

Referencing geostrophic velocities at a northern Adriatic section

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Summary. — We have addressed the merits of two familiar methods to calculate absolute geostrophic currents when applied to a shallow-water section of a marginal sea. The comparison was performed on the basis of monthly collected hydrographic data and relative geostrophic currents (calculated with respect to the 30 m level) at a transect in the northern Adriatic in 1992 and 2000. The computed geostrophic currents were also compared to current-meter data collected continuously in 1992, in the surface and bottom layer of a station on the same section, and filtered with cut-off period of 10 days. When relative currents were converted to absolute the Fomin method (requiring minimal kinetic energy in the water column) provided correction closer to filtered Eulerian currents in 21 out of 24 (12 surface plus 12 bottom) comparison pairs. Modification of the sections's position confirmed that the criterion of mass conservation over the entire section generates absolute correction more susceptible to the position and the extent of the section used in its calculation. Both approximations of absolute geostrophic current worked better when applied to data collected in the warmer part of the year.

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1. – Introduction

Since the beginning of the previous century the assumption of geostrophic balance has served as the basis of the dynamic method. The method has seen many fruitful applications despite its well-known limitations, not the least of which is the relative nature of the currents it enables one to calculate. One of the most recently proposed alternatives to the dynamic method is the steric height method of Hopkins [1] in which the only unknown constant is the sea level at the reference, deepest common point. Generally, geostrophy is more readily attained in case of large-scale, slowly varying ocean currents. Departures from it should be expected in relatively narrow/shallow and energetic flows, for which Rossby number is not as small as geostrophic balance would require.

Artegiani *et al.* [2] have recently analysed seasonal variability of the Adriatic Sea hydrographic properties. They focused on the climatological, baroclinic part of the Adriatic circulation seeking to elucidate the dominant horizontal structures from seasonally mapped hydrographic data. Dynamic height anomalies were calculated for each season relative to the annual mean. Due to the shallowness of the northern Adriatic its reference level was set to 30 m. Supić *et al.* [3] have recently calculated geostrophic currents relative to the same level in their study of the Istrian Coastal Counter Current. The authors also found it necessary to calculate the absolute geostrophic currents for which a requirement of mass conservation was invoked. The focus on the baroclinic part of the circulation in the former, and simplistic assumption about the barotropic reference in the latter paper illustrate well the problematic nature of referencing geostrophic velocities. The most straightforward approach would be to estimate the reference velocity from direct measurements, *e.g.* [4]. However, the major problem with direct estimates, away from deep waters in particular, is that all methods measuring absolute velocities include geostrophic as well as ageostrophic component. The latter may be significant, rendering the reference inappropriate. A surrogate approach is often easier applied than justified.

The problems outlined above as exemplified in northern Adriatic are the focus of the present paper. In particular, we compare two familiar methods for determining geostrophic reference velocity, using the hydrographic data collected at a northern Adriatic transect, as well as current-meter data registered on the eastern side of the same transect. The first method is that originally proposed by Wunsch [5], and later used by Benoit *et al.* [6], and more recently by Marinone and Ripa [7]. With this method the reference, barotropic current is obtained via an inverse procedure which renders weighted quadratic term (dimensionally equivalent to energy) as small as possible. In the inverse-method framework the missing barotropic component is interpreted more generally as the deviation of the (baroclinic) solution from initially assumed reference level. Formal general inversion can be avoided at the expense of more restrictive solution. The other method is due to Fomin [8]. Its central assumption is a set-up of the minimum of the total vertical kinetic energy when the current field is in geostrophic balance. More specifically, Fomin postulates that when the current is in a state of geostrophic balance, then the surface topography, density field and the bottom relief are mutually related in such a way that kinetic energy of any particular water column attains its minimum. Clearly, in both approaches one deals with minimisation of quadratic forms with the dimension of energy. To aid the comparison of the two approaches we also look at a time series of current meter velocities, low-pass filtered in order to be more indicative of the low-frequency geostrophic flow.

The paper is organised as follows. The data are presented in the second section, and the methods elaborated in the third. Results are presented and discussed in the fourth and summarised in the final section.

2. – Data

Data analysed in this paper were collected at a section in the northern Adriatic in the years 1992 and 2000. The 1992 set consists of twelve monthly samplings taken at six stations of the Po-Rovinj section (named RV001, SJ007, SJ105, SJ103, SJ101 and SJ108 in the Centre for Marine Research [CMR] oceanographic data base, and re-named here as 1, 2, 3, 4, 5 and 6, respectively) plus twelve-month series of current-meter data from a mooring at the station 2 on the same section. In the 2000 data set only hydrographic data are available; identical set of stations was occupied with the exception of station

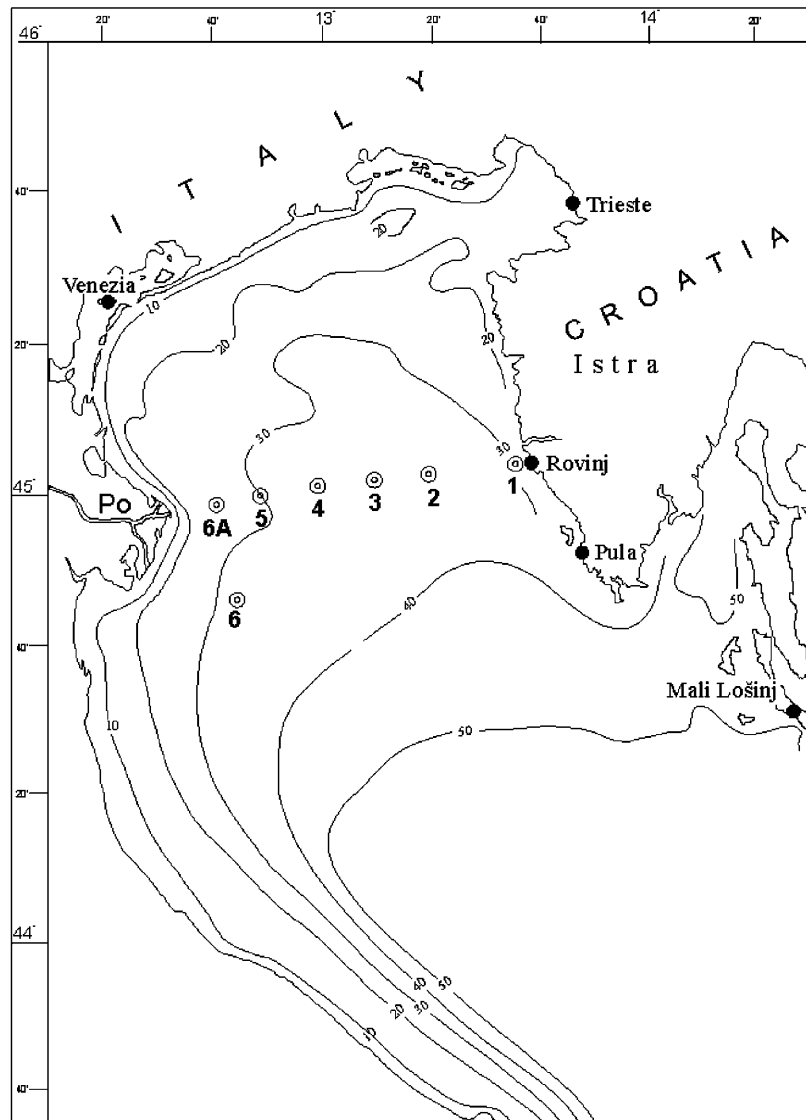


Fig. 1. – Map of the northern Adriatic with position of the hydrographic stations.

6A which was added to previous six. The station locations are shown in fig. 1.

Temperature was measured at standard depths (0, 5, 10 and 20 m) and 2 m from the bottom with protected reversing thermometer (Richter and Wiese, precision $\pm 0.01^\circ\text{C}$). Salinity was determined from bottle samples taken at the same depths using high-precision laboratory salinometers (YEOKAL 601Mk1V, precision ± 0.01). Currents were measured continuously (at 8 m and 30 m depth) with Aanderaa RCM4 current meters, and sampling interval set to 10 minutes. From original current meter data (current speed and direction) hourly u and v component velocities were calculated. Overall precision of ± 1 cm/s in magnitude, and $\pm 5^\circ$ in direction was achieved [9]. For the purpose of

comparison with geostrophic currents, the hourly values were low-pass filtered (cut-off period set to 10 days) and decimated to one value per day (set at noon).

3. – Methods

3.1. Basic assumptions. – In order to understand the problem of determination of absolute geostrophic currents, it is useful to start with the equations of motions

$$(1) \quad fv - \frac{1}{\rho} \frac{\partial p}{\partial x} = 0,$$

$$(2) \quad fu + \frac{1}{\rho} \frac{\partial p}{\partial y} = 0,$$

$$(3) \quad \frac{\partial p}{\partial z} = g\rho,$$

where u and v are current components in the x and y directions, respectively, f is equal to $1.03 \times 10^{-4} \text{ s}^{-1}$, p is pressure and ρ is water density. Integration of eq. (3) with respect to z , from the sea surface (ζ) to the depth z gives

$$(4) \quad p(x, z) = p_A + g \int_{-\zeta}^0 \rho_0 dz + g \int_0^z \rho(x, z) dz = p_A + g\rho_0\zeta(x) + g \int_0^z \rho(x, z) dz,$$

where p_A is the atmospheric pressure and ζ is the free surface. In the above relations the x -coordinate is written out to stress the variability along the lateral section. Inserting this equation at the first two equations of motion gives

$$(5) \quad u(x, z) = -\frac{g}{\rho f} \int_0^z \frac{\partial \rho}{\partial y} dz - \frac{g}{f} \frac{\partial \zeta(x)}{\partial y} = u_{\text{rel}}(x, z) + u_{\text{ref}}(x),$$

$$(6) \quad v(x, z) = \frac{g}{\rho f} \int_0^z \frac{\partial \rho}{\partial x} dz + \frac{g}{f} \frac{\partial \zeta(x)}{\partial x} = v_{\text{rel}}(x, z) + v_{\text{ref}}(x).$$

Consequently, geostrophic current consists of a relative (baroclinic) component that can be calculated from hydrographic data alone and the reference (barotropic) component that depends on unknown sea surface gradients. There are several different approaches to the estimation of u_{ref} and v_{ref} .

The simplest, often taken, approach is to assume the existence of the level of no motion, *i.e.* to select a vanishing reference velocity. With both Wunsch and Fomin methods a non-zero reference velocity is obtain via similar procedures that render respective quadratic forms minimal. More specifically, in the Wunsch method vanishing of the total net transport through a section is evoked, whereas in the Fomin method the minimum of the total kinetic energy of a water column is required. In the latter case the reference velocity turns out to depend on bottom topography, and deviation of the baroclinic part from its vertical mean. When measurements are available only along a section (as in our case) the problem reduces to calculation of the cross-section component.

3.2. Relative velocity. – Surface geostrophic currents, relative to 30 m, were computed by the standard dynamical method for each pair of the neighbouring stations on the section (1/2, 2/3, 3/4, 4/5 and 5/6) and for each cruise in the 1992. For the year 2000 geostrophic currents were computed following the same procedure except that the station 6 was replaced by station 6A.

Assuming that the origin of the coordinate system is located at the westernmost station, the relative dynamic depth of the 30 dbar surface with respect to the depth z (0 m, 5 m, 10 m, and 20 m; the difference $D_{30}(x) - D_z(x)$) was computed for each station (at distance x from the origin) and for each cruise as

$$(7) \quad D_{30}(x) - D_z(x) = \int_z^{30} \alpha(x, p) dp.$$

Specific volume of the seawater α at the pressure p ($\alpha = 1/\rho$ where ρ is the seawater density) was computed from the temperature and the salinity data according to international relations for the seawater. Relative currents at the location x and depth z , with respect to 30 m, were computed as

$$(8) \quad v_{\text{rel}}(x, z) = v_z(x) - v_{30}(x) = \frac{1}{f} \frac{\partial}{\partial x} (D_z(x) - D_{30}(x)).$$

For each cruise the absolute velocities $v(x, z) = v_{ij}$ for the i -th station pair ($i = 1, \dots, 5$) and the j -th level ($j = 1, \dots, 4$) were then computed from

$$(9) \quad v_{ij} = v_{\text{rel}}(x, z) + v_{\text{ref}}(x, z),$$

in analogy with (6).

3.3. Reference velocity. – To compute the reference velocity, we used two methods: one following Wunsch [5], but applied as in Supić *et al.* [3], and another according to Fomin [8]. In the rest of the paper the term “Wunsch method” will be taken to mean “Wunsch method as applied in Supić *et al.* [3]”.

Wunsch method. The criterion requiring mass conservation across the entire section ($\sum_{ij} v_{ij} A_{ij} = 0$) gives a constant reference velocity v_{ref} for the section

$$(10) \quad v_{\text{ref}} = \frac{\sum_{ij} v_{ij} A_{ij}}{\sum_{ij} A_{ij}} = -\frac{T_0}{P},$$

A_{ij} is the area associated with each v_{ij} , T_0 is the net geostrophic transport when $b_i = 0$ and P is the total area of the section. The formula is derived from a more complex relation with weights allowing more than one reference per section.

Fomin method. The criterion of minimum kinetic energy provides the reference velocity as

$$(11) \quad v_{\text{ref}}(x) = \frac{A_x^2(x) \bar{v}(x) + A_y^2(x) v'(x, H)}{A_x^2(x) + A_y^2(x)},$$

with

$$(12) \quad A_x(x) = \frac{1}{f} \frac{\partial H}{\partial x}, \quad A_y(x) = \frac{1}{f} \frac{\partial H}{\partial y},$$

$$\bar{v}(x) = \frac{1}{H} \int_0^H v'(x, z) dz, \quad v'(x, z) = \frac{g}{f\rho} \int_0^z \frac{\partial \rho}{\partial x} dz.$$

Values of bottom depth H and its gradients at the Po-Rovinj section were read from bathymetric charts. The reference velocity (barotropic component) here becomes a linear combination of the vertical mean of the baroclinic component and the bottom velocities where their contribution depends on the bottom gradient.

Intrinsically, both approaches assign a value to the reference velocity obtained from minimisation of a quadratic measure. The Wunsch original criterion provides reference velocities that depend on weights which require additional information and generalised inverse procedures. When such information is not available the procedure can be simplified, but the consequence is just one reference velocity for the whole section. The Fomin criterion is more physical, but its universal applicability is less obvious. At the expense of the minimal-kinetic-energy assumption the mass is conserved separately at each section segment, and the reference velocity uniquely determined.

4. – Results and discussion

4.1. *The year 1992.* – Hydrographic characteristics (temperature, salinity and density) of the northern Adriatic are strongly influenced by the Po river discharge. Generally, in winter the freshened Po-influenced waters are confined to the western coast, while during late spring and summer they spread over larger areas of the northern Adriatic. The water column is well mixed during winter and highly stratified in summer, *e.g.* [10].

The geostrophic currents in the northern Adriatic are generally weak in winter and strong in summer when waters of the Po origin spread over the northern Adriatic [3]. In keeping with that conclusion the relative geostrophic currents at the Po-Rovinj section were much weaker and less spatially variable in January 1992 (–3 cm/s–0 cm/s) than in June 1992 (–4 cm/s–6 cm/s) or September 1992 (–9 cm/s–5 cm/s). In all the three situations analysed there was an inflow of water in central/eastern part of the Po-Rovinj section and an outflow of water near both the eastern and western coast. The Wunsch method gave for each of the three cruises small and spatially uniform reference velocities (–0.3 cm/s in January, 0.6 cm/s in June and –0.2 cm/s in September). The reference velocities computed after Fomin were larger and station dependent (–0.1–0.0 cm/s in January, 0.0–1.9 cm/s in June and –1.6–2.3 cm/s in September). Consequently, the absolute currents at the section computed by the Wunsch method were generally closer to their relative values than the ones computed by the Fomin method.

Typical velocity measured at the station 2 in 1992 (mean absolute current value) amounted to 30 cm/s in the surface layer and about 15 cm/s at the bottom. Daily averaged currents ranged from 10 to 20 cm/s in the surface and from 5 to 10 cm/s in the bottom layer. The measured currents were much stronger than the geostrophic approximations, indicating strong ageostrophic component. Typical relative geostrophic current in 1992 amounted to 2–5 cm/s at the surface and to 1–2 cm/s at the bottom.

The ageostrophic contribution can come from a number of sources. For one, the current field in the northern Adriatic is under strong influence of winds. Bora and sirocco, the

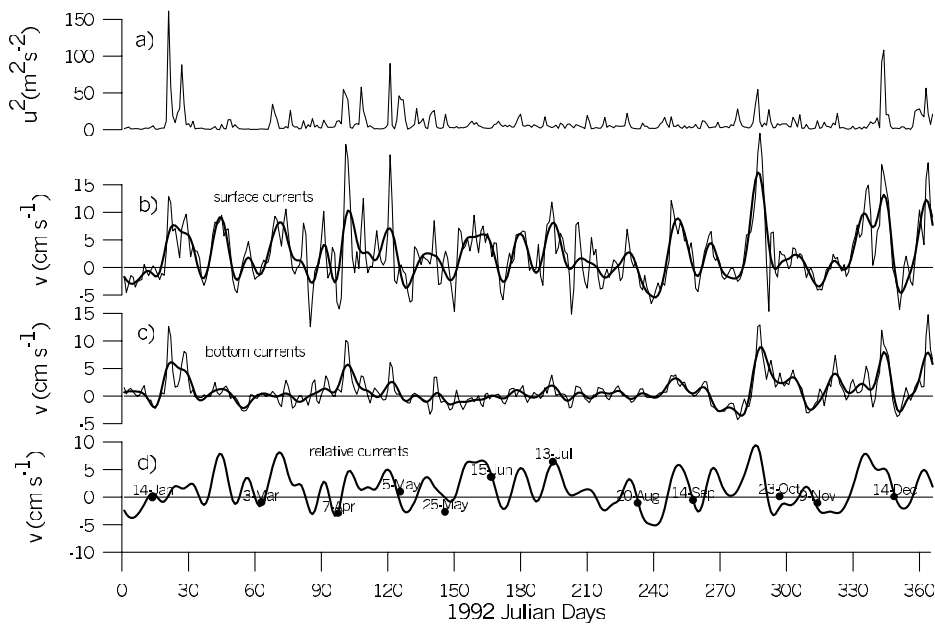


Fig. 2. – The year 1992 time series. a) Squared values of wind speed magnitude at Trieste; b) along-shore component of daily averaged (solid line) and ten-day filtered (heavy solid line) surface current measured at station 2; c) same as b) for the bottom; d) relative geostrophic approximation of current for the twelve dates in 1992 (filled circles).

strongest winds that blow in the region, can transform density gradients thus influencing the geostrophic flow; *e.g.*, during a bora episode fresh waters from the vicinity of the Po delta are transported towards the east [11]. The winds also generate inertial oscillations (in the stratification period) and wind-driven currents. In the northern Adriatic current field there exists also a tidal signal, one of the strongest in the whole Mediterranean. The currents measured continuously by current meters are the sum of all contributing components, geostrophic as well as ageostrophic, but the problem is how to single out only the geostrophical part of directly measured currents.

In order to select the most appropriate “geostrophic filter” to reduce this ageostrophic component to minimum, we compared surface geostrophic currents relative to 30 m depth at station 2 with filtered currents from the current-meter time-series. We used various filters with cut-off period from 5 to 20 days and found that sums of squared differences between geostrophic and filtered currents were minimal for period of 10 days. On fig. 2 the v -components of surface and bottom currents (daily mean with thin line and filtered values with thick line) as well as their differences are plotted along with squared values of daily mean wind speed at Trieste. Comparison suggests that strong currents often coincide with peaks in wind speed, although there are peaks in currents when there was not strong wind registered in Trieste. Those peaks could have been caused by the sirocco wind, not observed appropriately at the Trieste station. The bottom time series shows filtered relative currents (surface minus bottom values) with twelve dots marking the values of calculated relative geostrophic currents.

In order to address the merits of the Fomin and Wunsch solutions to the absolute geostrophic currents, we compared the above-described filtered Eulerian current data to

TABLE I. – Absolute current (cm/s) magnitude differences at the station 2 and two depths (8 m, 30 m) from the 1992 data: A: Wunsch-method corrected geostrophic current minus 10 day filtered measured current; B: Fomin-method corrected geostrophic current minus 10 day filtered measured currents; C: Wunsch-method solution minus Fomin-method solution.

Date		14/01	03/03	07/04	05/05	25/05	15/06	13/07	20/08	14/09	23/10	09/11	14/12
A	8 m	1.4	0.4	2.0	2.8	0.7	0.5	4.3	0.0	0.2	0.7	3.8	1.3
	30 m	2.3	0.3	2.2	0.2	1.4	0.7	3.5	0.2	0.3	2.6	1.0	1.3
B	8 m	1.2	0.3	1.9	2.4	1.0	0.2	3.4	0.2	0.3	0.5	3.2	0.9
	30 m	2.0	0.3	2.0	0.2	1.2	0.6	2.2	0.0	0.2	2.4	1.5	0.9
C	8 m	0.2	0.1	0.1	0.4	0.3	0.3	0.9	0.2	0.1	0.2	0.5	0.4
	30 m	0.2	0.1	0.1	0.4	0.3	0.3	0.9	0.2	0.1	0.2	0.5	0.4

both approximations (table I). In 21 out of 24 cases (12 surface and 12 bottom measurements) the Fomin approximation provided solution closer to the value of the directly measured current. However, the differences between the absolute currents computed after Fomin and those computed after Wunsch were small (0.9 cm/s in July and up to 0.5 cm/s in other months). The differences between the filtered Eulerian currents and absolute currents were up to 4.3 cm/s and 3.4 cm/s for the Wunsch and Fomin approximations, respectively. Discrepancy was more pronounced in January, April, May, July, October, November and December (2–4 cm/s) than in March, June, August and September (0–1 cm/s). Predictably, geostrophic approximation was generally closer to Eulerian measurements in the warmer part of the year, when the water column is stratified.

Furthermore, when relative currents are converted to absolute the Fomin method consistently provided correction closer to filtered Eulerian currents. However, the absolute currents discussed here were computed for a section not on a straight line between eastern and western coast of the northern Adriatic. That clearly affects the calculation of the reference velocity when the Wunsch method is used.

4.2. The year 2000. – For the year 2000 we analysed hydrographic conditions and geostrophic currents at the section 1-6A, connecting the city of Rovinj and the Po delta (fig. 1). As station 6A is very close to the Po delta it is more influenced by fresh-water discharge than station 6 used in previous section. Distribution of hydrographic parameters on the common part of both sections (stations 1-5) was in January, June and September of 2000 similar to multi-annual average (CMR, unpublished data). The relative geostrophic currents between stations 5 and 6A, however, were very strong — up to 16 cm/s in January, up to 18 cm/s in June and up to 28 cm/s in September. At the rest of the section (stations 1-5) the relative currents were much weaker. They were also less intense in winter (between 0 cm/s and 2 cm/s in January) than in stratification period (between –2 cm/s and 10 cm/s in June and between –4 cm/s and 4 cm/s in September). The reference velocities computed by the Wunsch method were again weaker than the ones computed after Fomin. The former method gave –0.3 cm/s in January, –1.3 cm/s in June and 0.9 cm/s in September *vs.* –0.1 to 0.4 cm/s in January, from –2.4 to 3.1 cm/s in June and from –0.8 to 2.6 cm/s in September obtained by the latter.

We additionally analysed hydrographic conditions and geostrophic circulation at the section 1-6 in 2000. As values of surface salinity at station 6 were generally higher

TABLE II. – *Absolute current magnitude differences at the station 2 from the 2000 data: A: Wunsch-method solution–Fomin-method solution at the 1-6A section; B: same as A, but at the 1-6 section; C: Wunsch-method solution at the 1-6 section minus Wunsch-method solution at the 1-6A section.*

Date	05/01	21/02	21/03	17/04	26/06	19/07	25/08	26/09	18/10	24/10	14/12
A	0.2	0.2	0.4	0.7	1.7	0.2	0.4	1.0	1.2	0.5	1.0
B	0.5	0.1	0.4	0.8	1.7	0.2	0.5	0.5	0.5	0.5	0.5
C	0.7	0.3	0.1	0.2	0.1	0.0	0.7	0.1	0.8	0.4	1.0

than those measured at station 6A, relative geostrophic currents between stations 5 and 6 were less intense than the ones between stations 5 and 6A. That affected reference velocity computation using the Wunsch method. However near-bottom velocities (30 m) thus obtained (0.4 cm/s in January, -1.3 in June and 0.7 in September) were very close to the ones computed for the 1-6A section and the same year. The reference-velocity differences computed at the two sections (1-6 and 1-6A) for all the cruises in 2000 are given in table II. They are small in most cases (up to 0.8 cm/s in October), and generally smaller than the differences computed separately by the two methods at each section (up to 1.7 cm/s in June). So our conclusion—based on data collected in 1992 on a section which does not lay on the straight line—that the Fomin method gives better approximations of absolute currents seems to be valid, in spite of possible errors.

5. – Conclusions

In this paper we have addressed the merits of two familiar methods for determining geostrophic reference velocity, *i.e.* for calculating absolute geostrophic currents. The methods (due to Wunsch and Fomin, respectively) have been applied to relative geostrophic currents calculated at a northern Adriatic section (Po-Rovinj) from the 1992 and 2000 hydrographic data. The 1992 calculations have been also compared to the results of an Eulerian current-meter data analysis, whereas a variation in the section's position has been exploited in the year 2000 analysis. Our goal has not been to provide observational evidence for geostrophic balance, but rather to examine the differences incurred by the two methods, while acknowledging the existence of the ageostrophic part of the flow. Some ageostrophy appears present throughout the year, but low-pass filtered current values stay rather close to the relative geostrophic current on the twelve dates for which hydrographic measurements were available. Understandably, in the warmer part of the year, the dynamic method provides relative currents closer to the filtered current-meter values, while thus calculated currents exhibit more discrepancy in the rest of the year. When relative currents are converted to absolute the Fomin method provides correction closer to filtered Eulerian currents in 21 out of 24 (12 surface + 12 bottom) cases. The experiment with the section position (possible in the year 2000, when measurements were available at both station 6 and 6A) further demonstrated the susceptibility of the reduced Wunsch method (as applied in [3]) to the size and/or position of the section used in calculating the correction.

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