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The case for 2d turbulence in Antarctic data(*)

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Summary. — In this paper we analyze the data that was collected at the British Haley Station in Antarctica on June 22, 1986. This data contains measurements of the temperature and wind velocity at three heights (5 m,16 m and 32 m). Using the Karahunen-Loeve algorithm we decompose this "raw" data into mean flow, waves and turbulent residuals. We then apply three tests to find if the turbulent field might represent "two-dimensional turbulence". The first of these tests was devised by Dewan (see *Radio Sci.*, **20** (1985) 1301), while the second relates to the scaling of the structure function (see Lindborg E., *J. Fluid Mech.*, **388** (1999) 259). To confirm further the results of these two tests, we show that around a frequency of 0.5 rad/s most of the spectral plots for the raw data exhibit a slope of −3. We also construct a scaling model in an attempt to interpret part of the high-frequency spectrum of this data which is almost flat and discuss its possible relation to Bolgiano "buoyancy range turbulence".

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PACS `47.27.-i` – Turbulent flows, convection, and heat transfer.
PACS `93.30.Ca` – Antarctica.

1. – Introduction

Two-dimensional turbulence has been the subject of intense theoretical research [1,2] and simulation experiments [3]. The reason for this interest stems from the fundamental differences between 3d isotropic and 2d turbulence. To begin with, vortex stretching is absent in 2d as a direct consequence of Navier-Stokes equations. Furthermore in 3d the energy cascade is from the large eddies to the small one but this process reverses itself in 2d and leads to the formation of large-scale coherent eddies. Another difference between two- and three-dimensional turbulence exists in the inertial range of the spectrum.

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Kraichnan showed [4] that in 2d in addition to Kolmogorov inertial range there is (due to ensotrophy conservation in zero viscosity) another scaling law in the form

$$E(k) = c\eta^{2/3}k^{-3} ,$$

where η is the ensotrophy dissipation rate.

While many simulations [5,6] confirm these theoretical predictions the actual observation and detection of 2d turbulence as a natural phenomena remains (as far as we know) an open question.

One of the objectives of this paper is to weigh in the evidence for 2d turbulence in the Antarctic data that was obtained by the British observation post at Haley Station in Antarctica on June 22, 1986. A complete and detailed description of the station location and the instrumentation used has appeared in the literature and will not be repeated here. See [7-11]. The data under consideration consists of 65000 simultaneous readings of the flow field and temperature at a fixed interval of 0.05 s. In a nutshell the gross features of the data have been described [12] as follows:

"strong surface inversions prevail over the slopes of the antarctic continent...The regional slope of the terrain at Halley is believed to be about 0.002... This has the effect of forcing persistent surface easterly wind and generates a low level jet in the wind profile..."

The importance of these measurements stem from the fact that the flow field $\mathbf{u} = (u, v, w)$ and the temperatures were measured simultaneously at three different heights viz. 5 m, 16 m and 32 m. These simultaneous readings enable us to apply a test devised by Dewan [13] for the detection of 2d turbulence. According to this test 2d turbulence is characterized by small values for the coherence [14] between the time series which represent the various meteorological variables at different heights. To confirm the results of this test, we computed also the second- and third-order structure functions and tested these against the theoretical predictions that were made recently by Lindborg [15]. Finally we examined also the spectrum of the data which shows clearly a slope of -3 for frequencies in the range of 0.1–0.8 rad/s.

From another point of view Antarctic data represents a stably stratified medium. Its modeling and study has implications for other stably stratified media such as the stratosphere. Furthermore similarities and contrasts between the Antarctic data and data about the nocturnal boundary layer is of great interest (in fact King [12] uses the same data that is being analyzed by us here). Moreover since Antarctic data represents a strongly stratified medium (according to mission records the temperature gradient with height can reach 1 K/m) several authors speculated in the past [16, 17, 13] that under these circumstances there might exist "buoyancy range turbulence" (BRT) which should lead to a flattening of the spectra in parts of the inertial range. In fact the spectrum of this data does show a range in which it is almost flat and thus support the theoretical arguments that were advanced for the existence of BRT. Further confirmation for BRT is offered by the second-order structure function for the temperature which exhibits a region in which it scales as $r^{2/5}$ as predicted by Bolgiano [16, 17].

As a first step in the analysis of this data we apply Karahunen-Loeve methodology (see [18] and references cited therein) to decompose the data into mean flow, waves and turbulence residuals. Although this method is not perfect it does lead to a decomposition where the presence of waves in the turbulent residuals is reduced considerably. This enables us to apply the aforementioned tests and draw conclusions with a reasonable degree of confidence.

TABLE I. – *Values of* m_1, m_2 *which were used to decompose the data to mean, waves and turbulent residuals.*

	m_1	m_2
u at 5 m	2	22
v at 5 m	2	22
w at 5 m	2	30
T at 5 m	2	22
u at 16 m	2	42
v at 16 m	2	40
w at 16 m	3	37
T at 16 m	2	41
u at 32 m	4	48
v at 32 m	1	40
w at 32 m	4	51
T at 32 m	2	42

Throughout the paper we use Taylor frozen turbulence hypothesis which is equivalent to saying that the time interval over which measurements were made is much smaller than the time scale for dynamical changes in the atmosphere to take place. This allows us [19] to plot the spectra *versus* the frequency (rather than the wave number) and compute the structure function as a function of the the time lag between the measurements.

The plan of the paper is as follows: In sect. **2** we introduce some theoretical background and present a model for the flattenings of the power spectrum of this data at high frequencies. In sect. **3** we describe the method used to filter out the mean flow and waves from the data and the tests that were applied to verify that the residuals actually represent turbulence. In sect. **4** we apply several tests for 2d turbulence and discuss their consequences. We end up in sect. **5** with some conclusions and counter arguments for the existence of 2d turbulence in this data. Thus in spite of the positive tests results obtained in this paper some open questions still linger regarding the final conclusions one can draw. Further research and additional data are needed to settle these questions.

2. – Theoretical considerations

The two-dimensional flow of an incompressible and inviscid fluid conserves both the energy E and the enstrophy Ω. For viscous fluid these quantities decay according to

$$-\epsilon = \frac{\partial E}{\partial t} = -2\nu\Omega, \quad -\epsilon_\omega = \frac{\partial \Omega}{\partial t} = -\nu \mid \overline{\nabla\omega} \mid^2 . \tag{2.1}$$

The energy spectrum is determined therefore by both parameters $\epsilon, \epsilon_\omega$ which leads to the definition of a length scale

$$L_\omega = \left(\frac{\epsilon}{\epsilon_\omega}\right)^{1/2} . \tag{2.2}$$

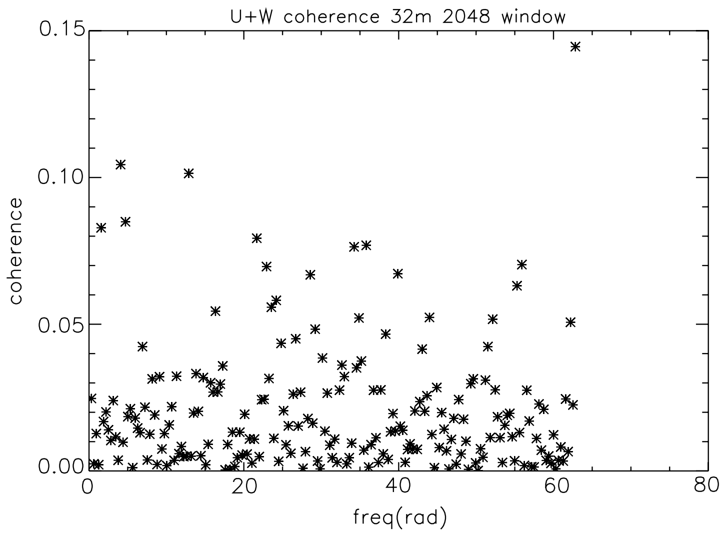

Fig. 1. – Coherence between the turbulent residuals of U and W at 32 m height.

From dimensional considerations one concludes then that [20] the energy spectrum in the inertial range must have the form

$$E(k) = f(kL_\omega)\epsilon^{2/3}k^{-5/3}\,, \tag{2.3}$$

where f is a function of the dimensionless variable kL_ω. If at one end of the inertial range only ϵ is essential (and the effect of ϵ_ω is negligible), then $f \cong$ const and the energy

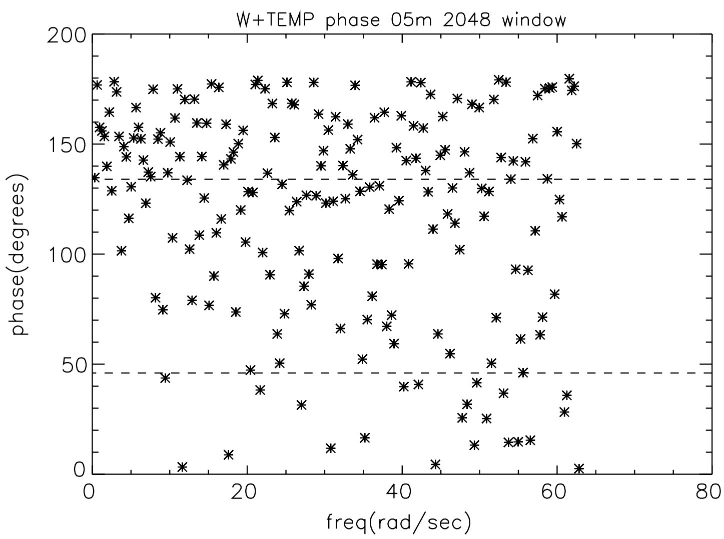

Fig. 2. – Phase between the turbulent residuals of W and temperature at 5 m height. (The wave sector is in between the dashed lines.)

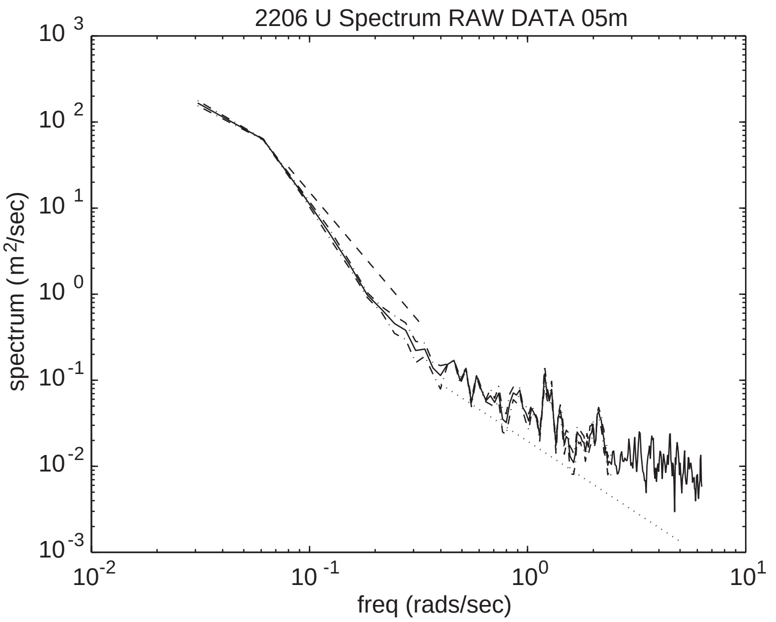

Fig. 3. – Low-frequency spectra of the raw data of U at 5 m height. 95% confidence is shown by the dash-dotted lines. The dashed and dotted lines have slopes of -3, $-5/3$, respectively.

spectrum obeys Kolmogorov 5/3 power law. If, on the other end of this range, ϵ is not essential, then f must have the form

$$f \cong (kL_\omega)^{-4/3} \tag{2.4}$$

and consequently

$$E(k) = C\epsilon_\omega^{2/3} k^{-3} \tag{2.5}$$

(where C is a constant).

For a stratified medium Obukov [21, 22] introduced the temperature inhomogeneity dissipation rate

$$\epsilon_T = 2\chi \int_0^\infty k^2 E_T(k)\mathrm{d}k\,, \tag{2.6}$$

where E_T is the temperature spectra and χ is the heat conductivity of the medium. He further postulated that the turbulent component of T is dependent on this parameter.

For the (stratified) Antarctic medium we would like to enlarge the domain of this postulate to include the velocity components of the flow. This enables us to introduce the buoyancy (length) scale [20, 21]

$$L_\mathrm{B} = (\alpha g)^{-3/2} \epsilon^{5/4} \epsilon_T^{-3/4}\,, \tag{2.7}$$

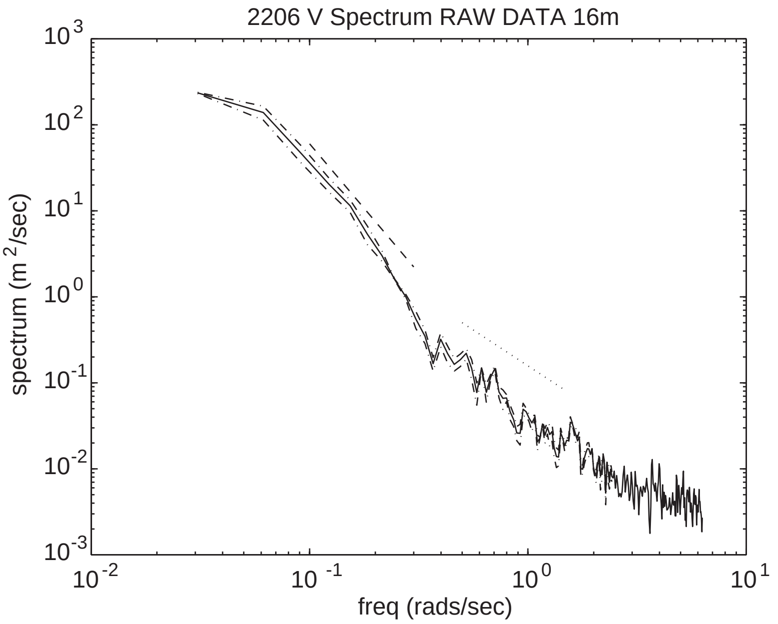

Fig. 4. – Low-frequency spectra of the raw data of V at 16 m height.

where (αg) is the buoyancy parameter. The existence of this second length scale for a stratified two-dimensional flow leads us to replace (2.3) by

$$E(k) = f(kL_\omega, kL_{\rm B})\epsilon^{2/3}k^{-5/3}\,. \tag{2.8}$$

However, since stratification and enstrophy conservation are independent of each other, we infer that f must have the form

$$f \cong (kL_\omega)^r(kL_{\rm B})^s. \tag{2.9}$$

It follows then that the spectral dependence on k is given by

$$E(k) \sim k^{r+s-5/3}. \tag{2.10}$$

We conclude therefore that various combinations of r, s are possible and this will lead to different spectral dependences on k.

Thus, if

$$E(k) \sim k^{-q}$$

and the dissipation ϵ is negligible, we must have then

$$r + s = 5/3 - q,\;\; \frac{r}{2} + \frac{5}{4}s + \frac{2}{3} = 0\,,$$

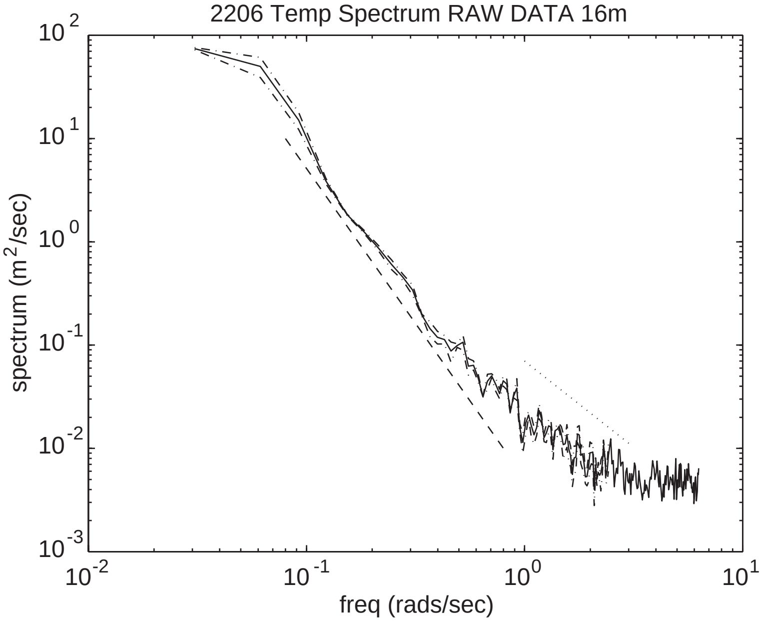

Fig. 5. – Low-frequency spectra of the raw data of temperature at 16 m.

which yields

$$r = \frac{33 - 15q}{9}, \quad s = \frac{15q - 18}{9}.$$

Thus the determination of q must be made from experimental data. It is interesting to note in this context that Kraichnan [4,23] already observed that the "energy spectrum of the flow depends on the details of the nonlinear interaction embodied in the equations that govern the flow and can not be deduced solely from the symmetries, invariances and dimensionality of the equations".

Finally we would like to observe that the data under consideration contains some discontinuities. These can change completely the asymptotic behavior of the spectrum. To demonstrate this assume that the data is described by

$$D(x) = CH(x - x_0) + g(x)\,, \tag{2.11}$$

where C is a constant, $g(x)$ is a smooth function whose Fourier transform (FT) decays exponentially and $H(x)$ is the Heaviside function

$$H(x) = \begin{cases} 1, & x \geq 0, \\ 0, & x < 0. \end{cases}$$

Differentiating (2.11) we have

$$D'(x) = C\delta(x - x_0) + g'(x) \tag{2.12}$$

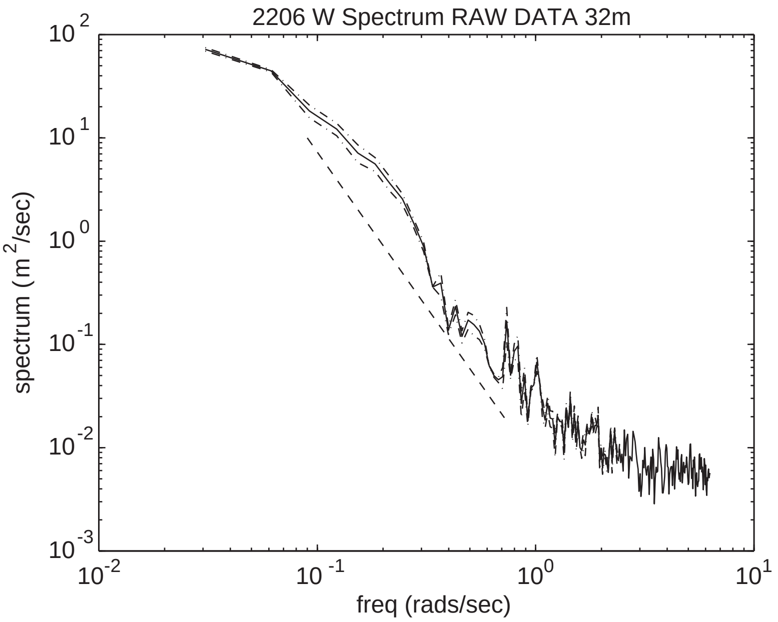

Fig. 6. – Low-frequency spectra of the raw data of W at 32 m height.

and the FT of (2.12) is

$$\tilde{D}'(k) = C + \tilde{g}'(k). \tag{2.13}$$

The FT of D is obtained then by dividing (2.13) by k which shows clearly that the asymptotic behavior of $\tilde{D}(k)$ is proportional to k^{-1}.

We conclude then that a proper filter for the removal of these discontinuities from the data is needed in order to obtain the true spectrum of the turbulent residuals. Such a filtering algorithm is given by the $K - L$ decomposition which was described in sect. **3**.

3. – Data decomposition

The statistical approach to turbulence splits the flow variables $\tilde{\mathbf{u}}, \tilde{T}$ (where $\tilde{T}$ is the temperature) into a sum

$$\tilde{\mathbf{u}} = \mathbf{u} + \mathbf{u}' + \mathbf{u}, \quad \tilde{T} = T + T' + t\,,$$

where $\mathbf{u}, T$ represent the mean (large-scale) flow, u', T' represent waves and u, t "turbulent residuals" [24].

To effect such a decomposition in our data we used the Karahunan-Loeve (KL) decomposition algorithm (or PCA) which was used by many researchers (for a review see [18]). Here we shall give only a brief overview of this algorithm within our context.

Let be given a time series X (of length N) of some geophysical variable. We first determine a time delay Δ for which the points in the series are decorrelated. Using Δ

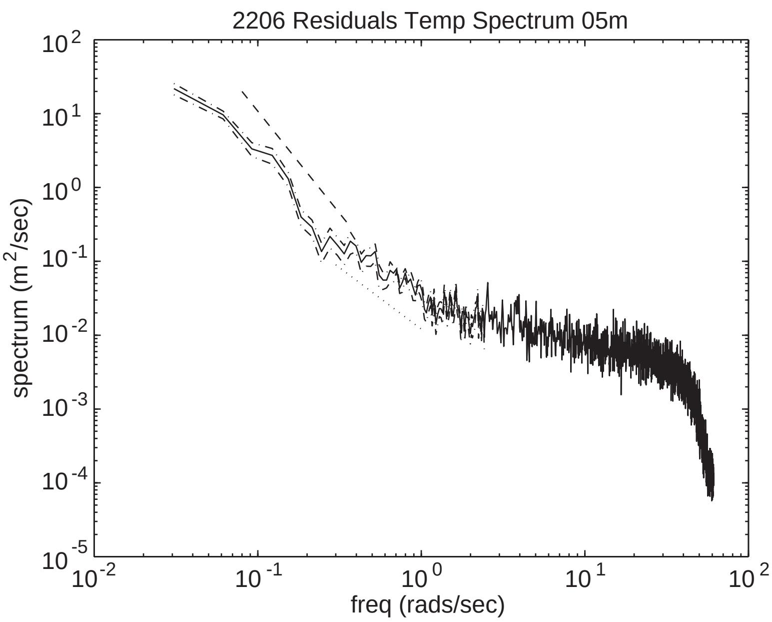

Fig. 7. – Spectra of the turbulent residuals of temperature at 5 m height.

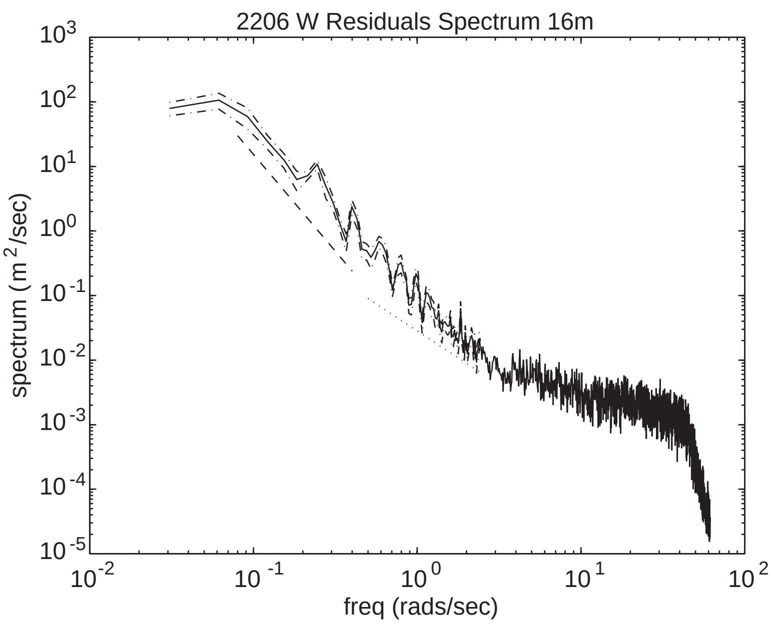

Fig. 8. – Spectra of the turbulent residuals of W at 16 m height.

we create n copies of the original series

$$X(k),\ \ X(d+\Delta),\dots,X(k+(n-1)\Delta).$$

(To create these one uses either periodicity or choose to consider shorter time-series.) Then one computes the auto-covariance matrix $R=(R_{ij})$

$$R_{ij}=\sum_{k=1}^{N}X(k+i\Delta)X(k+j\Delta). \tag{3.1}$$

Let $\lambda_0>\lambda_1,\dots,>\lambda_{n-1}$ be the eigenvalues of R with their corresponding eigenvectors

$$\phi^i=(\phi_0^i,\dots,\phi_{n-1}^i),\ \ i=0,\dots,n-1.$$

The original time series T can be reconstructed then as

$$X(j)=\sum_{k=0}^{n-1}a_k(j)\phi_0^k\,, \tag{3.2}$$

where

$$a_k(j)=\frac{1}{n}\sum_{i=0}^{n-1}X(j+i\Delta)\phi_i^k\,. \tag{3.3}$$

The essence of the KL decomposition is based on the recognition that, if a large spectral gap exists after the first m_1 eigenvalues of R, then one can reconstruct the mean flow (or the large component) of the data by using only the first m_1 eigenfunctions in (3.2). A recent refinement of this procedure due to [18] is that the data corresponding to eigenvalues between m_1+1 and up to the point m_2 where they start to form a "continuum" represent waves. The location of m_2 can be ascertained further by applying the tests devised by Axford [25] and Dewan [13] (see below).

Thus the original data can be decomposed into mean flow, waves and residuals (*i.e.* data corresponding to eigenvalues $m_2+1,\dots,n-1$ which we wish to interpret at least partly as turbulent residuals).

For the data under consideration we carried out this decomposition using a delay Δ of 1024 points (approximately 51 s) for all the geophysical variables. In table I we present the values of m_1, m_2 that were used in this decomposition for the flow variables at different heights. (In all cases $n=64$.)

The residuals of the time series which are reconstructed as

$$X^r(j)=\sum_{k=m_2+1}^{n-1}a_k(j)\phi_0^k \tag{3.4}$$

contain (obviously) the measurement errors in the data. However, to ascertain that they should be interpreted primarily as representing turbulence, we utilize the tests devised by [25] and [13]. According to these tests turbulence data (at the same location) is characterized by low coherence between u,v,w and a phase close to zero or π between

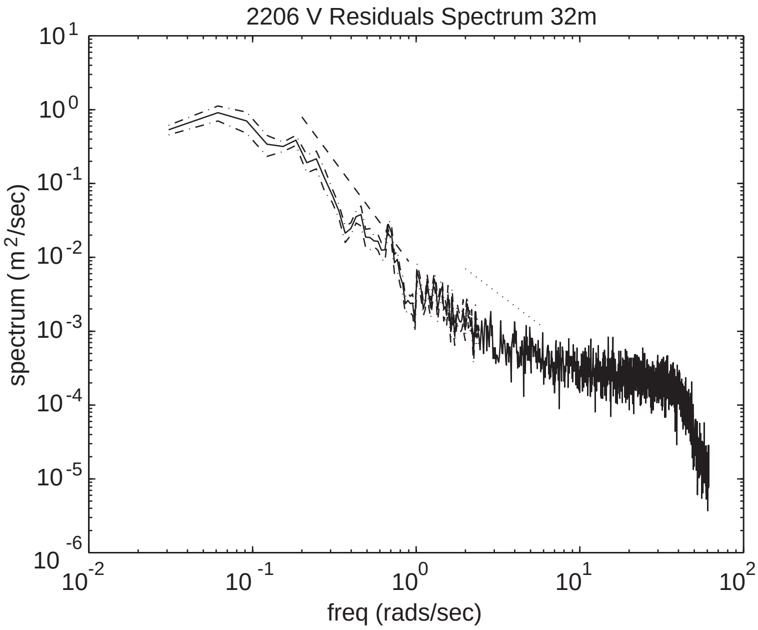

Fig. 9. – Spectra of the turbulent residuals of V at 32 m height.

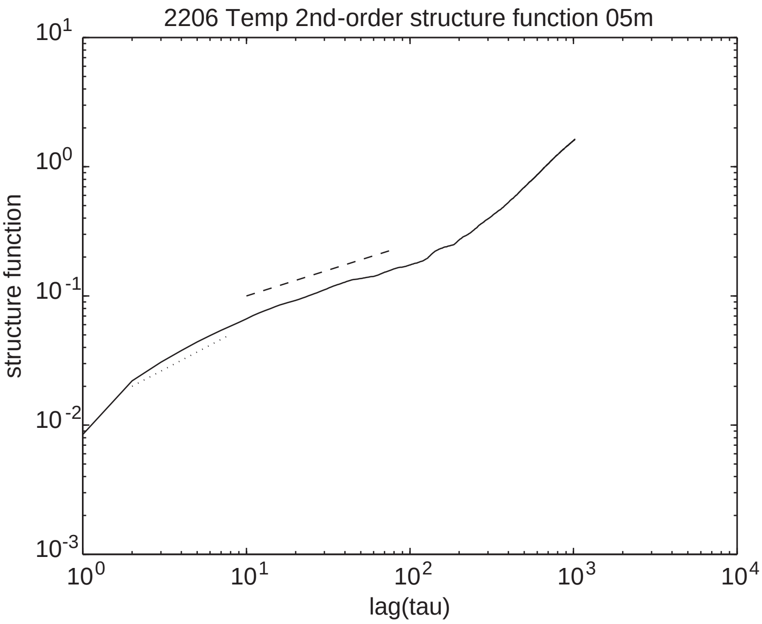

Fig. 10. – Second-order structure function for the (raw data) of the temperature at 5 m height. The dashed and dotted lines have slopes of $+2/5$ and $+2/3$, respectively. The section with slope 2/5 represents BRT as predicted by Bolgiano.

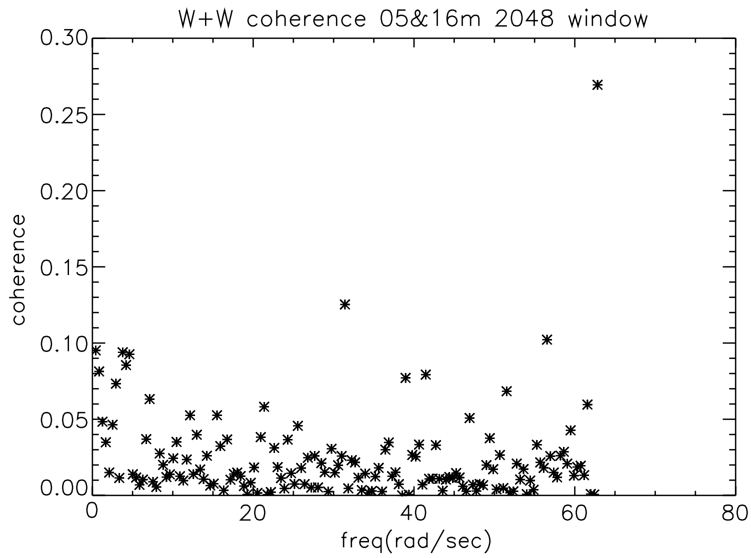

Fig. 11. – Coherence between the measurements of W at heights 5 m and 16 m.

w and t. (A phase close to $\pi/2$ is characteristic of waves.) Figure 1 shows a sample of the coherence between the residuals of u and w at 32 m. Similar plots were obtained for the other turbulent residuals of the flow. They demonstrate that for most frequencies the coherence is less than 0.1. Figure 2 gives a scatter plot of the phase between w and t at the height of 5 m. This figure is less definitive as there are still quite a few points in the wave sector $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ (in between the two dashed lines). However out of the 200 sample points in this plot 125 are in the "turbulence sector".

These tests show that to a large extent the residuals that were obtained from the KL decomposition represent actual turbulence.

4. – Tests for 2d turbulence

In today literature [19] a spectral slope of -3 in part of the inertial range is considered to be a strong indicator for 2d turbulence. However as noted already by Lily [5] "geophysical consideration" might modify this slope. As a matter of fact most of the spectral plots for both the "raw data" and the turbulent residuals field show exactly this type of dependence in the frequency range of 0.1–0.8 rad/s (see figs. 3, 4, 5, 6); in these plots the dashed line has slope -3 and the dotted one a slope of $-5/3$. (In these figures we plotted also the 95% confidence interval for the spectrum. This confidence interval is small due to the size of the time series for the data.) For higher frequencies the spectral plots for the turbulent residuals tend to flatten out (see figs. 7, 8, 9) and one can resort to the scaling model presented in sect. **2** for possible explanation of this peculiar behavior.

The existence of a spectral range where $E(k) \cong k^0$ in these plots can be interpreted as a confirmation for the existence of BRT as predicted by Bolgiano [16, 17]. A further confirmation for the existence of BRT in this data comes from the 2nd-order structure function for the raw data of the temperature at 5 m which is plotted in fig. 10. In this figure the dashed line has a slope of 2/5 and the dotted line has a slope of 2/3. The

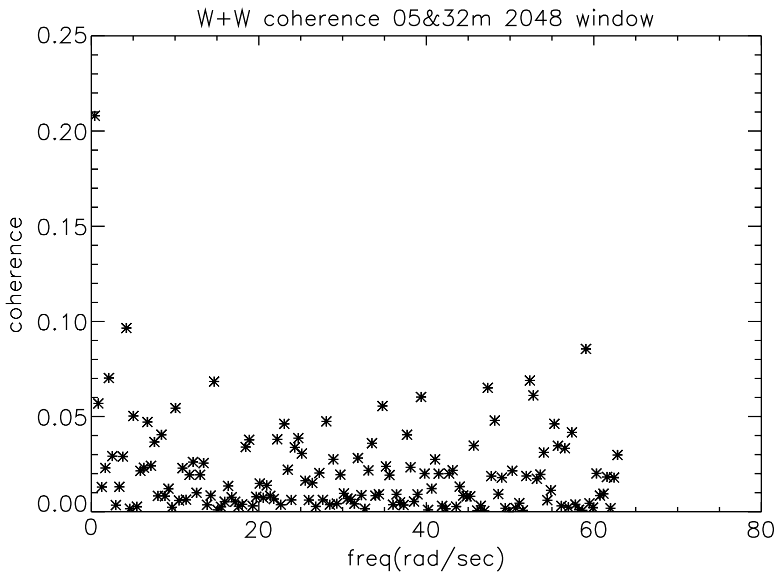

Fig. 12. – Coherence between the measurements of W at heights 5 m and 32 m.

range with slope 2/5 corresponds to BRT [16, 17], while the 2/3 range corresponds to Kolmogorov inertial range. (Similar possible detection of BRT in the upper troposphere was made recently in [26].)

As a first direct verification for the nature of the flow, we utilize a test devised by Dewan [13]. According to this test, inviscid two-dimensional turbulence is characterized by

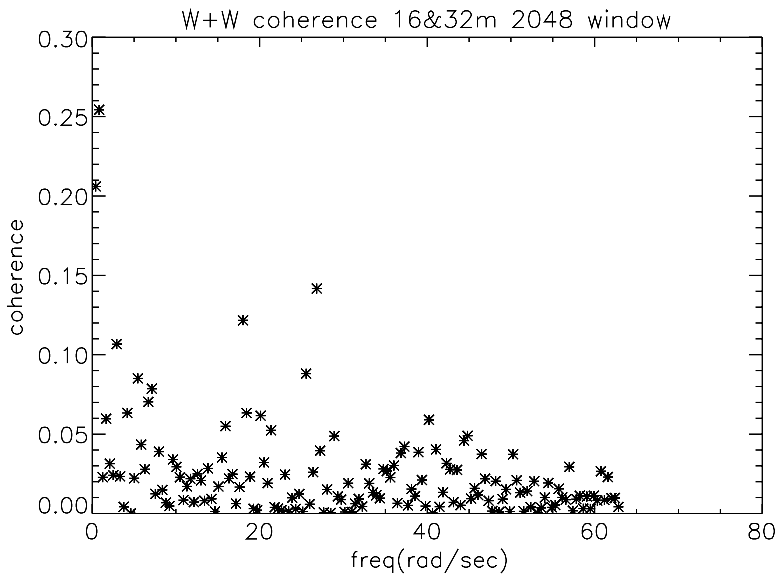

Fig. 13. – Coherence between the measurements of W at heights 16 m and 32 m.

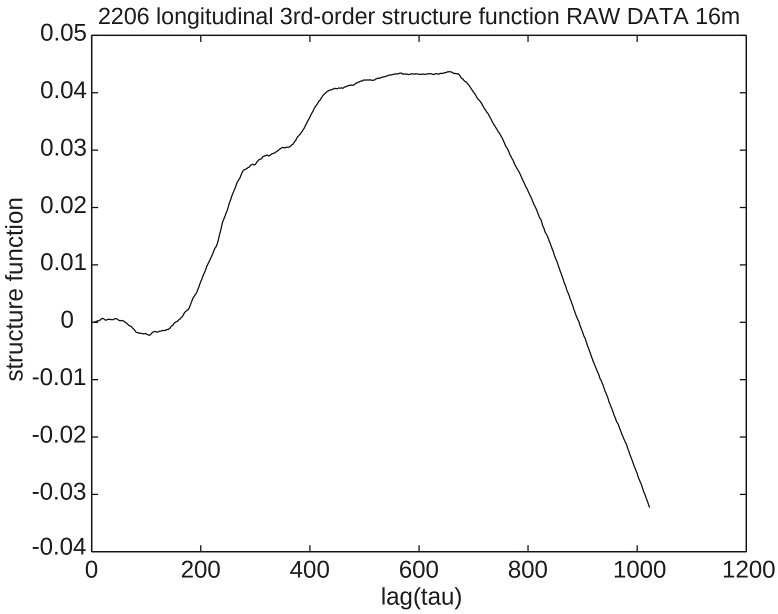

Fig. 14. – Third-order structure function for raw data of U at 16 m.

the fact that the temporal statistical coherency [14] between the time series representing the flow variables at different altitudes is zero. With viscosity taken into account some vertical separation of the order of (10 m for air) is needed for the coherency to become small. (Strong coherency with values close to one indicates a strong linear relationship between the two time series [14].)

Some typical plots for the coherency in the data are presented in figs. 11, 12, 13. In these plots the coherency for w between the different heights is plotted for different frequencies. We observe that for most sampled frequencies the coherency is well below 0.1 and according to [13] "these values constitute evidence for 2-d turbulence and against other types of fluctuations".

As a second test for two-dimensional turbulence we use the observation made recently by [15] about the third-order structure function for the longitudinal velocity component in the flow $s = \langle \delta \mathbf{u}_{\rm L} \delta \mathbf{u} \cdot \delta \mathbf{u} \rangle = \langle \delta u_{\rm L} \delta u_{\rm T} \delta u_{\rm T} \rangle + \langle \delta u_{\rm L} \delta u_{\rm L} \delta u_{\rm L} \rangle$, where $u_{\rm L}$, $u_{\rm T}$ are the longitudinal and transverse components of $\mathbf{u}$ (in our data $u_{\rm L} = u$). According to this observation this structure function (in a flow devoid of waves) has a negative slope for three-dimensional turbulence (which reflects the energy cascade from large to small eddies), while for two-dimensional turbulence the slope is positive. In the flow under consideration there is a strong activity of waves and as a result this structure function for the raw data does not conform to this theoretical prediction for large r. Nevertheless for mid-range values of r, s is positive and increasing. (See, *e.g*, fig. 14.) However, when we compute this structure function for the turbulent residuals, we obtain an excellent agreement with the theory (fig. 15). Not only that the structure function is increasing with r but it scales as r^3 exactly as Lindborg predicted for the enstrophy inertial range. (This is true for the three heights in which measurements were made.) Similar agreement is obtained for the second-order structure function $\langle \delta \mathbf{u} \cdot \delta \mathbf{u} \rangle$ which scales as r^2 (fig. 16).

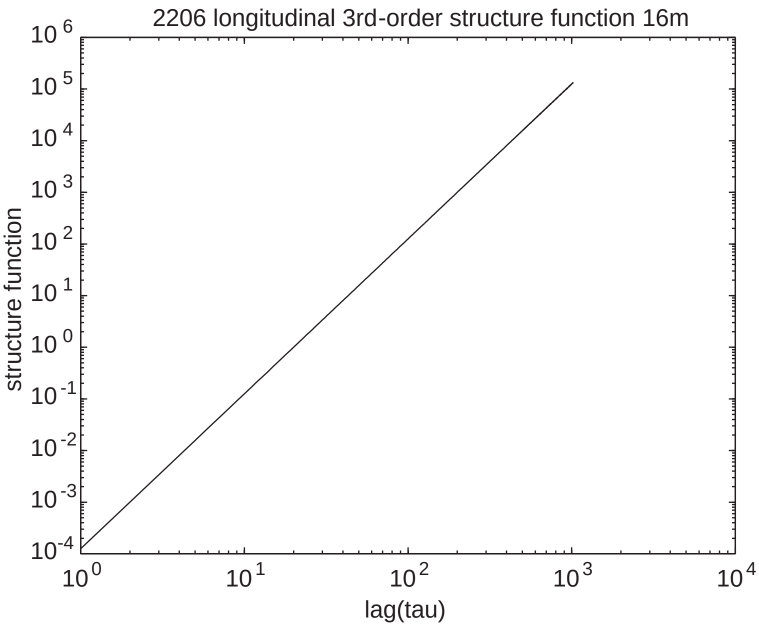

Fig. 15. – Third-order structure function for turbulent residuals of U at 16 m.

It should be observed that for other components of the flow the second- and third-order structure functions exhibit a more complex behavior. This might reflect either deviations from 2d flow or contamination of waves in the data.

Further analysis and information about the flow can be obtained from the spectral plots and the three level measurements.

To begin with, we compute the averaged Brunt-Vaisala frequency

$$\langle N \rangle = \left[\frac{g}{T} \left(\left\langle \frac{\mathrm{d}T}{\mathrm{d}z} \right\rangle + \frac{g}{C_p} \right) \right]^{1/2} . \tag{4.1}$$

To estimate $\left\langle \frac{\mathrm{d}T}{\mathrm{d}z} \right\rangle$ in this expression we use the second-order accurate, three-point difference formula for the derivative (which can be obtained using Taylor expansion around $h = 16$ m)

$$\frac{\mathrm{d}T}{\mathrm{d}z}(t) \mid_{16 \text{ m}} = \frac{1}{4752} [121 T_{32}(t) + 135 T_{16}(t) - 256 T_5(t)] , \tag{4.2}$$

where $T_{32}(t)$, $T_{16}(t)$, $T_5(t)$ are the readings of the temperature at heights 32 m, 16 m and 5 m, respectively at time t. From these formulas we obtain

$$\langle N \rangle \cong 0.035 \text{ s}^{-1} . \tag{4.3}$$

To estimate the averaged dissipation rate $\langle \epsilon \rangle$ we use the spectral plots in the region where Kolmogorov $-5/3$ law holds. In this region we have

$$E(k) = c_1 \epsilon^{2/3} k^{-5/3} , \tag{4.4}$$

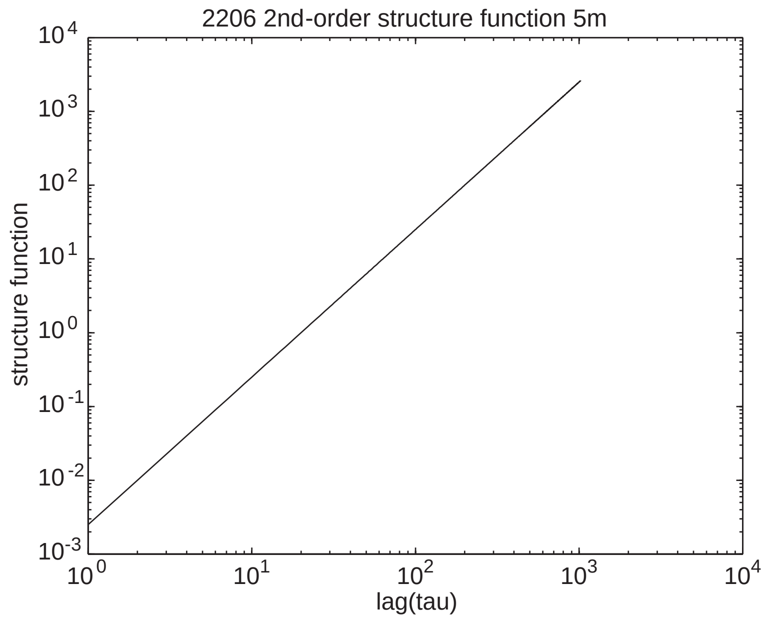

Fig. 16. – Second-order structure function for turbulent residuals of U at 5 m.

where $E(k)$ is the spectral intensity, c_1 is the Komogorov constant (approximately 0.2 in air). Hence (at $h = 16$ m)

$$\langle \epsilon \rangle = \left[\frac{\langle E(k) k^{5/3} \rangle}{c_1} \right]^{3/2} \cong 1 \times 10^{-4} \mathrm{J/s}\,. \tag{4.5}$$

Using these estimated values of $\langle N \rangle$ and $\langle \epsilon \rangle$ we can compute the averaged Ozmidov length at 16 m [20]

$$\langle L_{\mathrm{O}} \rangle = \left[\frac{\langle \epsilon \rangle}{\langle N \rangle^3} \right]^{1/2} \cong 0.67 \text{ m}\,. \tag{4.6}$$

Similarly the averaged shear length can be estimated using

$$\langle L_{\mathrm{sh}} \rangle = \left[\frac{\langle \epsilon \rangle}{\langle \frac{\mathrm{d}u}{\mathrm{d}z} \rangle} \right]^{1/3} \cong 0.125 \text{ m}, \tag{4.7}$$

where we used a formula similar to (4.2) to estimate $\mathrm{d}u/\mathrm{d}z$.

5. – Conclusion

In this paper we applied three independent tests to the data collected at Halley station on June 22, 1986 to examine if the turbulence field of this flow can be interpreted as two-dimensional turbulence. In the first test we examined the spectral plots for the raw data and the turbulent residuals. In both cases we recovered a slope of -3 for a spectral segment around 0.5 rad/s. As a second test we used Dewan coherency test between the measurements taken at different heights. According to this test the low coherency between the different time series for w is a strong indicator for 2d turbulence. Finally we computed the second- and third-order structure functions for the longitudinal component of the turbulent field in the flow (which in this case is represented by the turbulent residuals of u) and found that they scale as r^2 and r^3, respectively. This conforms exactly to the theoretical predictions made by Lindborg for these functions in the enstrophy inertial range.

In spite of these results some counter arguments can be advanced against making final conclusions regarding the existence of 2d turbulence in this data. We enumerate hereby some of these arguments:

1) The slope of -3 in the data is at small scales where some 3d effects might be present (in spite of the strong stratification). Furthermore the influence of the ground and the boundary layer dynamics might have an impact on the data.

2) One may argue that the vertical coherence between the data time series is not necessarily a sign of two dimensionality but of strong vertical gradients.

3) The decomposition of the data into mean flow, waves and turbulent residuals has an impact on our results (*e.g.*, the computation of the structure functions). However we used several ("classical") tests to justify this decomposition.

4) We did not build nor identify a dynamic model that supports the arguments for 2d turbulence.

We conclude therefore that some doubt may still linger regarding the final conclusions that can be drawn from this data. Further research and data are needed to settle this issue.

We also observed that the spectral plots have a segment at high frequencies in which they are almost flat. In an attempt to explain this peculiar behavior we introduced a scaling model that takes into account the stratification in the flow. This model shows that when buoyancy effects are taken into account different slopes of $E(k)$ in the inertial range are possible (as predicted by Lily). Thus we introduced a model which allows for the possible interpretation of these parts of the spectra as those belonging to BRT.

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