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Does Cointegration Matter?

An Analysis in a RBC Perspective

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Does Cointegration Matter? An Analysis in a RBC Perspective*

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Abstract

The aim of this paper is to verify if a proper SVEC representation of a standard Real Business Cycle model exists even when the capital stock series is omitted. The argument is relevant as the common unavailability of sufficiently long medium-frequency capital series prevent researchers from including capital in the widespread structural VAR (SVAR) representations of DSGE models - which is supposed to be the cause of the SVAR biased estimates. Indeed, a large debate about the *truncation* and *small sample* bias affecting the SVAR performance in approximating DSGE models has been recently rising. In our view, it might be the case of a smaller degree of estimates distorsions when the RBC dynamics is approximated through a SVEC model as the information provided by the cointegrating relations among some variables might compensate the exclusion of the capital stock series from the empirical representation of the model.

JEL CLASSIFICATION: E27, E32, C32, C52.
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1 Introduction

The purpose of this paper is to show that it is possible to overcome the limitations exhibited by the Structuralized Vector Autoregressions (SVAR) approach

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in the identification of the theoretical predictions of Dynamic Stochastic General Equilibrium (DSGE) models by means of a Structuralized Vector Error Correction (SVEC) representation. Several literature contributions have focused on the bias implied by the estimates of finite-order SVARs(p) when theoretical DSGE models are tested against empirical data, whereas little attention has been paid so far to the potential of the SVEC based evidence.

Most of the literature we refer to argues that the long-run SVAR approach - which implies an identification strategy of the structural shocks consistent with the economic theory for the long-run - may yield results which are only imperfect approximations of the model predictions, when the long run effects of a technology or a monetary shock has to be represented within a standard Real Business Cycle (RBC) framework. Chari et al. (2008), Erceg et al.(2004) and Ravenna (2007) argue that, when a DSGE model has to be empirically represented, but some model variables are unobservable and then omitted from the SVAR specification, it may perform poorly when estimated over the sample periods normally available. This is because, when the recursive dynamic equilibrium of a standard DSGE model admits the following state-space representation:

$$
\begin{aligned}
&y_t = Px_{t-1} + Qz_t \\
&x_t = Rx_{t-1} + Sz_t \\
&Z(L)z_t = \varepsilon_t
\end{aligned}
$$

where y_t is the endogenous variable vector with dimension r x 1 , x_t is the observable state variables vector with dimension n x 1 , z_t is the m x 1 exogenous state variables vector and ε_t is an m x 1 vector of stocastic variables such that $\mathrm{E}(\varepsilon_t) = 0$, $\mathrm{E}(\varepsilon_t\varepsilon_t^{'}) = \Sigma$, $\mathrm{E}(\varepsilon_t\varepsilon_j^{'}) = 0$ for $j \neq t$ and where Σ is a diagonal matrix, if $n < m$, the only finite order representation of the model is a VARMA (p,q) representation[1]. It is argued that if the number of the exogenous variables exceeds the number of the observable state variables, the VARMA (p,q) can still be approximated by a finite order VAR (p), provided that the VAR is characterized by a sufficiently high value of p.

Unfortunately, the short lenght sample data problem prevents economists from including in the VAR a sufficient number of lags in order to obtain a reliable model representation. The so-called *truncation bias* - due to the fact that researchers are forced to estimate only *truncated* SVARs of small[2] order p - has been largely discussed by Chari et al. (2008) and Ravenna (2007), whereas Faust and Leeper (1997) and Erceg et al. (2004) highlighted the *small sample bias* as the major shortcut of the SVAR approach when applied to DSGE models.

The aspect of the debate this paper focuses on comes from Fry and Pagan (2005) which discuss the points of Chari et al. (2008) and Erceg et al. (2004) regarding the SVAR limitations in approximating DSGE models. Fry and Pagan

[1] See Chari et al. (2008) and Ravenna (2007).

[2] Tipically, with existing data lengths, researchers are forced to deal with VAR(4).

claim that the problems arising from the finite order VAR (p) performance - when it is only an approximation of the true VARMA representation of the model - crucially depends on "*which variables are retained in the VAR and which are deleted"*. Indeed, within the DSGE class of models, if one wants to model only a subset of the original variables dealing correctly with a finite order VAR (p), he has to ensure that: i) the omitted variable doesn't Granger causes the rest of the variables included; ii) the eliminated variables must be connected to the retained variables through an identity which cannot contain lagged expression of the dropped variable. Within a standard RBC framework, these conditions are met when the capital stock variable is not ruled out from the VAR variables set.

As a matter of fact, it might be difficult to get or build data on the capital stock even if Del Negro et al.(2005) show how to recover the capital stock series out of model parameters and the investment data or Ireland (2004) shows how to "*take DSGE models to the data*". Nonetheless, strong limitations of the Ireland approach have been recentely highlighted by Juselius and Franchi (2007).

In this paper we want to enhance the potentiality of the SVEC approach compared to the SVAR results when a technology shock effects as predicted by a standard RBC model have to be represented. We find also insightful to compare the two approaches with respect to the capital stock series inclusion/exclusion debate mentioned above. As within the RBC model permanent relations among variables are driven by the technology dynamics, we argue that the SVEC representation takes advantage of that by considering the role of cointegration, whereas a SVAR model might be misspecified when cointegration emerges[3].
In the spirit of Erceg et al.(2004) we run a RBC model and get the theoretical predictions about the effects of a technology shock upon output, consumption, investment and capital stock. By doing that, we generate a 10,000 artificial observations series for each variable. Then, in a Monte Carlo fashion, we sample 999 subsamples - the typical amount chosen in this kind of experiments - 200 observations long each - that we consider as 200 quarters series - The sampled artificial series are employed in order to estimate a SVAR and a SVEC model from which we get the relative impulse response functions (IRFs) to a technology shock which is identified following a long run approach in both cases. In order to address the argument by Fry and Pagan (2005), we re-estimate the SVARs and SVEC models, omitting the capital stock series from the variables set and get new different IRFs. The comparison between the theoretical and the estimated SVEC and SVAR IRFs allows to assess the performances associated with the different empirical representations of the model.

The paper is organized as follows. Section 2 outlines the RBC model and its specifications. In the same section we also briefly comment the theoretical impulse response to a technology shock (2.2). Section 3 unveils the role of cointegration within the RBC model and outlines the methodology employed in order to build the artificial dataset and the sampling (3.2); the identification

[3]A fundamental contribute to the argument is given by King et al. (1991) which made evident as "vector error-correction models" represent an ideal environment for testing RBC models.

schemes of the SVAR (3.3) and the SVEC (3.4) are also presented. The results obtained in terms of IRFs are presented and discussed in Section 4. Section 5 concludes.

2 The RBC model

2.1 The environment

In this section we layout our baseline Real Business Cycle (RBC) model which is an extended version of the benchmark stochastic neoclassical growth model in Uhlig (1999). We define the fundamentals of the economic environment. For the whole model analytical solution and loglinearization see Appendix B.

The *preferences* of the representative agent are as follows:

$$U = E_t \sum_{t=0}^{\infty} \beta^t e^{u_t^\beta} \left[\log C_t - e^{u_t^N} \frac{N_t^{1+\eta}}{1+\eta} \right] \tag{1}$$

where C_t is consumption, N_t is labour, $0 < \beta < 1$ is the private discount factor, η is the inverse of Frish labour supply elasticity . Agent's preferences are influenced by $e^{u_t^\beta}$ and $e^{u_t^N}$ whose dynamics is ruled by an I(1) stationary stochastic processes $e^{u_t^\beta} = e^{\rho_\beta u_{t-1}} e^{\epsilon_\beta}$ and $e^{u_t^N} = e^{\rho_N u_{t-1}} e^{\epsilon_N}$ where both the error terms follow a white noise process. The first shock might be interpreted as a shock affecting the intertemporal consumption vs. saving choice, whereas the second shock affects the intratemporal labor-supply choice.

The *technology* is characterized as a standard Cobb-Douglas production function:

$$Y_t = e^{u_t^a} K_{t-1}^{\alpha} N_t^{1-\alpha} \qquad \text{where } 0 < \alpha < 1 \tag{2}$$

where capital stock - K_t - and labour - N_t - are the input factors and α stands for the *capital share*. Capital stock equation is given by:

$$K_t = I_t + (1-\delta) K_{t-1} \qquad \text{where } 0 < \delta < 1 \tag{3}$$

where δ is the *depreciation rate* and where the investment is specified by an increasing and convex equation with capital adjustments costs which are subject to a stationary I(1) shock ($e^{u_t^K} = e^{\rho_K u_{t-1}} e^{\epsilon_K}$):

$$I_t = e^{u_t^K} f\left(\frac{K_t}{K_{t-1}}\right) K_{t-1} \tag{4}$$

Output (Y_t) is absorbed completely by consumption, investment and by an exogenous government spending component[4] which follows the following stationary I(1) process: $e^{u_t^G} = e^{\rho_G u_{t-1}} e^{\epsilon_G}$:

[4] See Smets and Wouters (2007) for this aggregate resource constraint specification. We allow the government spending to be just a small persistent exogenous disturb.

$$Y_t = I_t + C_t + e^{u_t^G} \tag{5}$$

In this paper, we assume that technology $e^{u_t^a}$ may evolve following two alternative stochastic processes[5]. Given the general expression for an I(2) trending non-stationary technology (in logarithms) :

$$u_t^a = u_{t-1}^a + \rho_a du_{t-1}^a + \epsilon_a$$

which is a unit-root I(2) process where $du_{t-1}^a = u_{t-1}^a - u_{t-2}^a$ and $\epsilon_a \sim i.i.d.(0, \sigma^2)$, we're going to consider both the specifications where $\rho_a = 1$ and where $\rho_a = 0$ which implies the pure random-walk case:

$$u_t^a = u_{t-1}^a + \epsilon_a.$$

2.2 Theoretical impulse responses

We look at the theoretical responses of output (y), consumption (c), investment (i), capital stock (k) and employment (n) to a technology shock in terms of log-deviations from their steady state growth path, according to both the specifications of the TFP. Our analysis will specifically aim at comparing the theoretical responses of y, c, i and k to the those derived by SVAR and a SVEC evidence.

As it is displayed in Appendix A (Figg.1 - 2) the IRFs obtained by our RBC model cover 40 periods afterwards the occuring of the shock and are distinguished on the basis of the specification of the technological process. It is worth spending a few words on the alternative specifications for technology and the different impulse responses obtained, as this aspect deals with the vivid debate arisen after that Galì seminal paper challenged the prediction power of RBC models[6]. Later, several authors aimed both at supporting his results (e.g. Basu, Fernald and Kimball (2001), Francis and Ramey (2003)) and at questioning them (e.g. Altig et al. (2002), Christiano, Eichenbaum and Vigfusson (2003)), but here we address in particular to those contributions as Lindé (2004) or Rotemberg (2003) which looked for the necessary conditions in order to reconcile RBC models and empirical evidence. Indeed, according to them, if one allows for a technical progress which diffuses its effects at sufficient slow rates (i.e. a technological shock which is correlated over time in growth terms) the RBC model provides results which satisfy the evidence pointed out by Galì as far as labour productivity and hours worked responses are concerned. The same does not hold if a stationary-in-differences random walk process is chosen.

The I(2) results we present in Fig.2 (where a non stationary technology growth rate is assumed), differently from those achieved through the I(1) specification, are in line with these arguments as it is shown that investment, capital

[5] The role played by an alternative specification for technology will be clearified in Subsection 2.4. Henceforth, we consider the I(1) and the I(2) TFP driven models, separately.

[6] Contrary to RBC predictions, Galì found that when a technology shock is the only source of disturbance in the economy, according to data, the labour productivity rises and the hours worked fall after a positive realization of it.

and employment fall after a positive realization of the shock. The economic intuition behind is that when it takes longer for the technological progress to diffuse, for the representative agent the current marginal utility of wealth is lower. Therefore, he will find more convenient to consume, to enjoy leisure and to wait until labor effort will become more effective. This intuition is consistent with the fall in investment that we found - see Fig. 2 - but also with the results obtained in terms of diminishing hours worked and rising labor productivity provided by Lindé (2004) and Rotemberg (2003). Nonetheless, we found also that investment, capital and hours worked come to positive values in the steady-state in a very short run.

As far as the I(1) specification is concerned, we find that all the variables (y, c, i, k and n) share the same positive sign both at the impact and all along the following pattern. Nonetheless, while output and investment reach the new steady-state level a few periods after the shock, consumption, employment and capital paths take longer. In particular, the capital stock shows a very little log-deviation value at the impact, that seems to increase steadily until the 30th period, after which it keeps on rising but at a decreasing rate. Investment and employment are the only variables dropping after a positive impact reaction.

3 The SVAR and the SVEC specification

Before evaluating the SVAR and the SVEC performances in terms of theoretical responses approximation, we want to discuss more extensively the intuition according to which the SVEC approach might overcome the SVAR models limitations even when the capital stock series is omitted from the empirical representation, addressing the debate recalled above. Then, after having briefly recalled some theory about the SVAR and the SVEC models, we discuss the shock identification rationale employed to recover the empirical IRFs.

3.1 The CI space

Within the RBC model a not trivial source of information is represented by the presence of the *common trends* induced by the technological dynamics [7] which implies the stationarity of the following great ratios: C_t/Y_t and I_t/Y_t. From an econometric perspective, the existence of CI relations between y - c and between y - i can be exploited in order to catch the permanent relations established among y, c, i and k as well.

We address the argument of Fry and Pagan (2005) that discuss the contributions by Chari et al. (2008) and Erceg et al. (2004) that pointed out the limitations of the SVAR approach. Fry and Pagan state that the necessary conditions to be ensured in order to deal with a correct finite order VAR (p)

[7] The so-called "*Common Trends*" approach has been originally proposed by Stock and Watson (1988).

though modelling only a subset of the original model variables, imply that the capital stock must not be ruled out from the VAR variables set, if the model is a RBC model. According to them, the crucial role played by k_t is due to the fact that it sets the model dynamics being either the endogenous Granger causal variable and being linked to the rest of the variables through its lagged value. The empirical problem is that, as sufficient long medium-high frequency time series for capital stock are normally scarcely available, economists are often induced to omit it from the empirical representation of DSGE models.

Our point rests on the intuition that taking account of the CI relations the SVEC representation should perform better than a SVAR even when the series of k is omitted from the empirical representation of the RBC model, so that one could even disregards the "necessary conditions" pointed out by Fry & Pagan[8].

The intuition comes from the RBC steady state solution[9] which highlights the relation linking y - i, y - c and i - k. The relationship between output, investment and consumption derives directly from the resources contraint:

$$Y_t = C_t + I_t$$

which in steady state, once having normalized with respect to Y_t, results as:

$$\overline{C} = 1 - \overline{I}$$

and symmetrically as:

$$\overline{I} = 1 - \overline{C} \tag{6}$$

(where $\overline{C}$ and $\overline{I}$ are stationary stochastic processes) and from the fact that in steady state the allocation of time between work and leisure is kept constant. Besides, from the capital accumulation equation we get the relationship between capital stock and investment :

$$\begin{aligned} K_t &= e^{u_t^K} f\left(\frac{K_t}{K_{t-1}}\right) K_{t-1} + (1-\delta) K_{t-1} \\ I_t &= e^{u_t^K} f\left(\frac{K_t}{K_{t-1}}\right) K_{t-1} \end{aligned}$$

which in steady state results as:

$$\begin{aligned} \overline{K} &= (1-\delta)\overline{K} + \overline{I} \\ \overline{I} &= \delta \overline{K} \end{aligned} \tag{7}$$

[8]Leaving aside our specific issue, the better performance of the SVEC models over the SVAR in presence of CI relations is analitically described also in Pagan & Pesaran (2008). Indeed, when a permanent/ transitory decomposition of the shock intervenes, the SVECM provide useful information to restrict the structural identifying equations system which is missed within a SVAR context.

[9]See Appendix for the complete steady state solution of the model.

As the technology dynamics determines CI relations between y - c and y - i , it is evident that, holding (10), the relations between y - i and between y - k are proportional. As a consequence, i and k result to be driven by the same stochastic trend. Then, it might be reasonable to expect that a satisfying SVEC approximation of the RBC dynamics can be obtained even excluding k from the variables set since its dynamics is preserved through the presence of investment.

It now appears more evident the reason why the inclusion of the series for k appears essential within a framework which cannot exploit the proportionality that links k and i, as the SVAR framework is: if the dynamic endogenous variable is dropped, the model dynamics cannot be recovered alternatively.

3.2 Methodology

The methodology we adopt to compare the SVAR to the SVEC approximating performances of the true responses is in the spirit of Erceg et al. (2004) or Chari et al. (2008) which perform a similar exercise but with different aims. We run 10 000 stochastic simulations of the RBC model for each TFP specification, obtaining a 10 000 observations series for each variable in both cases.[10]. Then, we employ a Monte Carlo procedure by sampling 999 series, 200 observations long, out of each variable artificial dataset. In the end, we obtain 999 subsamples, 200 quarters long for each variable, for both the TFP specifications.

We then, respectevely, estimate 999 four-variables (y, c, i, k) VARs and VEC models employing the artificial series sampled out of the models simultaions[11]. Then, we identify the technology shock using a long run aproach in both the SVAR and the SVEC framework and compare the IRFs with the theoretical ones. We repeat the procedure omitting k from the variable set in order to compare the SVAR and the SVEC different performances in terms of approximation of the theoretical predictions. At last, the estimated IRFs[12] and the RBC theoretical responses are compared and their gap is measured by means of the MAE and the RMSE statistics.

3.3 The long-run SVAR approach

Given the following reduced form Vector Moving Average (VMA) representation recovered from the inversion of a stationary Vector Autoregressive Representation (VAR):

$$y_t = A_i^{-1}(L)u_t \qquad u_t \sim N(0, \Sigma)$$

[10]Model simulation and artificial data generating process are obtained by using Dynare 4.02 for Matlab.

[11]For the AR(1) model specification, we estimate a VAR (1) with differentiated data and a VECM (1) with non-differentiated data, as suggested by the Schwarz Info Criterion . Similarly, in the AR(2) case, we estimate a VAR(2) with differentiated data and VECM(2) with non-differentiated data, according to the same criterion.

[12]In particular, we calculate an average IRF for each group of 999 estimated IRFs to the technology shock, for each SVAR and the SVEC estimation. The average IRFs - one for every variable- are then compared with their respective RBC theoretical responses.

where y_t is the vector of the variables included in the model; $A_i^{-1}(L)$ is the inverted dynamic coefficients matrix; u_t is the vector of the reduced-form error terms, we define $A^{-1}(L) = \Phi(L)$ and obtain a process expressed as a linear combination of the past innovations in accordance with the Wold decomposition:

$$\begin{aligned} y_t &= \Phi(L)u_t = \sum_{h=0}^{\infty} \Phi_h u_{t-h} \\ \text{where} \quad \Phi_0 &= I_m \end{aligned} \tag{8}$$

But, in order to recover the unobservable relevant shocks (ε_t) out of the observable reduced form innovations (u_t), a structural VAR representation has to be considered and a set of restrictions has to be imposed. Given the following structural VAR form:

$$A_0 y_t = \sum_{i=1}^{p} A_i^* y_{t-i} + B\varepsilon_t \qquad \varepsilon_t \sim N(0, I_m) \tag{9}$$

where A_o is the (m x m) contemporaneus effects matrix; A_i^* is the (m x m) lagged effects matrix and B is the (m x m) structural shocks "short-run response" matrix. What follows is the system of structural equations linking u_t to ε_t which we have to restrict in order to univocally identify them

$$u_t = A_0^{-1} B\varepsilon_t \tag{10}$$

Among the existent different identification strategies - short-run restrictions on B, Cholesky triangularization on A_0, restrictions of both A_0 and B (Amisano Giannini (1997)) - we choose the long-run SVAR approach (Blanchard-Quah (1989)): the contemporaneous matrix is orthonormalized ($A_0 = I_m$), the short-run B matrix is let totally unrestricted while some restrictions are imposed on the long-run response matrix $C(1)$[13] in accordance with the long-run theoretical predictions about the effects of the original economic shocks hitting the economy.

A growing recent literature estimates structural VARs by using this approach. Within the RBC framework a typical restriction is that only technology shocks affect labor productivity in the long run (Galí (1999); Francis and Ramey (2003; 2005); Christiano, Eichenbaum, and Vigfusson (2003); Galí and Rabanal (2004)); but there are also contributions dealing with different indentification schemes as the absence of a permanent technological effect on hours (Shapiro-Watson (1988); Gamber-Joutz (1997); Fleischman (2000); Francis and Ramey (2003)), or with the absence of a permanent effects of demand shocks - as goverment spending, preferences shifting - on output, hours worked (Shapiro-Watson (1988); Gamber-Joutz (1997); Fleischman (2000)) or real wages (Gamber-Joutz (1997)).

The long-run structuralization we choose is in line with part of this literature. Indeed, we assume that the technology shock is the only shock having

[13] The long-run matrix is defined as the sum of the short-run cumulated responses. This is a B-model approach.

permanent effects on all the variables. At first, we deal with a 4 variables SVAR - output (y_t), consumption (c_t), investment (i_t) and capital-stock (k_t) - which are represented in the following vector of stationary series:

$$x_t = [y_t \quad c_t \quad i_t \quad k_t]'$$

On total, the number of restrictions we have to impose is: $m(m-1)/2 = 6$ - since we deal with a 4 variables SVAR -which are imposed on the long-run C(1) matrix, according to a Cholesky triangular factorization:

$$C(1) = \begin{bmatrix} c_{11} & 0 & 0 & 0 \\ c_{21} & c_{22} & 0 & 0 \\ c_{31} & c_{32} & c_{33} & 0 \\ c_{41} & c_{42} & c_{43} & c_{44} \end{bmatrix}$$

This recursive scheme is in line with the theoretical assumptions which are relevant for our purpose i.e. that on the long-run output is affected exclusively by the technological shock which permanently influences also consumption, investment and capital. Accordingly, we set c_{12}, $c_{13}, c_{14} = 0$.which are the only restrictions theoretically justified as we want to identify only the technological dynamics due to ε_t^a, i.e. the supply shock. The rest of the restrictions imposed - $c_{23}, c_{24}, c_{34} = 0$ - are not pinned down economically: we need the related shocks just in order to have a full identification of the system.

When we remove k_t from the variables set we have a 3 variables (y_t, c_t, i_t) SVAR and only 3 restrictions to be imposed on the long-run C(1) matrix:

$$C(1) = \begin{bmatrix} c_{11} & 0 & 0 \\ c_{21} & c_{22} & 0 \\ c_{31} & c_{32} & c_{33} \end{bmatrix}$$

where the relevant theoretical predictions are the same as in the four variable case and imply $c_{12} = 0, c_{13} = 0$.

3.4 The SVEC approach

3.4.1 VEC model

The basic concept of the VEC approach is *cointegration* among non-stationary or I(1) series where a set of I(1) series driven by the same stochastic trend is said to be *cointegrated* if there exists a linear combination of them which is stationary or I(0). As shown by Granger (1981) and Engle and Granger (1987), the best representation in a CI context is provided by the error correction models (ECM). The VEC representation is obtained by a reparametrization of a VAR(p) which yields a VEC (p-1):

$$\Delta y_t = \Pi y_{t-1} - \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-1} + u_t \qquad u_t \sim N(0, \Sigma_u) \tag{11}$$

where y_t is a vector of m dependent variables, $\Gamma_i = -(A_{i+1} + ... + A_p)$ refers to the *short-run* parameters, u_t is an independent stochastic vector of the unobservable error terms and $\Pi = -(I_m - A_1 - .... - A_p)$ is a matrix of rank m which can be factorized (Johansen 1995)) as the product of a *cointegration matrix* (β) and a *loading coefficient matrix* (α) :

$$\underset{m\times m}{\Pi} = \underset{m\times r}{\alpha} \cdot \underset{r\times m}{\beta'}$$

where $rk(\alpha) = rk(\beta) = r$. Assuming that the original series were I(1), the VEC form obtained above contains only I(0) terms as Δy_t and Δy_{t-1} are stationary terms by definition and, even tough Πy_{t-1} includes I(1) variables, it contains also the CI relations which are stationary. The Πy_{t-1} term identifies the *long-run* parametres.

As it is well known, if the matrix Π is invertible - i.e. if $rk(\Pi) = m$ - it means that all the m variables are stationary. Whereas if $rk(\Pi) = r < m$, then the I(1) variables are linked by r CI relations which are I(0) and which represent the long-run equilibrium relations existing among the variables. The whole statistical equilibrium would be driven by $m - r = k$ stochastic trends.

As the fundamental contribution of King et al. (1991) proved, the VEC framework is particularly suitable for the RBC environment which, in our case, is characterized by the presence of four variables (y, c, i and k) and three CI relations (y - c; y - i and y - k) as c, i and k are driven by the same technological stochastic trend set by the TFP non-stationary dynamics.

3.4.2 Cointegration tests and CI space restrictions

We test for the presence of CI relations by running the Johansen test (Johansen 1995)) for each subsample which consists of 999 four-variable vectors[14]: the results we obtain confirm at a confidence level of 95% the presence of 3 CI vectors for the groups including[15] k and the presence of 2 CI vectors in both the groups not including[16] k,as we expected.

Provided that we need $r^2 = 9$ restrictions in order to correctly identify β, we consider the two CI vectors in β' which define the steady state relations between output and, respectively, consumption ($c_t - \beta_{11} y_t$) and investment ($i_t - \beta_{21} y_t$). The third CI vector define the following $k_t - \beta_{11} i_t$ ratio and rests on the theoretical permanent relation:

$$\overline{I} = \delta \overline{K}$$

Having normalized accordingly three vectors, we impose the remaning 6 restrictions on β on the basis that each cointegrated variable (c, i and k) is not included in the steady-state relations involving the other ones. Eventually we obtain the following:

[14] We have 2 groups of sampled series generated by the alternative TFP models specifications. Each group is considered both with and without the series of K so that we have 4 subgroups of samples.

[15] For the 91% of the I(1) subsamples and for the 88% of the I(2) subsamples.

[16] For the 91% of the I(1) subsamples and for the 89% of the I(2) subsamples.

$$\underset{rxm}{\beta}{}^{'}\underset{mx1}{y_{t-1}} = \begin{bmatrix} \beta_{11} & 1 & 0 & 0 \\ \beta_{21} & 0 & 1 & 0 \\ 0 & 0 & \beta_{33} & 1 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ c_{t-1} \\ i_{t-1} \\ k_{t-1} \end{bmatrix}$$

3.4.3 Identification of the SVEC representation

When we consider a vector y_t generated by a reduced form VECM (11), it can be expressed in the followinfg VMA form (Johansen (1995)):

$$y_t = C(1)\sum_{i=1}^{t} u_i + C^0(L)\, u_t + y_0 \tag{12}$$

where y_0 depends on the initial conditions of non-stationary variables y_t, $C(1) = \beta_{\perp}(\alpha_{\perp}^{'}(I_m - \sum_{i=1}^{p-1}\Gamma_i)\beta_{\perp})^{-1}\alpha_{\perp}^{'}$ contains the permanent components and $C^0(L) = \alpha(\beta^{'}\Gamma\alpha)^{-1}\beta^{'}$ is the transitory component which contains the instantaneous coefficients. We know that $C(1)$ has rank $m - r$ when this model is characterized by r transitory and stationary components and by $m - r$ permanent and non-stationary components. In order to identify the structural shocks hitting the system (ε_t) we have to switch from the reduced form to the structural representation by replacing u_t by $A_0^{-1}B\varepsilon_t$ as in the SVAR case:

$$y_t = C(1)\sum_{i=1}^{t} A_0^{-1}B\varepsilon_t + C^0(L)\, A_0^{-1}B\varepsilon_t + y_0 \tag{13}$$

The long-run effects of shocks are captured by the common trends in $C(1)\sum_{i=1}^{t} u_t$. Consistently with the long-run approach (King et al. 1991), we restrict $C(1)\,A_0^{-1}B$ which gives the long-run effects of the structural innovations.

In order to fully structuralize the SVEC model we need to impose $m(m-1)/2$ $= 6$ restrictions as $m = 4$. Considering the CI relations and provided that $C(1)$ matrix can admit at most a rank equal to $(m-r) = 1$, we impose $(m-r)r = 3$ restrictions by setting the last three columns equal to zero, consistently with the presence of the unique technology structural shock having permanent effects:

$$C(1) = \begin{bmatrix} c_{11} & 0 & 0 & 0 \\ c_{21} & 0 & 0 & 0 \\ c_{31} & 0 & 0 & 0 \\ c_{41} & 0 & 0 & 0 \end{bmatrix}$$

where the relevant restrictions are imposed on the elements c_{12}, c_{13} and c_{14} that are set equal to zero. These hypotesis are consistent to the popular theoretical presumption that only supply shocks have permanent effects on productivity (Blanchard & Quah (1989), Galì (1999), Francis & Ramey (2005)).

The remaning three restrictions are imposed on the short-run impact matrix B in order to exactly identify the transitory components, not relying exculsively on a recursive structure for C(1)[17] (Stock and Watson (1988); King et al.(1991); Warne (1993)). Indeed, as proved by Vlaar (2004), a more flexible identification strategy implies one can successfully identify the common trend space by imposing any set of $(m-r)(m-r-1)/2$ independent linear zero restrictions on either the contemporaneous or the permanent impact matrix. In that spirit, we restrict the B matrix by employing cross coefficients restrictions recovered by the calibrated policy functions generated by our model[18]. We impose $r(r-1)/2$ = 3 cross-elements restrictions on the 4 x 4 B matrix which are implied by the following steady state relationships: *i)* $(1-\varphi)c_t + \varphi i_t = y_t$ which is the log-linearized resources constraint where φ is the steady state value of the investment share of output which corresponds to $\frac{\alpha\beta\delta}{1-\beta+\beta\delta}$; *ii)* $k_t = \delta i_t + (1-\delta)k_{t-1}$ which is the log linearized capital accumulation equation where δ is the capital depreciation rate.

If we consider the transitory component affecting consumption the following first two cross coefficient proportionality restrictions - which flow directly from both the relationships stated above - are derived, whereas the third restriction comes directly from the resources constraint and is obtained by considering the transitory component affecting the investment[19]:

$$\begin{aligned} b_{32} - 0.022b_{42} &= 0 \\ b_{12} - 0.706b_{22} - 0.294b_{32} &= 0 \\ b_{13} - 0.706b_{23} - 0.294b_{33} &= 0 \end{aligned}$$

When we exclude k from the variable set, our model is left with only three variables (y, c and i) and two CI vectors ($y-c$ and $y-i$) so that the number of necessary restrictions drops to 3. Accordingly, the CI space is identified as follows:

$$\underset{rxm}{\beta}' \underset{mx1}{y_{t-1}} = \begin{bmatrix} \beta_{11} & 1 & 0 \\ \beta_{21} & 0 & 1 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ c_{t-1} \\ i_{t-1} \end{bmatrix}$$

The restrictions on $C(1)$ become:

[17] If one must distinguish temporary (common trends) from permanent (cointegrating) components only through the long-run C(1) matrix restrictions, its recursive structure is necessarily required. See Vlaar 2004.

[18] The cross-coefficient restrictions approach is adopted in Vlaar (2004) where the existing proportionality between the impact responses of infaltion and real money balances is exploited. As far as we know, the "policy function approach" - which we adopt - is quite an original one within the existing impact matrix restriction strategies.

[19] It has been recognized that this identification scheme doesn't guarantee the theoretical identification of the transitory shocks, since the same cross restriction can be employed for different shocks and that the theoretical constraints depend on the model calibration. However, as we only look for a structural identification, this is not a problem for our analysis (see also Tancioni Giuli 2009).

$$C(1) = \begin{bmatrix} c_{11} & 0 & 0 \\ c_{21} & 0 & 0 \\ c_{31} & 0 & 0 \end{bmatrix}$$

where the two restrictions implied by the cointegrated variables are imposed on the elements c_{12}, $c_{13} = 0$. And the linear restriction derived by the resources constraint relation when a positive consumption shock is considered is imposed on the 3 x 3 B matrix

$$b_{12} - 0.706b_{22} - 0.294b_{32} = 0.$$

4 Results

Now we comment the results found in terms of estimated IRFs to a technological shock compared to the model responses. In particular we consider the mean of the 999 estimated responses getting an average IRF for each SVAR and SVEC specification. Primarily, we review the results in terms of the graphs reporting the estimated and the true responses. Secondly, we discuss the results by meansin terms of the MAE and the RMSE statistics which quantify numerically the distance between the theoretical and the estimated responses.

4.1 Graphical analysis

In each graph, the green line represents the model response, the blue line stands for the IRF gained by the SVAR estimation and the red one stands for the SVEC estimated responses. On the X-axes we have the periods (20 quarters) after the technological shock, while the Y-axes the values the IRFs are indicated.

Figures 3 and 4 both refer to the SVAR and SVEC estimations obtained without the omission of the capital stock series, in particular they display, respectively, the responses obtained by using the series generated by the I(1) and the I(2)model.

Figure 3 suggests that the SVAR and the SVEC performances are quite similar in terms of the true model representation. The y response is badly reproduced by both, even if the SVEC, similarly to what happens in the c case, is more precise both at the impact and for the first 3 quarters. SVEC display a much higher precision along the pattern after the first five quarters both in the i and in the k case, whereas by the same horizon onwards the SVAR performs poorly. The k impact is well catched by both.

In the AR (2) case (Figure 4), the SVEC model predictions, compared to the SVAR ones, are closer to each of the model variable responses both at the impact and along their path. The impacts of the shock and the subsequent 5 periods are always perfectly catched by the SVEC which, differentely from the SVAR, also catches the negative reponse of k.

Figures 5 and 6 refer to the SVAR and SVEC predictions when the capital stock series is omitted in their specifications. These are the results we are largerly interested in, as we want to assess the ability of the two econometric models when the dynamic variable is not included in their specifications.

Figure 5 shows the I(1) results which provide evidence of a quite poor performance of the SVAR which definetely misses the impact variables responses and predicts a flat pattern after only the first two periods following the shock. This is in contrast with the true responses which returns to the steady state after at least 8 periods after the shock. Similarly to the SVAR, the SVEC misses the y response along its rising pattern but twith a smaller impact prediction error. Concerning the c and the i the SVEC clearly performs better than the SVAR which completely misses the impacts and whose IRFs, as in the y case, become flat at horizon two after the shock.

The I(2) model predictions are compared to the IRFs derived by the omitting capital stock estimations in Figure 6. Here, the SVEC is closer to the RBC evidence in each case. In particular, the impacts ant the short run of the true responses are better reproduced by the SVECM which, as in Fig.2, catches the negative sign of the investment response. In this case, even if the SVAR gains efficiency on the long run, misses completely the sign of the responses at the impact.

To sum up, the graphical inspection shows that the SVEC advantage over the SVAR is more evident in the I(2) case and is reinforced especially when one looks at the IRF gained by the estimations performed by omitting the capital stock series. This result suggests that the lack of information related to that omission is more crucial for the SVAR than for the SVEC ability of reproducing the theoretical evidence.

4.2 MAE and RMSE statistics

The gap between the SVAR and the SVEC IRFs and our baseline model responses is measured by means of two commonly used statistics, the Mean Absolute Error (MAE) and the Root Mean Squared Errors (RMSE). Both of them are computed for each estimation we run and each model specification.

As it is known, differentely from statistics as the the Mean Error (ME) or the Mean Percentage Error (MPE), the MAE typically measures the average magnitude of the errors without considering their sign. Moreover, differently from the RMSE - which gives a relatively high weight to large errors, being a quadratic scoring rule - the MAE is a linear score which assigns an equal weight to each gap between the forecast and the true sample. In the present framework, we consider both of them for the sake of completeness.

Technically, we compute an average IRF out of each group of variable impulse responses, computed both through the SVAR and the SVEC estimation, according to each model specification. Then, we compare each average IRF to the respective theoretical response: the MAE and the RMSE statistics are computed on the basis of the first 20 periods[20] of the two. The MAE is obtained

[20] Technically, we firstly sum up the values each variable estimated response assumed in each period for each of the four model estimations. Then the sum obtained in each model is divided by 999 - the number of estimated subsamples IRF in each model specification - which

by dividing the sum up to 20 of the absolute value of the difference between the average and the true responses, by the length of the selected interval[21], i.e. $t = 20$ in order to get the MAE:

$$MAE = \frac{\sum_{i=1}^{20} \left| \overline{IRF}_i - \widetilde{IRF}_i \right|}{20}$$

where $\overline{IRF}$ stands for the average response time-series and $\widetilde{IRF}_i$ represents the theoretical response series. For the RMSE case, we consider the root of the squared and averaged difference between the average estimated responses and the theoretical ones, over the same time horizon:

$$RMSE = \sqrt[2]{\frac{\sum_{i=1}^{20} \left(\overline{IRF}_i - \widetilde{IRF}_i \right)^2}{20}}$$

Both the indicators measure the bias of the performed estimations with respect to the true responses: larger values of the statistics mean a smaller precision of the estimates. Tables reported in Appendix A display both the MAE and RMSE values computed for each variable and for every SVAR and SVEC performed estimation. We also report the differences between the SVAR and the SVEC statistics to get an immediate insight of their distance and comparison. The overall indicators are then displayed, they result from the sum of the MAE and the RMSE statistics calculated over all the variables in order to get a global score of the estimations for every model specification.

Table 1 and 2 refer to the estimates computed for the I(1) and the I(2) cases when the SVAR and the SVEC specifications include the capital stock series. Both tables enhance the better fitting performance of the SVEC over the SVAR for each variable - with the exception of the response of output in the I(1) case - as the MAE and the RMSE attached to the SVAR are always larger than those ones attached to the SVEC.

In particular, Table 1 shows that the estimates performances are almost equivalent in the y, c and k case, while the advantage of the SVEC over the SVAR is enhanced especially in the i case (4% in terms of RMSE difference and almost 1% in terms of MAE difference). Compared to that, Table 2 reports proportionally higher results in favour of the SVEC fitting performance (the distance is equal to 11% in the y case and to 5% in the c case, in terms of RMSE and, respectevely, to 1% and 1.1% MAE).

Table 3 and 4 refer to the case of our larger concern which is the SVEC and the SVAR estimates computed when the capital-stock series are omitted. Both in the I(1) and in the I(2) case the SVEC fits always better the model (the apparent negative advantage of the SVAR in the I(1) y case is not relevant

provides an average estimated IRF for each variable and estimation.

[21] The analysis over the first 20 periods allows to focus on the model dynamics over a short/medium-run as, given we deal with artificial quarterly data, the first 20 periods of simulation captures ideally the first 5 years of observations which is the typical lenght of time during which fluctuations are observed and measured.

because both the models fail in catching the true response (see Fig.5)). The distances in terms of RMSE range from 3% to 8% and, in terms of MAE they are at least equal to 0.8%. The limited advantage of the SVECM in the investment response in the I(2) case is quite misleading since, as Figure 6 shows, the SVAR prediction completely fails in catching the negative impact of the response.

The overall statistics assess and confirm the results gained from the indicators computed for each single variable. The SVEC fits on the whole better than tha SVAR; its largest advantage over the SVAR is gained in the I(2) case when capital is included, while its smallest result is verified in the correspondent I(1) case.

In general, the comparison between the models bias confirm what is highlighted at graphical level. We are allowed to claim that, compared to a long-run SVAR approach, taking account of the information contained in the CI relations among variables by means of a SVEC model allows to obtain a more confident representation of the RBC theoretical predictions, when a technology shock is considered.

5 Conclusions

The aim of this paper is to shed a new light on the debate regarding the performance of the long-run SVAR approach when a DSGE model has to be empirically tested and some variables are omitted. Given the debate about the SVAR approach bias, we address to Fry-Pagan (2005) and their arguments according to which the necessary conditions that must be satisfied in order to overcome the SVAR limitations in a RBC framework essentially imply the inclusion of the capital-stock series in the SVAR variables set. We suggest to consider the SVEC model as an emprical strategy alternative to the SVAR approach when a standard RBC model has to be empirically replied as we want to verify if, in such a framework, the omission of the capital stock series turns out to be not a compromising problem.

What we learn from our results is that, within the RBC framework, the SVEC empirical estimates of the economy responses to a technology shock are quite more precise than the SVAR forecastings either when the capital stock series is included and when it is omitted - which is more relevant for this paper concern. By our perspective, this result is due to the fact that the SVEC representation exploits the information implicit in the CI relations that crucially links y to c and i respectevely, which is not wasted even when the capital stock series is omitted. Indeed, the long run k and i proportional ratio implies that information contained in the investment series is also informative of the long run capital-stock evolution.

The fact that our results show the theoretical responses to be better approximated following a SVEC approach rather than a long-run SVAR, does not imply that the SVEC represents a univoque solution to the debated bias affecting the long run SVAR approach. Nonetheless, it might be insightful to verify if the degree of distorsion implied by the *truncation* or the *small sample bias* within the SVAR models is larger or smaller than, *ceteris paribus*, the distorsion that emerge when the CI relations are ignored, i.e. when the SVAR rather than the SVEC approach is employed. The statistics we provide should help in evaluating the relative advantage of the SVEC approach over the SVAR which, in our perspective, reflects the importance of the role played by the CI relations within the RBC models. In analogy with this way of proceeding, there might be room for further research aiming at comparing directly the distorsion involved by the *truncation* and the *small sample bias* with the loss of precision in terms of model approximation when the SVEC specification is disregarded.

As Ravenna (2007) has already pointed out, DSGE models mainly aim at accounting for the correlations among macroeconomic variables and it is therefore crucial for them to be tested against data representations the more "model consistent" as possible. Our results provide evidence that a SVEC approach turns out to be a more reliable model-consistent representation compared to the SVAR. In spite of that, there is still much to be said up to this concern, since we are still far from being able to establish a ranking among the alternative econometric procedures. This is why further research on this way is encouraged.

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A Appendix

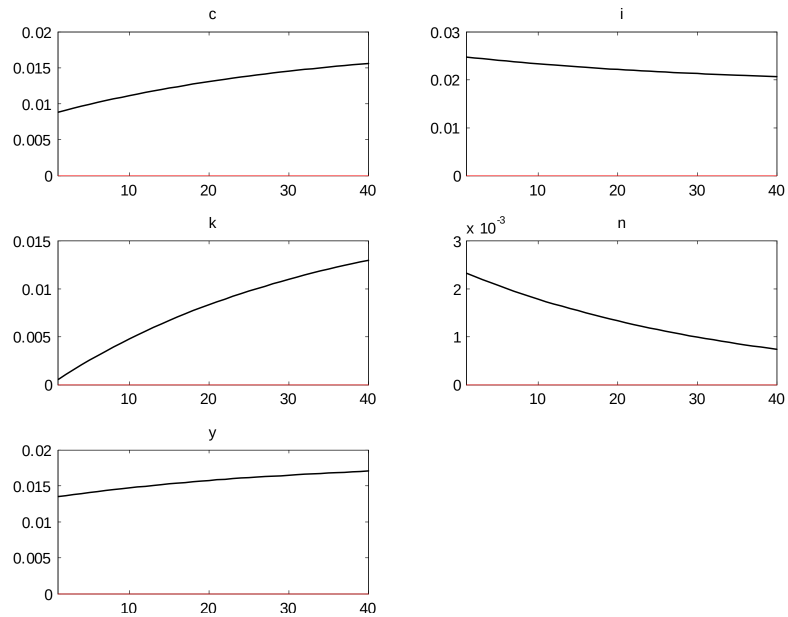

Figure 1 - Impulse Responses to an I(1) Technological Shock - RBC model

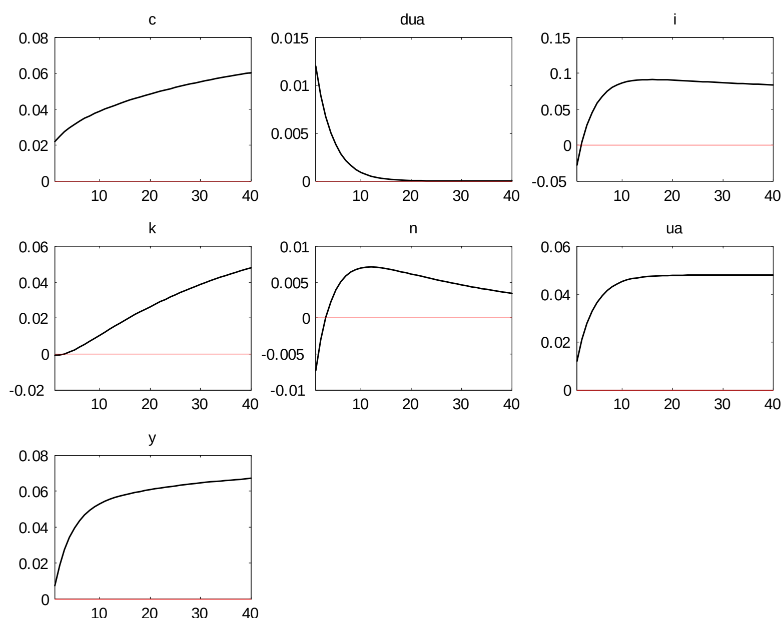

Figure 2 - Impulse Responses to an I(2) Technological Shock - RBC model

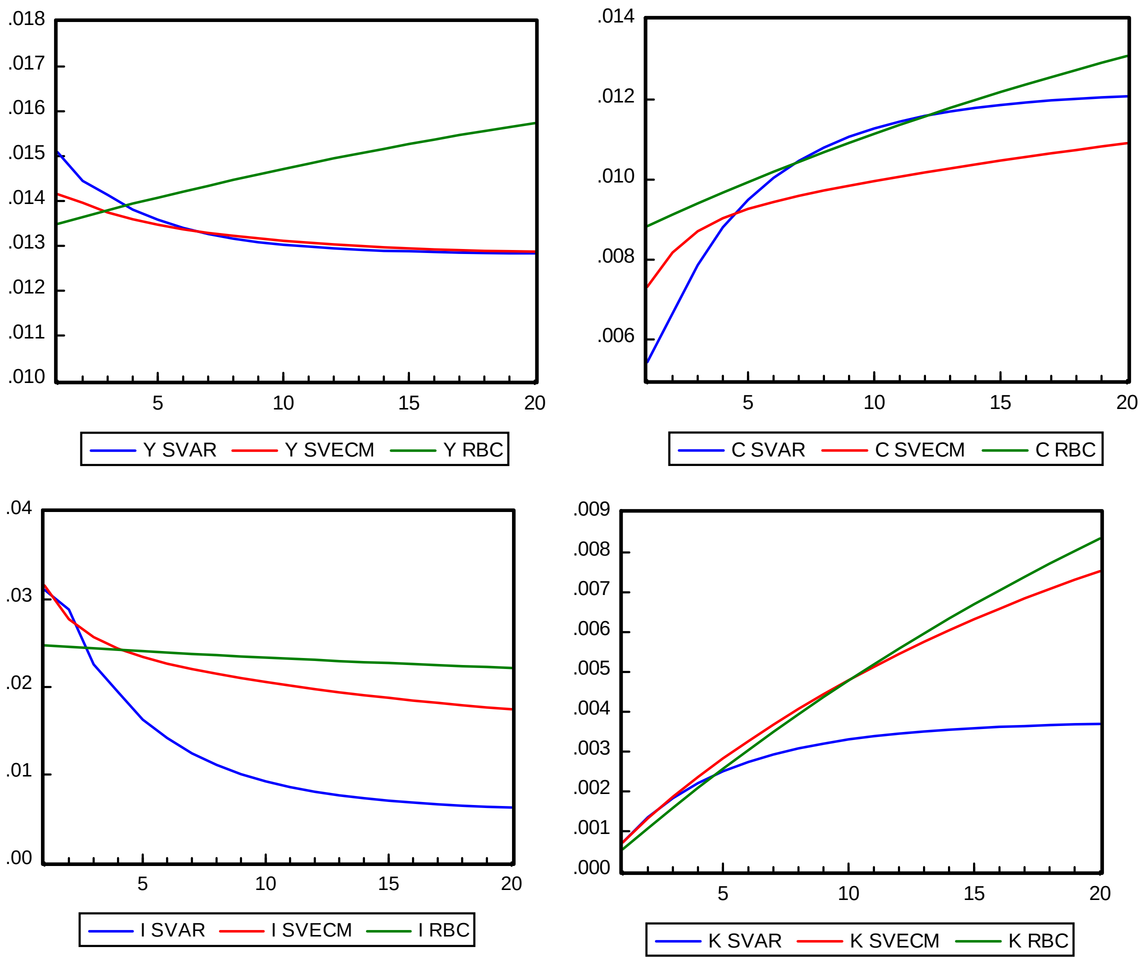

Figure 3: Estimated and theoretical IRFs - I(1) model - Capital stock series included

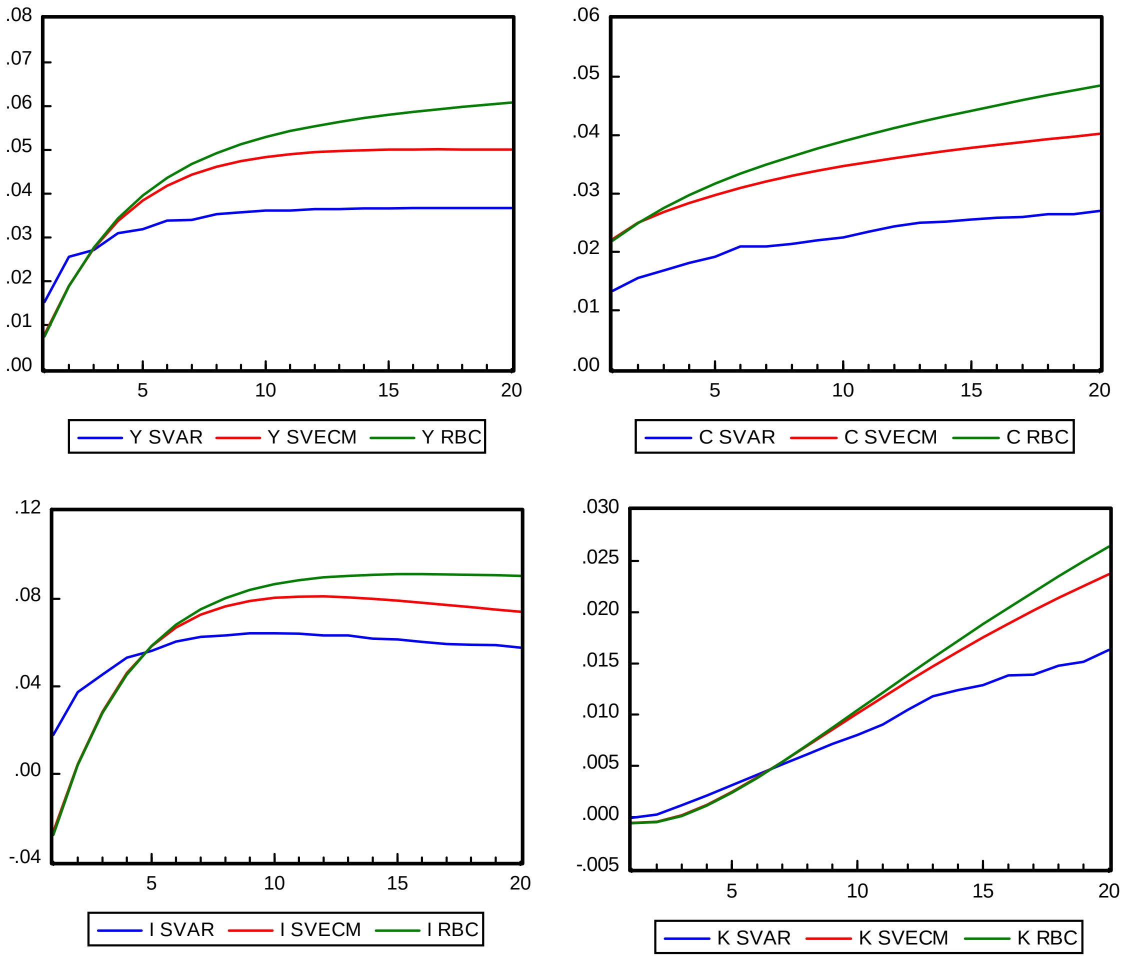

Figure 4: Estimated and theoretical IRFs - I(2) model - Capital stock series included

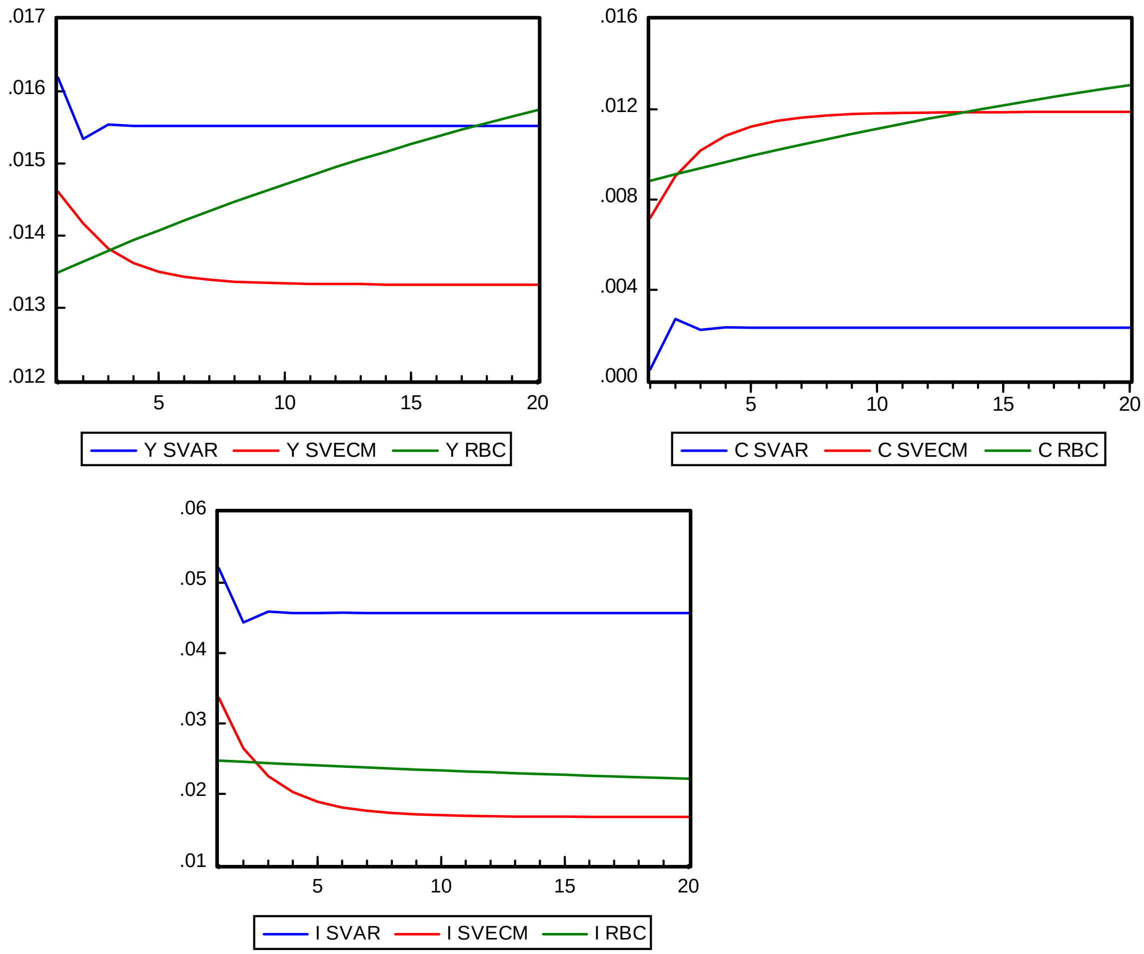

Figure 5: Estimated and theoretical IRFs - I(1) model - Capital stock series omitted

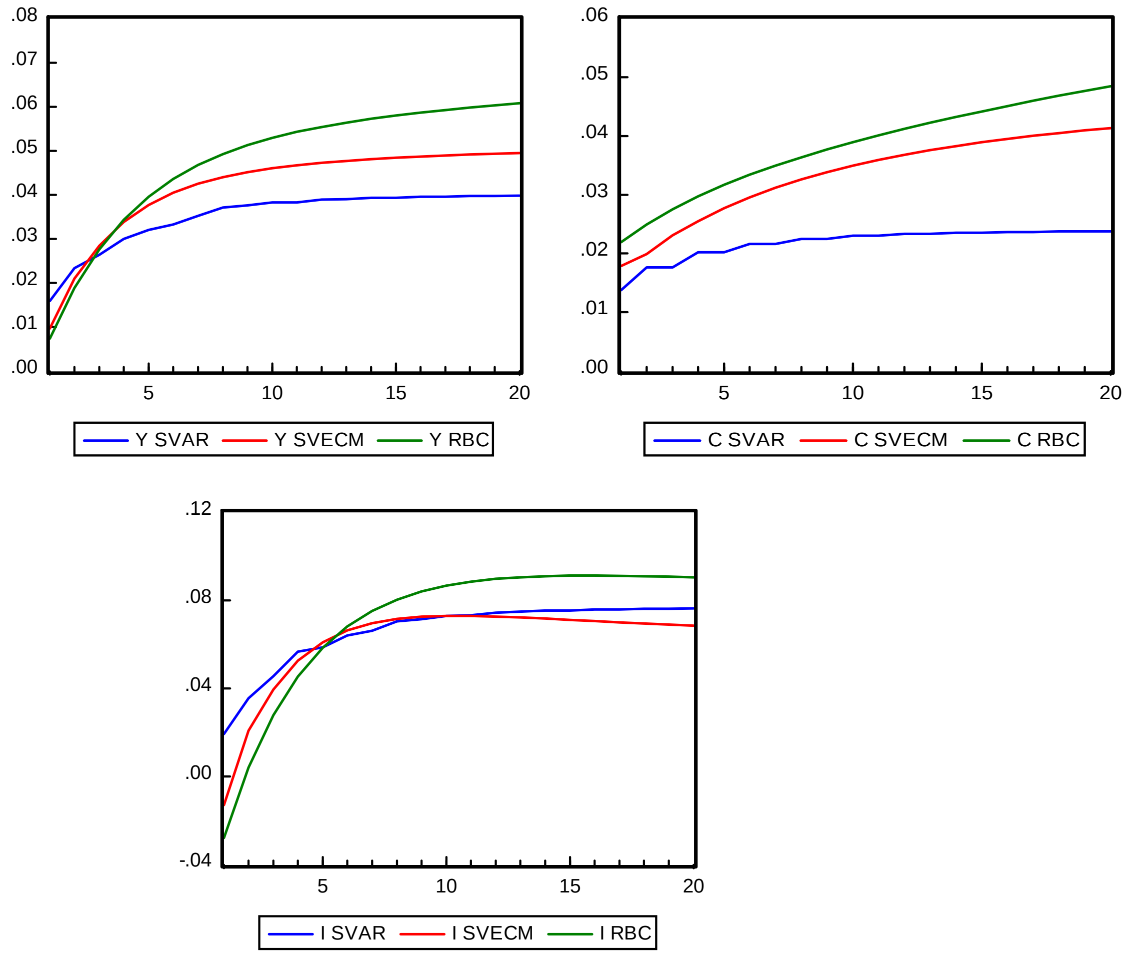

Figure 6: Estimated and theoretical IRFs - I(2) model - Capital stock series omitted

Table 1: I (1) specification - capital-stock included (% values)				
	Y		**C**	
	MAE	RMSE	MAE	RMSE
SVAR	0.17	0.7	0.006	0.03
SVECM	0.11	0.5	0.01	0.06
TOTAL difference	*0.06*	*0.2*	*0.005*	*-0.03*
	I		**K**	
	MAE	RMSE	MAE	RMSE
SVAR	1,2	5	0.18	0.8
SVECM	0.03	1	0.004	0.1
TOTAL difference	*0.9*	*4*	*0.177*	*0.7*

Table 2: I (2) specification - capital-stock included (% values)				
	Y		**C**	
	MAE	RMSE	MAE	RMSE
SVAR	1.5	13	1.5	7
SVECM	0.5	2	0.4	2
TOTAL difference	*1*	*11*	*1.1*	*5*
	I		**K**	
	MAE	RMSE	MAE	RMSE
SVAR	2.5	11	0.4	1.7
SVECM	0.7	8	0.08	0.3
TOTAL difference	*1.8*	*3*	*0.32*	*1.4*

Table 3: I (1) specification - capital-stock omitted (% values)						
	Y		**C**		**I**	
	MAE	RMSE	MAE	RMSE	MAE	RMSE
SVAR	0.08	0.3	0.9	4	2.3	10
SVECM	0.1	0.7	0.008	0.3	0.5	2
TOTAL difference	*-0.02*	*-0.4*	*0.82*	*3.7*	*1.8*	*8*

Table 4: I (2) specification - capital-stock omitted (% values)						
	Y		**C**		**I**	
	MAE	RMSE	MAE	RMSE	MAE	RMSE
SVAR	1.3	6	1.6	7.3	1.6	7
SVECM	0.6	2.9	0.5	2.1	1.4	6.5
TOTAL difference	*0.7*	*3.1*	*1.1*	*5.2*	*1.2*	*0.5*

Table 5: I (1) specification - overall indicators (% values)				
	capital-stock included		**capital-stock omitted**	
	MAE	RMSE	MAE	RMSE
SVAR	1.55	6.53	3.28	14.3
SVECM	0.15	1.66	0.61	3
TOTAL difference	*1.4*	*4.87*	*2.67*	*11.3*

Table 6: I (2) specification - overall indicators (% values)				
	capital-stock included		**capital-stock omitted**	
	MAE	RMSE	MAE	RMSE
SVAR	5.9	32.7	4.5	14.9
SVECM	1.68	12.3	2.5	11.5
TOTAL difference	*4.22*	*20.4*	*2*	*3.4*

B Appendix: RBC Model

B.1 The Social Planner problem and equilibrium solution

Max $U = E_t \sum_{t=0}^{\infty} \beta^t e^{u_t^\beta} \left[\log C_t - e^{u_t^N} \frac{N_t^{1+\eta}}{1+\eta}\right]$

s.t.

$C_t + I_t + e^{u_t^G} = +Y_t$

$Y_t = e^{u_t^a} K_{t-1}^{\alpha} N_t^{1-\alpha}$

$K_t = e^{u_t^K} f\left(\frac{K_t}{K_{t-1}}\right) K_{t-1} + (1-\delta) K_{t-1}$

$I_t = e^{u_t^K} f\left(\frac{K_t}{K_{t-1}}\right) K_{t-1}$

where $f\left(\frac{K_t}{K_{t-1}}\right)$ is a positive, concave installation function with the following steady state properties: $f(\overline{K/K}) = \delta$, $f'(\overline{K/K}) = 1$, and $f''\ (\overline{K/K}) = \psi$ where $\psi > 0$ denotes the degree of capital adjustment costs.

All the shocks follow a stationary I(1) process, i.e.:

$e^{u_t^j} = e^{\rho_j u_{t-1}} e^{\epsilon_j}$ where $0 < \rho_j < 1$ and $\epsilon_j \sim i.i.d.(0, \sigma^2)$ for $j = \alpha, \beta, N, G, K$

Setting the Lagrangian:

$L : E_t \sum_{t=0}^{\infty} \beta^t \left\{ e^{u_t^\beta} \left[\log C_t - e^{u_t^N} \frac{N_t^{1+\eta}}{1+\eta}\right] - \lambda_t \left[C_t + e^{u_t^K} f\left(\frac{K_t}{K_{t-1}}\right) K_{t-1} - e^{u_t^a} K_{t-1}^{\alpha} N_t^{1-\alpha}\right]\right\}$

The FOCs are the following:

$$\frac{dL}{dC_t} : \lambda_t = \frac{e^{u_t^\beta}}{C_t} \tag{14}$$

$$\frac{dL}{dN_t} : -e^{u_t^\beta} e^{u_t^N} N_t^{\eta} + \lambda_t \left[(1-\alpha)^{u_t^a} K_{t-1}^{\alpha} N_t^{-\alpha}\right] = 0 \tag{15}$$

$$\frac{dL}{dK_t}:\lambda_t e^{u_t^K} df\left(\frac{K_t}{K_{t-1}}\right)K_{t-1}\frac{1}{K_{t-1}} =$$
$$-\beta E_t\lambda_{t+1}\left\{e^{u_{t+1}^K}\left[f\left(\frac{K_{t+1}}{K_t}\right)-df\left(\frac{K_{t+1}}{K_t}\right)\frac{K_{t+1}}{K_t}\right]-\alpha e^{u_{t+1}^a}K_t^{\alpha-1}N_{t+1}^{1-\alpha}\right\} \tag{16}$$

$$\frac{dL}{d\lambda_t}:C_t+e^{u_t^K}f\left(\frac{K_t}{K_{t-1}}\right)K_{t-1}-e^{u_t^a}K_{t-1}^{\alpha}N_t^{1-\alpha}=0 \tag{17}$$

Combining FOCs 1-2 provides:

$$\begin{aligned} e^{u_t^\beta}e^{u_t^N}N_t^\eta &= \frac{e^{u_t^\beta}}{C_t}\left[(1-\alpha)e^{u_t^a}K_{t-1}^{\alpha}N_t^{-\alpha}\right] \\ N_t &= \left[\frac{(1-\alpha)Y_t}{C_t e^{u_t^N}}\right]^{\frac{1}{1+\eta}} \end{aligned} \tag{18}$$

So that the resulting relevant equations are as follows:

$$Y_t = e^{u_t^a}K_{t-1}^{\alpha}N_t^{1-\alpha} \tag{1.1}$$

$$C_t + I_t + e^{u_t^G} = Y_t \tag{1.2}$$

$$K_t = e^{u_t^K}f\left(\frac{K_t}{K_{t-1}}\right)K_{t-1}+(1-\delta)K_{t-1} \tag{1.3}$$

$$\lambda_t e^{u_t^K} df\left(\frac{K_t}{K_{t-1}}\right) =$$
$$-\beta E_t\lambda_{t+1}\left\{e^{u_{t+1}^K}\left[f\left(\frac{K_{t+1}}{K_t}\right)-df\left(\frac{K_{t+1}}{K_t}\right)\frac{K_{t+1}}{K_t}\right]-\alpha e^{u_{t+1}^a}K_t^{\alpha-1}N_{t+1}^{1-\alpha}\right\} \tag{1.4}$$

$$N_t = \left[\frac{(1-\alpha)Y_t}{C_t e^{u_t^N}}\right]^{\frac{1}{1+\eta}} \tag{1.5}$$

$$\lambda_t = \frac{e^{u_t^\beta}}{C_t} \tag{1.6}$$

Table 7 - RBC calibrated parameters			
α	β[22]	σ	δ
0.36	0.995	2	0,022

[22] The discount rate β is set so that the annual real interest rate is 2% accordingly to a quarterly data simulation.

B.1.1 Steady state results

By normalizing the model by Y, we have:

$$\overline{Y} = 1 \tag{2.1}$$

From the resources constraint: $\overline{Y} = \overline{C} + \overline{I}$ then it is: $\overline{C} = 1 - \overline{I}$ and $\overline{I} = 1 - \overline{C}$

From 1.3 we get the steady state equation: $\overline{K}{=}\overline{I}{+}\left(1-\delta\right)\overline{K}$ that is:

$$\overline{I} = \delta\overline{K} \tag{2.2}$$

We can rewrite 1.4 as:

$$\begin{aligned}
\lambda_t e^{u_t^K} df\left(\frac{K_t}{K_{t-1}}\right) &= -\beta E_t \lambda_{t+1}\left[\frac{dI_t}{dK_t} - \frac{dY_t}{dK_t}\right] \\
\lambda_t e^{u_t^K} df\left(\frac{K_t}{K_{t-1}}\right) &= \beta E_t \lambda_{t+1}\left[\frac{dY_t}{dK_t} - \frac{dI_t}{dK_t}\right]
\end{aligned}$$

where: $R_{t+1} = \left[\frac{dY_t}{dK_t} - \frac{dI_t}{dK_t}\right]$ so that:

$$\lambda_t {=}\ \beta\lambda_{t+1} R_{t+1}$$

which in steady state implies:

$$\overline{R}{=}\frac{1}{\beta} \tag{2.3}$$

From the steady state value of 1.5 we obtain:

$$\overline{N} = \left[\left(1-\alpha\right)\overline{C}^{-1}\right]^{\frac{1}{1+\eta}} \tag{2.4}$$

By the definition of R_t in steady state we get**:**

$$\boldsymbol{\alpha}\frac{\overline{Y}}{\overline{\overline{K}}}{+}\left[1-\delta\right]{=}\overline{R}$$

which combined with 2.3 yields:

$$\begin{aligned}
\alpha\frac{1}{\overline{\overline{K}}}{+}\left[1-\delta\right] &= \frac{1}{\beta} \\
\overline{K} &= \frac{\alpha\beta}{1-\beta+\beta\delta} \qquad (2.5)
\end{aligned}$$

and combined with 2.2 provides:

$$\overline{I} = \delta\left(\frac{\alpha\beta}{1-\beta+\beta\delta}\right) \tag{2.6}$$

Given the resources constraint, we get the expression for $\overline{C}$:

$$\overline{C}= 1-\left(\frac{\alpha\delta\beta}{1-\beta+\beta\delta}\right) \tag{2.7}$$

B.2 The loglinearized solution

For $Y = e^{u_{a_t}} K_t^{\alpha} N_t^{1-\alpha}$:

$$y_t = u_t^a + \alpha k_t + (1-\alpha)\, n_t \tag{3.1}$$

For $C_t + I_t + e^{u_t^G} = Y_t$:

$$\overline{C}c_t + \overline{I}i_t + u_G = y_t \tag{3.2}$$

For $K_t = e^{u_t^K} f\left(\frac{K_t}{K_{t-1}}\right) K_{t-1} + (1-\delta)\, K_{t-1}$:

$$k_t = \delta i_t + (1-\delta)k_{t-1} \tag{3.3}$$

From $\lambda_t e^{u_t^K} df\left(\frac{K_t}{K_{t-1}}\right) = -\beta E_t \lambda_{t+1}\left\{e^{u_{t+1}^K}\left[f\left(\frac{K_{t+1}}{K_t}\right) - df\left(\frac{K_{t+1}}{K_t}\right)\frac{K_{t+1}}{K_t}\right] - \alpha e^{u_{t+1}^a} K_t^{\alpha-1} N_{t+1}^{1-\alpha}\right\}$ we derive:

$$k_t = \frac{\lambda_{t+1} - \lambda_t + \beta\psi k_{t+1} + \psi k_{t-1}}{(1-\alpha)(1-\beta+\beta\delta)} - \frac{(\psi+\beta\psi)\, k_t}{(1-\alpha)(1-\beta+\beta\delta)} - \frac{u_t^k - \beta\delta u_{t+1}^k}{(1-\alpha)(1-\beta+\beta\delta)} + \\ + n_{t+1} + \frac{u_{t+1}^a}{(1-\alpha)} \tag{3.4}$$

For $N_t = \left[\frac{(1-\alpha)Y_t}{C_t}\right]^{\frac{1}{1+\eta}}$:

$$n_t = \frac{1}{1+\eta}(y_t - c_t - u_t^n) \tag{3.5}$$

For $\lambda_t = \frac{e^{u_t^\beta}}{C_t}$ we have:

$$\lambda_t = u_t^\beta - c_t \tag{3.6}$$

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