLE I.

	α	E, energy	S, area	P, amplitude	T, duration
E	1.37		0.80	0.50	0.55
S	1.42	1.25		0.67	0.71
P	1.61	2.00	1.50		1.10
T	1.66	1.82	1.40	0.91	

energy of annihilation itself be the source of turbulent motion of plasma capable of annihilation of magnetic elements. Hereby, it is necessary to identify the "trigger" of the avalanche. Therefore, during the next stage, the model introduces 2 elements of equal value and opposite charge into a randomly chosen cell, which corresponds to the emergence of the magnetic tubes from inside of the Sun.

The given example is a self-organized critical model, *i.e.* it evolves to a critical point independent of the initial status, in which it is described by a power dependence. To avoid effects of the system geometrical sizes limitations, we used a grid of the biggest size process able with our computer (512×512). For statistical purposes we have made 3×10^7 steps after the model has achieved its critical point.

3. – Results

Beside the energy, amplitude and duration of flares, we were interested in their size and the number of local peaks of power during the life of a flare. Now let us consider the results of the modeling. Figure 1 (top) represents the data on the flares distribution based on the amount of energy discharged. In the dual logarithmic coordinates one can clearly see linear stretches, corresponding to the power law distribution. In order to obtain its value, let us resort to the standard procedure of scaling [14]. Let us assume that the probability of flares of energy E with a given Q is set by the scaling formula

$$P(E) \sim Q^{-\mu} f(EQ^{-\beta}) , \tag{2}$$

whereby $f(x) \sim x^{-\alpha}$ in moderate values and is considerably reduced in big values. In the zone of intermediate energy values, the rate of probability should not depend upon Q, therefore $\alpha\beta - \mu = 0$ which allows a precise estimation of the α value. At the bottom is a chart $p(E)Q^{\mu}$ subject to EQ^{β} with scaling values, ensuring the best overlapping of the charts: $\beta = 1.50$ and $\mu = 2.05$. Thus, $\alpha = 1.37$. The distribution of values of other characteristics, listed in the second column of table I, is obtained in a similar way. Beside individual characteristics of avalanche distribution it is interesting to know their correlation. Although there is no direct dependence between them, one can speak of an average. Figure 2 shows the distribution of average flare energy in duration. It is a straight line in dual logarithmic scale with a tilt 1.82. Items of similar dependences in other flares characteristics are listed in the table. It should be noted that the indexes, which link energy values and flare amplitude and duration, are in very good consistency with the observational results [6].

It should also be noted that the values obtained are lower than those really observed. The reason, we guess, is as follows. Our model does not cover one important condition. In the evolution of a flare, the heating of plasma should result in the displacing of the activity outside the boarders of the flare and the motion of magnetic elements out of the center as well. Since this motion is unilateral, it results in a kind of inertia. As is shown in the analysis, the inertia concerned will increase the distribution of values. However, more detailed results will be published later. Presently, we shall just make a brief theoretical research into the model, which, beside anything, allows to forecast the above-mentioned inertia effect. Small-scale flares can develop incidentally. However, the large ones spring out only if their magnetic elements are depleted. Since a magnetic element can only move as a result of annihilation with an oppositely charged element, the number of steps made by the element during its lifetime can be estimated as Q. And since its motion is purely diffusive, the displacement amounts to

$$\Delta\ell \sim \sqrt{Q}\,. \tag{3}$$

As soon as the length of the evolving flare front acquires this value, the flare ceases to

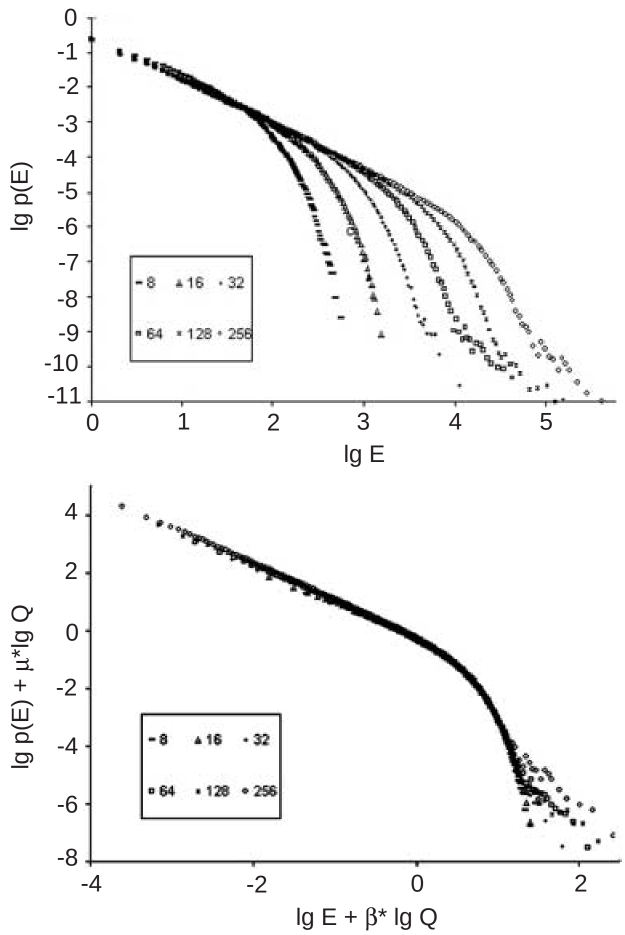

Fig. 1. – At the top we show the distribution of energy flares under various values of Q in dual logarithmic scale. At the bottom there is the scaling of the charts with $\beta = 1.5$ and $\mu = 2.05$, which ensures their exact overlapping.

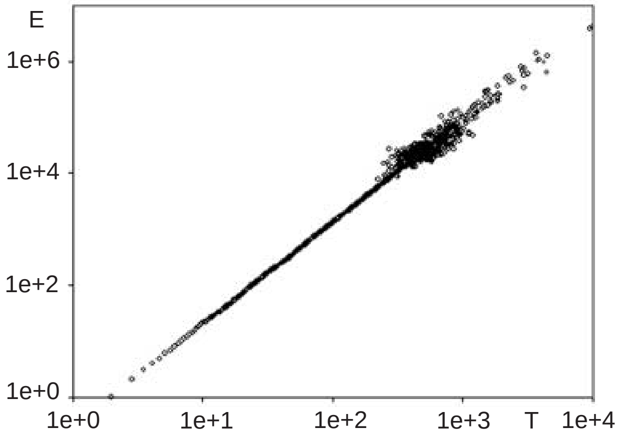

Fig. 2. – Average energy of the flare in the duration $Q = 256$. Tilt: 1.82.

behave as a rigid unit and compensation of local shortages of magnetic elements from the neighboring zones becomes impossible. To interrupt a flare, the number of disappeared magnetic elements should amount to the length of the flare front $\Delta n \sim \ell$. In this case, their linear density will be reduced by a finite value. And since one disappearance of element occurs approximately once in Q annihilations, $E_c \sim Q\Delta n$, which together with formula (3) gives $E_c \sim Q^{3/2}$, *i.e.* $\beta = 3/2$. Hence, $\mu = 2$ and $\alpha = \mu/\beta = 1.33$, which perfectly complies with the results of the modeling.

If the flare possesses considerable energy, the magnetic elements move mostly in one direction and correlation (3) is described as

$$\Delta\ell \sim Q \tag{4}$$

making $\beta = 2$, $\mu = 3$ and $\alpha = 1.5$, which matches the observations data.

The value of $\alpha = 1.5$ corresponds to the branching process with independent particles, whereby one can neglect the possibility of the connection of magnetic elements, which have already been connected in the evolution of the avalanche concerned. It is equivalent to the case of infinitely dimensional space, *i.e.* the value of inertia is to some extent equal to the increase in the dimensions of space.

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