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On the dynamics of quasi-geostrophic intergyre gyres

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Summary. — An important aspect of the present climatic change concerns the wind-stress anomalies over the ocean. It is possible to associate to them a special current field, which appears between the subtropical and the subpolar gyres and is known as intergyre gyre. In the present paper we investigate its dynamics by including recent models of stochastic wind field into the classical model of ocean circulation at the basin scale of Rhines and Young. In the framework of an analytical approach, developed at the geostrophic level of approximation, we explore the circulation patterns of this recently discovered characteristic of double gyres.

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1. – Introduction

In the last years an increased attention has been devoted to the role of the climate anomalies in the wind-stress field on the air-sea interactions, which play an important role in the climate variability. In particular, the anomalies in the wind field have the property to create changes in the wind-driven ocean circulation in a longer time scale than the time scale associated to the same anomalies [1] and, consequently, to the redistribution of heat in the atmosphere by the water masses [2]. At the same time they create an ocean-atmosphere feedback effect proportional to the Sea Surface Temperature (SST) anomaly field. In particular, recent studies [3] showed that, due to the weak feedback at the mid latitudes, the non-linear dynamics in the ocean acquires importance.

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In the North Atlantic phenomenology these anomalies are associated to the North Atlantic Oscillation (NAO), the primary mode of atmospheric low-frequency variability over the North Atlantic basin [4].

The anomalies have a stochastic distribution in time, but coherent in the spatial pattern. In particular, over the North Atlantic basin they are centred at the latitude of zero wind-stress curl line at the middle latitudes. Their latitude pattern is less extended than the latitude extension of the total basin. This implies that the linear combination between the northern zero wind-stress curl line of the anomalies and the zero wind-stress curl line associated to the climatic wind field, creates a northward migration of the total northern zero wind-stress curl line.

It is possible to associate to the wind-stress anomalies an anomalous ocean circulation. For its meridional extension, located between the subtropical and the subpolar gyres, and its role in the heat exchange between the two gyres, this anomalous circulation has acquired the name of intergyre gyre. As shown in the literature [5] the existence of the intergyre gyre can be observed through the modulation of the dipole-like anomaly in the air-sea heat flux by the anomalous advection of heat by the ocean circulation.

The sign of the circulation of the intergyre gyre is determined by the sign of the anomalies associated to a positive (NAO(+)) or negative (NAO(−)) index of the NAO [4].

In this paper we show how the properties of the intergyre gyre can be studied in a comfortable way using the quasi-geostrophic, baroclinic ocean circulation model, introduced in 1982 by P. Rhines and W. Young [6, 7]. See also [8, 9], here applied for a continuously stratified fluid.

This is a circulation model based on the conservation of the quasi-geostrophic potential vorticity, applied at basin scale to a fluid in stationary flow.

The validity of the stationary flow condition can be assumed due to the fact that the baroclinic modes of the ocean can be excited just by a low-frequency forcing [3]. In this way the ocean acts as a low-pass filter for the atmospheric forcing frequencies.

Thus, the sign of the intergyre gyre circulation can be assumed coherent on decadal time scales, or even in time scales larger than the spin up time associated to the "turn on" of the anomalies.

This is allowed by the observation [4] of the tendency of the NAO index to stay on positive values.

An important issue of the Rhines-Young model is the inherent linearity of the equation that defines the conserved potential vorticity: this allows to see the deformation of the streamlines due to the wind-stress anomalies as a simple linear combination of the streamlines of the flow given by the unperturbed wind-stress field and the one given by the anomalies. This is an important facilitation for the study of the dynamics of the intergyre gyre.

One of the most interesting results of the model is the evaluation of the maximum downward propagation of the baroclinic circulation into the ocean interior. This depends strictly on the density stratification of the ocean. Here, we compare the maximum depth of the intergyre gyre as a function of the intensity of the wind-stress anomalies, in the presence of a constant and an exponential buoyancy frequency profile [10]. It is possible to see that the nonlinearity introduced by this second density profile allows the anomalous circulation to propagate deeper than that given by the linear (constant) density stratification. An interesting case is the one in which the pycnoclyne is sufficiently close to the interface between the Ekman layer and the geostrophic interior. In this case the maximum depth of the motion is achieved for a constant buoyancy frequency.

In the next sections we develop the equations for the ocean circulation model; in sect. **3** we develop the equations for the climate anomalies and, finally, we apply the theory to the intergyre gyre on the basis of the model of Rhines-Young.

2. – Ocean model equations

We premise that we deal with the nondimensional version of the model of Young and Rhines and start by fixing the fluid domain $D = [0 \leqslant x \leqslant 1] \times [0 \leqslant y \leqslant 2]$ on a certain beta-plane in which a steady, quasi-geostrophic double gyre (subtropical plus subpolar) is included. We assume a non-diffusive and isentropic flow, so that

$$\frac{\mathrm{D}\rho}{\mathrm{D}t} = 0\,, \tag{1}$$

where $\mathrm{D}/\mathrm{D}t$ is the Lagrangian derivative operator and ρ is the total density, given by the sum of a standard term $\rho_S(z)$ and a term of density anomaly $\rho'(x, y, z)$. Under hypotheses of adiabatic flow above, Young and Rhines [6, 7] and Pedlosky [9] showed that the non-dimensional equation for the conservation of total vorticity can be written in the form

$$y + \frac{\partial}{\partial z}\left(\frac{1}{N^2}\frac{\partial \psi}{\partial z}\right) = y_0\,, \tag{2}$$

where y_0 is the line of zero wind-stress curl that separates the subtropical gyre from the subpolar gyre. In (2), y is the latitude-dependent part of the non-dimensional planetary vorticity, $\frac{\partial}{\partial z}\left(\frac{1}{N^2}\frac{\partial \psi}{\partial z}\right)$ is the so-called thermal vorticity, $\partial\psi/\partial z$ is the density anomaly at the geostrophic level of approximation and $N(z)$ is the buoyancy frequency. In the scaling the planetary and thermal vorticity are both set as $O(1)$, so that their values result comparable. Note that the conserved planetary vorticity does not include the term of relative vorticity, term that can be neglected at basin scale. Equation (2) is referred to a steady flow. Whatever the Brunt-Väisälä frequency $N(z)$ may be, it satisfies the following requests:

$$N\,(0) = 1\,, \qquad \frac{\partial N}{\partial z} \geqslant 0\,, \tag{3}$$

where the first comes from the scaling while the second is requested by static stability. Equation (2) is a linear differential equation for the unknown ψ in which $N\,(z)$ is a known function. Hypothesis (1) implies that, due to the presence of the term of perturbation of density, each material volume of fluid of the geostrophic interior can acquire an ageostrophic (subscript 1) vertical velocity, say w_1, which matches the Ekman pumping vertical velocity w_{E} at the top of the geostrophic interior in $z = 0$. Hence, from (1) we have

$$w_{\mathrm{E}} = -\frac{1}{N^2} J\left(\psi, \frac{\partial \psi}{\partial z}\right)_{z=0}\,. \tag{4}$$

Equations (2) and (4) define a flow extending between $z = 0$ and the maximum depth $z = -h\,(x, y)$, to be determined, below which the flow disappears. Along the surface $z + h\,(x, y) = 0$, the boundary conditions for the stream function are

$$\psi = 0\,, \qquad \frac{\partial \psi}{\partial z} = 0\,. \tag{5}$$

A double integration of (2) from $-h$ to z gives

$$\psi\left(x,y,z\right)=\left(y_0-y\right)\left(\int_{-h(x,y)}^{z}\theta N\left(\theta\right)^2\,\mathrm{d}\theta+h\left(x,y\right)\int_{-h(x,y)}^{z}N\left(\theta\right)^2\,\mathrm{d}\theta\right).\tag{6}$$

Equation (6) poses an implicit dependence of ψ on the surface $z=-h\left(x,y\right)$ (also called *bowl*). The equation for h is found by substituting (6) into (4), whence

$$\frac{\partial h}{\partial x}\int_{-h}^{0}zN\left(z\right)^2\,\mathrm{d}z=-\frac{w_{\mathrm{E}}}{\left(y_0-y\right)}.\tag{7}$$

In (7), the known functions $N(z)$ and $w_{\mathrm{E}}(x,y)$ characterize the structure of the bowl $h(x,y)$. In turn, this last is included into (6) to single out the stream function, that is to say the solution of the circulation problem posed by the model. As we have anticipated in the Introduction, Garrett and Munk [10] found, on best-fit arguments applied to observational data, the simple exponential buoyancy frequency below the mixed layer:

$$N\left(z\right)=\exp\left[z\right].\tag{8}$$

This choice allows us to evaluate (6) to find

$$\psi=\frac{y_0-y}{2}\left\{\left(z+h\right)\exp\left[2z\right]-\frac{1}{2}\Big[\exp\left[2z\right]-\exp\left[-2h\right]\Big]\right\}.\tag{9}$$

From (7), (8) and the boundary condition $h(1,y)=0$, the following trascendental equation in the unknown $h(x,y)$ turns out to hold:

$$h\Big[1-\exp\left[-2h\right]\Big]=-4w_{\mathrm{E}}\frac{1-x}{y_0-y}.\tag{10}$$

Unlike the intricate form of (10), the constant buoyancy frequency $N(z)=1$ easily leads to the simple result [6, 7, 9]

$$h\left(x,y\right)=\left(-6w_{\mathrm{E}}\frac{1-x}{y_0-y}\right)^{1/3}.$$

3. – Climate anomalies in the mid-latitude wind field

The model describing the climate anomalies in the wind field has been developed by Jin [11], Neelin and Weng [12], Marshall, Johnson and Goodman [5] and Dewar [3]. In this model the wind-stress anomalies are composed of two terms. The first one is a temporally stochastic component, with a coherent spatial structure. Its amplitude can be comparable to the amplitude of the unperturbed field. The second component is given by the feedback of the ocean to the wind-stress field and its amplitude is proportional to the SST anomalies. Due to this proportionality, its amplitude is one order less than the one of the stochastic component.

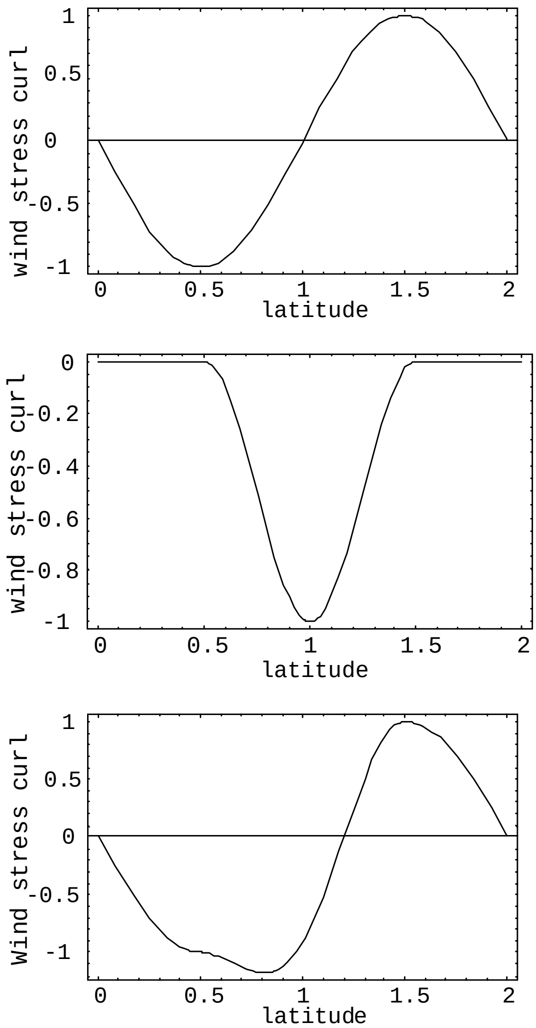

Fig. 1. – Ekman in the absence of climate anomalies (upper panel). Perturbation pumping (middle panel). Pumping associated to a period of positive wind-stress anomaly (lower panel). $A(t) = 0.9$.

Referring to the already introduced double-gyre domain we can describe the total wind-stress curl field as a linear sum of an unperturbed component and an anomalous component (fig. 1):

$$w_{\mathrm{E}}(y,t) = w_{\mathrm{E0}}(y) + w'_{\mathrm{E}}(y,t) \ .$$

The unperturbed wind-stress curl field is a one-dimensional, sinusoidal function [13]

$$w_{\mathrm{E0}}(y) = -\sin(\pi y) \ . \tag{11}$$

We describe the temporal part of the anomalies as a one-dimensional, sinusoidal function, centred at the middle-domain latitude and with a limited latitude extension:

$$w'_{\mathrm{E}}(y,t) = \frac{A(t)}{2}\cos(2\pi y), \qquad \frac{1}{2} \leqslant y \leqslant \frac{3}{2}. \tag{12}$$

The limited latitude extension can reproduce the observed migration of the total zero wind-stress curl line, while the one-dimensionality of the field cannot reproduce the rectification of it.

In (12) the scaled coefficient $A(t)$ is defined as

$$A(t) = -\left(kS(t) - \frac{1}{2}f\Delta T(t)\right), \tag{13}$$

where $S(t)$ is the temporal part of the stochastic component of the anomalies and

$$\Delta T(t) = T_{\mathrm{n}}(t) - T_{\mathrm{s}}(t) \tag{14}$$

is the net temperature difference between the northern and southern SST lobes [1]. The time evolution of (14) is discussed in [3]. Typical values of ΔT are of $O(10^{-1})$.

In (13) k and f are set to make the entire coefficient dimensionless.

Following Crisciani and Cavallini [14] we can put the following assumptions for an Ekman pumping dependent just on the latitude y:

$$w_{\mathrm{E}}(0) = w_{\mathrm{E}}(y_0) = w_{\mathrm{E}}(2) = 0, \qquad \lim_{y\to y_0}\frac{w_{\mathrm{E}}(y)}{y_0 - y} < \infty. \tag{15}$$

For a climatic wind-stress curl (11), assumptions (15) are satisfied for $y_0 = 1$. If the Ekman pumping is instead affected by climatic anomalies, assumptions (15) are satisfied for a value of y_0 dependent on the amplitude of the same anomalies.

4. – The intergyre gyre

It is now possible to define the anomalous circulation associated to the climate anomalies in the wind-stress field. This anomalous circulation can be analysed putting the form of the wind-stress curl anomalies (12) into the equations for the stream-function (6) and the maximum downward propagation of the motion (7). In particular, in the case of the exponential stratification (8), this happens putting (12) into eqs. (9) and (10).

Figure 2 shows how the streamlines of the anomalous advection create an exchange of water masses between the subtropical and subpolar gyre. The streamlines at different depths (here not shown) show the absence of the migration to the North-West or South-East corner of the basins that are instead visible for, respectively, the subtropical and subpolar gyre [6, 7, 9]. This is due to the symmetry for reflection to the plane $y = y_0$ of the bowl associated to the intergyre gyre (fig. 3).

From the shape of the bowl associated to the circulation driven by the total wind-stress (fig. 4) it is possible to see how the anomalies introduce changes in the convexity of the surface $z = -h(x, y)$. This is caused by the asymmetry introduced in the wind-stress curl by the climate anomalies. Figure 4 shows how the bowl associated to the subtropical gyre is a monotonic-decreasing function in the variable y and a monotonic-increasing function in the variable x, showing a minimum at the North-Western corner

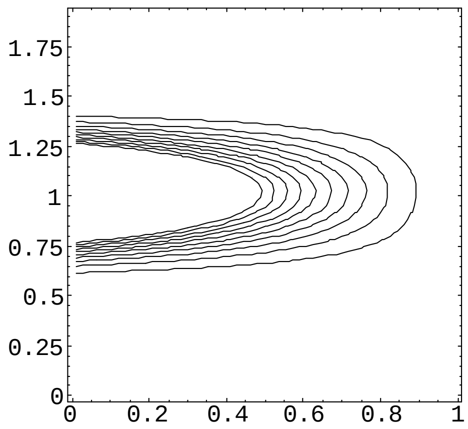

Fig. 2. – Streamlines of the transport function for the intergyre gyre. $A(t) = 0.9$. The beta-plane frame of reference is understood.

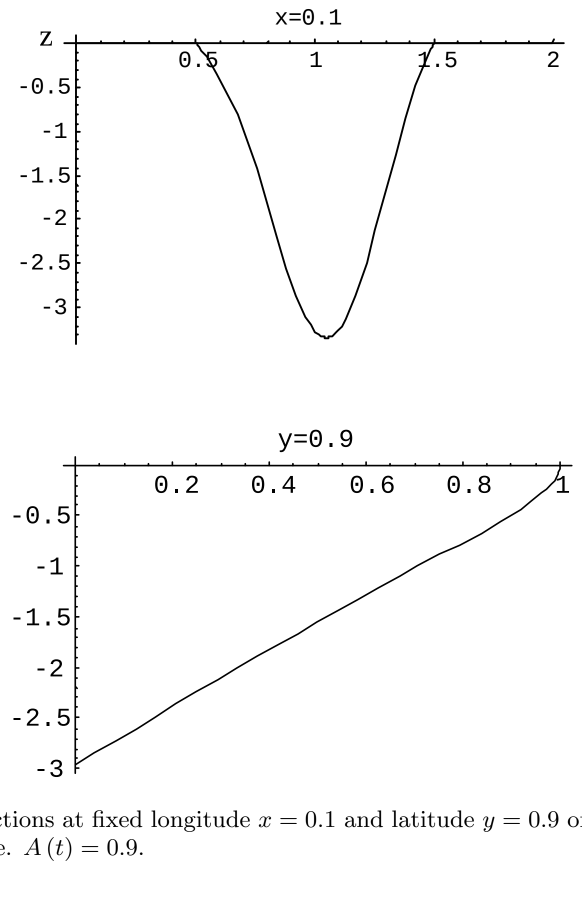

Fig. 3. – Vertical sections at fixed longitude $x = 0.1$ and latitude $y = 0.9$ of the bowl associated to the intergyre gyre. $A(t) = 0.9$.

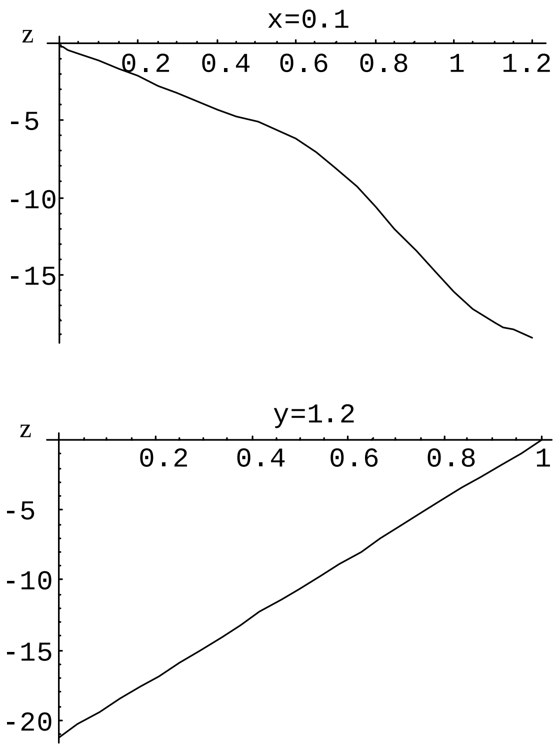

Fig. 4. – Vertical sections at fixed longitude $x = 0.1$ and at fixed latitude $y = 1.2$ of the bowl associated to the subtropical gyre affected by anomalies. $A(t) = 0.9$.

of the domain [6, 7, 9]. We see in fig. 3 that for the intergyre gyre, h conserves its monotonicity in the variable y only in the subdomains $D_1 = [0 \leqslant x \leqslant 1] \times [0 \leqslant y \leqslant 1]$ and $D_2 = [0 \leqslant x \leqslant 1] \times [1 \leqslant y \leqslant 2]$.

The maximum depth of the flow is found at the point $(0, y_0)$ of the bowl. A derivation of the equation for the maximum penetration depth of the interface can be found, for example, in [14]. The lack of monotonicity of the intergyre gyre bowl requires that the derivation assumed by [14] must be restricted to one of the subdomains, D_1 or D_2. Following [14], due to the symmetry for reflection to the plane $y = y_0$ and to the monotonicity of h, in each of these subdomains holds:

$$h_{\max} = \max_{x,y} h(x, y) \ .$$

The substitution of (12) into (7) shows how the value of the maximum downward propagation of the flow depends on the amplitude of the anomalies through the parameter $A(t)$. This parameter should vary slow enough to preserve the condition of adiabatic flow (1): changes in the anomalies amplitude are, in fact, correlated to changes in the density anomaly field through (4). Sudden changes in the density anomaly field can thus create non-adiabatic, cross-isopycnal fluxes. In this way we can consider $A(t)$ as a phenomenological constant and study different configurations of the circulation associated to different values of it.

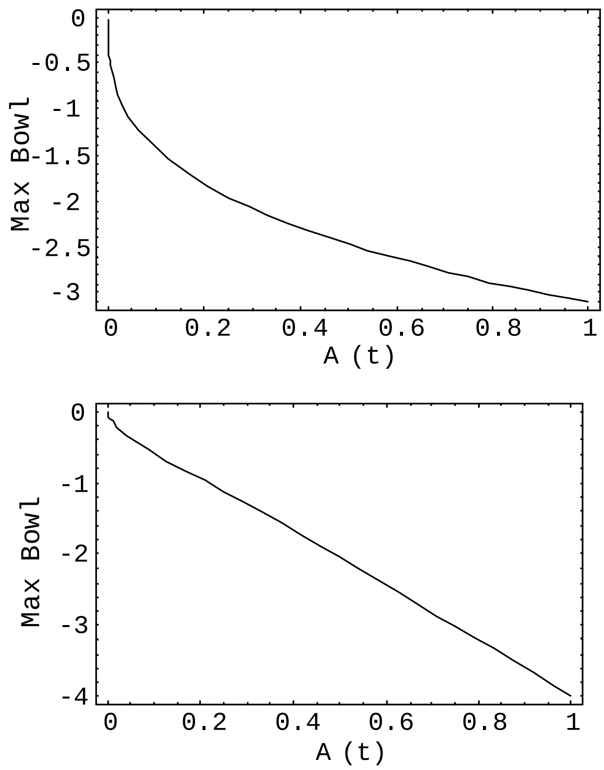

Fig. 5. – Maximum depth of the intergyre bowl as a function of the anomalies amplitude $A\,(t)$ for constant (upper panel) and exponential (lower panel) Brunt-Väisälä frequency.

Figure 5 shows the behaviour of the value of the maximum downward propagation of the flow in function of the anomalies amplitude $A\,(t)$, both for the cases of constant (upper row) and exponential (lower row) profiles of the Brunt-Väisälä frequency. The values of the maximum depth of the intergyre gyre for an exponential stratification can be found with a numerical solving of (10).

TABLE I. – *Non-dimensional maximum depth of the bowl for the climatic wind field (upper row) and for the anomalies-affected field (lower row). In the middle row is reported the value of the non-dimensional maximum depth of the bowl associated to the intergyre.* $A\,(t) = 0.9$*, exponential stratification.*

Non-dimensional maximum depth of the unperturbed ($A\,(t) = 0$) bowl	-11.7
Non-dimensional maximum depth of the intergyre gyre bowl ($A\,(t) = 0.9$)	-3.4
Non-dimensional maximum depth of the total bowl, given by the non-linear combination of the unperturbed and intergyre gyre bowls	-21

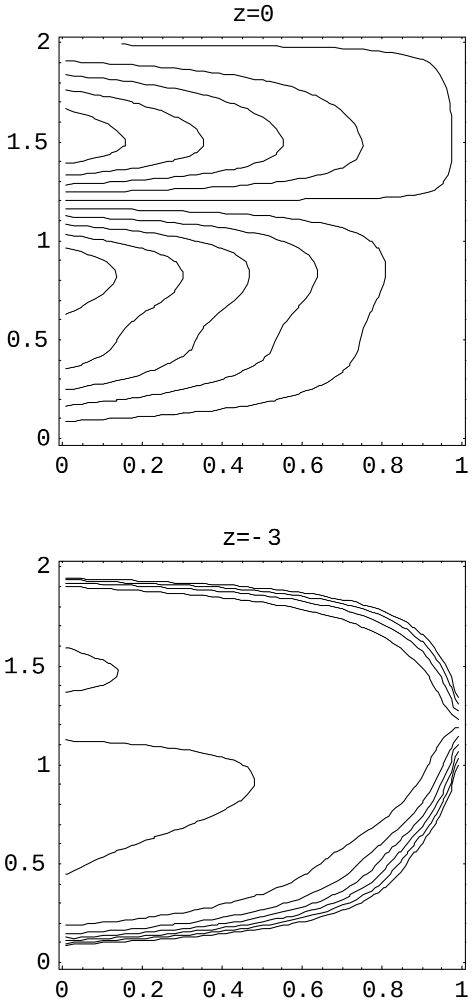

Fig. 6. – Double-gyre streamlines for the total wind-stress affected by climatic anomalies at the non-dimensional depths $z = 0$ and $z = -3$. $A(t) = 0.9$. The beta-plane frame of reference is understood.

The comparison between the two figures shows how the non-linearity introduced by the exponential Brunt-Väisälä allows a deeper downward propagation of the circulation induced by the anomalies inside the ocean interior. The non-linearity has also a second interesting effect: the total depth of the motion cannot be considered anymore as a linear combination of the depth of the unperturbed case added to the depth associated to the anomalies (see table I).

It is possible to see that the non-dimensional depths are really larger ($O(10)$) than we should expect for a real ocean. This case is discussed in [9] and is ascribed to the fact that the wind field depends only on latitude.

An interesting case is the behaviour of the bowl for values of z close to zero. If we linearize about $z = 0$ in (7) [14], we obtain the equation for the bowl already cited at the end of sect. **2**, *i.e.*

$$h\,(x,y) = \left(-6w_{\mathrm{E}}\frac{1-x}{y_0-y}\right)^{1/3}. \tag{16}$$

This indicates that for depths close to the interface between the Ekman and the geostrophic layers the circulation almost does not depend on the density stratification and the maximum depth of the circulation given by the sum of the climatic and the anomalous wind field behaves like in the case of linear stratification. At the same time this allows the weak anomalies, associated to not deep bowls, to propagate into the ocean interior independently of the stratification.

On the other hand, at greater depths the circulation depends strongly on the buoyancy profile.

Finally, fig. 6 shows the streamlines of the circulation affected by anomalies, for a value of the amplitude of these of $A(t) = 0.9$. The linearity of (2) for the variable ψ allows to see how the deformation of the streamlines of the total circulation is simply given by linear combination of the streamlines of the unperturbed case and the streamlines associated to the anomalies. From the figure it is possible to see a northern deformation of the streamlines of the subtropical gyre and a pronounced eastward deformation of the streamlines of the subpolar gyre. These deformations are caused by the asymmetry introduced by the anomalies in the Ekman pumping. The same asymmetry shapes the behaviour of the streamlines at greater depths, where the streamlines of the subpolar current show a faster extinction as a function of depth than the streamlines of the subtropical gyre. At these depths we have so an eastward predominance of the subtropical current over the subpolar current.

The shape of the streamlines is confirmed by observations [15, 16].

5. – Conclusion

An independent approach, consisting in the use of the baroclinic circulation model of Rhines and Young in a typical observational context, has been used to investigate the anomalous circulation created by climate anomalies in the wind-stress field.

This allowed to study the property of the intergyre gyre, in particular the dependence of the maximum depth of the motion inside the ocean interior as a function of the amplitude of the anomalies. This dependence can be linear or non-linear, according to the kind of stratification chosen for the ocean. Non-linearity allows the anomalous circulation to propagate deeper into the ocean interior, while weak anomalies propagate independently of the choice of buoyancy frequency.

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