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Discussion of a didactic proposal on quantum mechanics with secondary school students

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Summary. — Within some research projects a proposal for the teaching of quantum mechanics in secondary school has been carried out, and some didactic material has been prepared in order to illustrate it, offering resources for its class experimentation (www.fisica.uniud.it/URDF/). In order to study in depth the critical points, which cause learning difficulties for the students in this field, a pilot activity was carried out for a restricted group of students with which the crucial points were discussed. Some interesting elements emerged, such as for example the fact that the major problems in understanding the concept of quantum state are linked to the meaning of incompatible observables.

PACS 01.40.Fk – Physics education research (cognition, problem solving, etc.).
PACS 03.65.-w – Quantum mechanics.

1. – Introduction

The introduction of quantum mechanics elements (QM from now on) in secondary school is foreseen in all European schools and most recent text-books. The role that QM plays as a reference theory for the microscopic description of reality is generally recognised [1, 2], but there is still not a similar sharing of the disciplinary approach to adopt, also because of divergences on the interpretative level [3-5].

The objective of researchers is to determine the strategies able to allow the students to deal with the theoretic foundations, even if in limited contexts, using the methodological instruments that characterise it and that can therefore represent a cultural contribution to the formation of citizens, not only of those who intend to carry on their studies in physics.

The challenge is to build competences on fundamental disciplinary knots, such as the superposition principle, but also on other aspects which are still problematic from the

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interpretative point of view, such as the conflictual relation between the classic macro-world and the quantum micro-world [1] or the role of formalism in attributing a meaning to entities in QM [6, 7].

An idea of the radical change in the interpretative point of view has to be given to the students in rigorous terms by using the characteristics of the theory itself. In simple phenomenological contexts, like the one of optical physics, it is possible to deal with the most important elements of a didactic proposal: the deep revision that concepts such as the state and property of a system, and even more the attribution of a property, must undergo; the characterisation of the quantum state concept; the superposition principle; the concept of incompatible observables [8-15].

Centred on previous conceptual aspects, we developed, in the past ten years, a didactic proposal for the introduction of quantum mechanics in secondary school [16], within research projects carried out in collaboration with other Italian universities or locally(1)(2)(3). Its aim is the formation of theoretic thought, the building of the meaning of physical entities and the introduction of elements of formalisation, with the Dirac vectorial approach [17, 18], which are easily generalised starting from simple bi-dimensional cases [19].

Materials for teacher formation—which are available on the web—have also been planned, and experimentations of the proposal have been carried out in pilot classes [20, 21].

Work strategies have been set up in order to connect the quantitative analysis of the macroscopic phenomenology under study (polarisation), with the one proposed in ideal situations through the use of informatics' tools [22] and didactic materials that illustrate the didactic proposal and offer resources for class experimentation have been set up (`www.fisica.uniud.it/URDF/`) [23, 24] (2).

The cross-experimentation of especially planned Formation Modules for Teachers, carried out in 2001 and 2002, has allowed to achieve various class experimentations and apprenticeship activities (3) [25]. Up to today, the proposal has been experimented in 12 classes of the last year of high school, for a total of 300 students.

To study in depth the critical aspects, that cause difficulties in the learning of QM,

(1) ESP C project of the FISISS–MURST–1997/98. The FISISS research (In Service Formation of Secondary School Teachers), coordinated at a national level by Guidoni, following a MURST proposal to which the universities of Ferrara, Milan, Modena, Naples, Padua, Pavia, Turin, Udine took part, was carried out for two years with three experiments, to each of which the University of Udine contributed. In particolar the ESP C experiment was focused on the planning of paths and proposals for the introduction of modern physics in secondary school.

(2) SeCiF–MIUR Project 2000-2001. It is a research project co-financed by MIUR, coordinated at the national level by Paolo Guidoni. Its title is "Studying and Understanding Physics", and it also defines its acronym. The research units of Naples, Milan, Palermo, Bicocca, Padua, Pavia, Turin, Udine took part in this project, followed by those of Bologna, Ferrara, Modena and Triest. The project ended in February 2001 and produced didactic materials for teacher formation, which are available on-line at the following address `http://pctidifi.mi.infn.it/secif/` for the part concerning the national group, and at `www.fisica.uniud.it/URDF` for the part concerning the University of Udine.

(3) FFC–MIUR Project 2002-2003. The SeCif materials were them re-processed within the FFC project–Physics for Cultural Formation, always coordinated by Paolo Guidoni, with the purpose of preparing Modules for Formative Intervention (MIF) for teacher formation. The two year project ended in 2003 and involved the groups of Bologna and Rome, as well as the groups that carried out the SeCif project.

shown in the literature [26-30] and/or arisen as research results in class experimentations and during teacher formation activities [20,21,25], a pilot activity of conceptual discussion has been carried out with a limited group of students from local schools.

We here present the research results emerging from this pilot activities.

2. – General aspects of activity and research questions

The activity was carried on discussing these main topics:

– phenomelogical framework contruction (esploration of light interaction with Polaroids and birefringent crystal, Malus law and trasmittivity law).

– From phenomenology to probabilistic interpretation (validity of Malus law for single-photons experiment).

– The construction of quantum state concept (interpretative hypothesis on superposition states).

– First step of formalisation (state vector and superposition principle).

– Second step of formalisation (observables and operators; non-commutativity and connection with uncertainty relations).

– Non-locality (discussion proposed by D. Mermin [31]).

The problems with which students were faced had been organised in mono-conceptual units, using for each of them a work sheet. These work sheets were developed in the previous recalled researches [19-24], were planned with the purpose to set up PEC-type strategies (Prevision-Experiment-Comparison) [32, 33] and they were based on inquiring methods [34].

In the activity we use two types of work sheets: a) sheets for the collection and elaboration of experimental data, for the building of phenomenological laws starting from experimentally collected data (EX sheets from now on); b) sheets for conceptual building, which favour the formulation of hypotheses regarding specific situations, in an ideal context, through the use of questions (WS sheets from now on).

For each step the discussion followed this order: written individual answers to the questions asked; group discussion and specific analysis of some answers; revision of the sheets (made recognizable); conclusion. In some cases students completed the sheet compilation as homework. The activity was monitored by taking notes on the dynamics discussion.

Seventeen high-school students voluntarily took part in the pilot activities on QM. The discussion consisted of six 1 hour and half long meetings, for a total of 9 hours.

The crucial points of the research were discussed with them, in order to study the following aspects:

– the role played by the link with the explored context in students' learning (which is the level of knowledge required to understand the proposal? In what measure do the notions acquired depend on the context?).

– The ways in which students passed from classic to quantum interpretations (which aspects cause change? Is it possible to talk about conceptual change and, if so, in what sense?).

– The role played by formalism in learning (how do students deal with it? Does formalism represent an obstacle to learning?).

3. – The dynamics of the discussion: the outline of work followed and the instruments used

The crucial points discussed by the students with reference to the work sheets used are illustrated beneath.

I) *Building of the phenomenological framework*

The EX1 sheet suggests observing the light of a luminous board transmitted by one, two or more polaroids. The questions asked are meant to stimulate the operative recognition of the polarised light's main characteristics and in particular its transversal property. Phenomenological laws (Malus, transmissivity), which describe the main phenomenology to which we are referring, have been put on quantitative bases through the use of light intensity measures with on-line sensors([4]) and the use of two work sheets (EX2-EX3).

II) *From phenomenology to probabilistic interpretation*

The students, using a work sheet (WS1) mainly structured on closed questions, discussed the experiment's outcomes and re-interpreted them in probabilistic terms as interactions of single photons with the apparatus considered. They were also able to link the real experiments with the ideal ones with single photon using sw simulation (five simulations on an electronic sheet and one implemented java [22-24]).

III) *Building the concept of quantum state*

The concept of physical state of a system can be defined on the basis of a measurement process [35]. The request for continuity in the description (at least in a non-relativistic context) causes the fact that measures repeated in a short spatial and temporal succession of the same dimension of a physical system have the same outcome. This allows to attribute to a quantum-mechanical system properties that characterise the state in which it is, at least as a work hypothesis or, in other words, staying within a specific interpretative framework [35-37]. However, the knowledge of these properties allows to make definite previsions only in certain cases (measures on system that are in auto-state). In general, since the system is in a state of superposition of the possible conditions which it can acquire after a measurement, the outcomes of single measurement processes are foreseeable only in probabilistic terms. The properties related to the state of preparation are not compatible with those that can be attributed to the observation system immediately after the measurement itself. This aspect characterises the peculiarities of the concept of quantum state, causing for example quantistic indeterminism (non-epistemic) and the entanglement of quantum phenomena, experimentally confirmed during the past 20 years [38, 39], and differentiates it from any classical interpretation [1, 35].

In order to explore the interpretative hypotheses concerning the aspects that characterise the principle of superposition, and the consequences related to them, an iconographic representation of the photons' properties has been suggested, since it is considered to be a good instrument for the building and discussion of hypotheses (fig. 1).

Two work sheets (WS2, presented in the following paragraph, and WS3), structured in questions with open, closed and map answers [40], have established which aspects of the interpretative hypotheses on the state of superposition should be analysed.

([4]) Light sensor Pasco—black series CI-6504A.

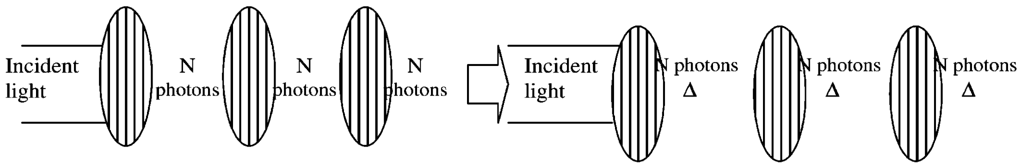

Fig. 1. – The photons transmitted by the first polaroid are always transmitted by the others. A property can be associated to them (36). Such property can be indicated with the symbol Δ.

IV) *Building of formalism*

The vectorial representations (limited to the real case) of the quantum state and of the superposition principle, which appear as formal translations of the concepts focalised upon in the previous phase (fig. 2), have been recognised as effective in giving account of the phenomenology under study. In particular Malus' law, re-interpreted as the outcome of the interactions between single photons, provides the law on the probability of transition between two states, expressed as the square of the scalar product of the two state vectors. The quantum interference appears to be a peculiar differentiation from the interpretation of classical physics. The students engaged in the first rudiments of formalism, acquiring its physical meaning, linking it with the phenomenological context of polarisation suggested by the questions of the work sheet handed out (WS4).

V) *Observables and operators*

The operational representation of physical observables was built starting from the recognition that it is possible to establish an analogy between the projector $\mathbf{uu}\,\cdot$ and a measurement apparat, like the Polaroid, that selects the state represented by the state vector $\mathbf{u}$. This provides an incentive to consider more general operators, for example linear combinations of projectors. Their physical meaning has been built by evaluating the expectation value of the polarisation variable. The work sheet (WS5), on which the students worked, implements this procedure basing itself on the result achieved in phase IV, where the transition probability between the two states is evaluated with the scalar product of the corresponding state vectors.

The relation between the non-commutativity of the linear operators and the incompatibility of the physical observables was also discussed (fig. 3). The students, using the formalism without the work sheet guide, recognised the non-commutativity of the vertical and 45° polarisation operators. Another topic discussed was how formalisation incorpo-

Iconographic representation	*Formalisation of concepts*
Photons with property *	state represented by the versor **H'**
Photons with property Δ –	state represented by the versor **V'**
Transition probability **H'→V'**: P(**H'→V'**)=(**H'** • **V'**)=0 → **H'** ⊥ **V'**	
properties * and Δ mutually exclusive	Mutually othogonal state vectors **H'** ⊥ **V'**
property ◊ incompatible with property * (Δ)	Superposition state: $\mathbf{u_{45}}$ = (1/v2) (**H'**+**V'**)

Fig. 2. – Connection between the iconographic representation of the properties attributed to photons and the vectorial representation of the states.

The operator: $\mathbf{u_{45}}\,\mathbf{u_{45}}\cdot$, which represents polarisation at 45°, can be expressed with the vectors **H'** and **V'** in the following way:

$\mathbf{u_{45}}\,\mathbf{u_{45}}\cdot = 1/2\,(\mathbf{V'\,V'\cdot + H'\,H'\cdot + V'\,H'\cdot + H'\,V'\cdot})$

If **u=aV'+bH'** is a generic vector, then:

$[(\mathbf{V'\,V'}\cdot)\,(\mathbf{u_{45}\,u_{45}}\cdot)]\,\mathbf{u} = [(\mathbf{V'\,V'}\cdot)\;1/2\,(\mathbf{V'\,V'\cdot + H'\,H'\cdot + V'\,H'\cdot + H'\,V'\cdot})]\,\mathbf{u} =$

$= ½\,(a\,\mathbf{V'} + b\,\mathbf{V'})$ (in any case // **V'**)

$[(\mathbf{u_{45}\,u_{45}}\cdot)\,(\mathbf{V'\,V'}\cdot)]\,\mathbf{u} = [1/2\,(\mathbf{V'\,V'\cdot + H'\,H'\cdot + V'\,H'\cdot + H'\,V'\cdot})\,(\mathbf{V'\,V'}\cdot)]\,\mathbf{u} =$

$= ½\,(a\,\mathbf{H'} + a\,\mathbf{V'})$ (in any case // $\mathbf{u_{45}}$)

Therefore the operator: $\mathbf{u_{45}\,u_{45}}\cdot$ does not commute with the operator $\mathbf{V'\,V'}\cdot$

$[(\mathbf{V'\,V'}\cdot)\,(\mathbf{u_{45}\,u_{45}}\cdot)]\,\mathbf{u} - [(\mathbf{u_{45}\,u_{45}}\cdot)\,(\mathbf{V'\,V'}\cdot)]\,\mathbf{u}$?0

Fig. 3. – Non-commutativity of operators representing incompatible measures, in the simplified formalism adopted.

rates the indetermination relations of Heisenberg, which are wrongly often considered the founding principle of theory.

VI) *Non-locality*

By analysing an EPR-type experiment, suggested by D. Mermin [31] the non-locality of quantum processes and of entanglement were discussed. The work sheet (WS6), in which formalism is not used, placed the students in a condition to explore hypotheses, making them recognise the connected implications.

4. – The instruments: the work sheets

In this section, as an example, two of the work sheets are presented following the parts in which it is divided.

Work sheet WS2 – Interaction of photons with a polaroid

A1. With the purpose of recalling the phenomenological framework of reference and of acquiring information on how the students deal with it, the analysis of ideal experiments on the transmission of photons through two or three polaroids is proposed: V, H, 45°, which means that they polarise the light respectively vertically, horizontally and with a 45° inclination. We asked to fill in the tables with data concerning the light's intensity/the probability of transmission of single photons.

A2. The hypothesis on the possibility to associate a property to a photon only after having measured it is formulated. This hypothesis is adopted as a working assumption and operatively introduced.

A3. Definite cases are analysed in order to introduce the idea of mutual exclusivity. The iconographic representation of the photons' properties is also introduced (fig. 1). A table on the transmission probabilities and outcomes of the measurements, as well as multiple choice questions is presented.

A4. With the objective of discussing the concept of superposition, the analysis of the transmission of 45° polarised photons through polaroids V and H is suggested.

A PEC cycle is activated, inviting the students to formulate their possible interpretative hypotheses on the phenomenon. In particular, previsions are asked on the following: nature of the superposition; role of the Polaroid.

A4_1_From the experiments' outcomes which hypotheses can be formulated on the composition of the ensemble of photons with the ◊ property?
- half are photons with the Δ property and half are photons with the ∗ property
- half are photons with the ◊ and Δ properties and half are photons with the ◊ and ∗ properties.
- all the photons have only the ◊ property
- other

A4_2_What is the role of the polaroid?
It is a passive filter (it absorbs some photons and transmits others)
It is an active filter (it absorbs some photons and transmits the others after having interacted with them)

Fig. 4. – A4 questions of the W2 work sheet.

With multiple choice answers, the students have to choose among the possible interpretations that emerged spontaneously in previous researches [19, 20], as interpretative hypotheses on the 45° polarisation state (fig. 4).

A5. In particular, the level of consistency they have with the experimental context is explored, through the use of multiple choice questions, and the consequences the various interpretative hypotheses imply (quantum indeterminism and identity of quantum systems). Questions with open answers or with two options (yes/no) are used, with a discussion/explanation of the answers given.

A6. With the objective of completing the Pec cycle and activating a new one, with open questions, the summary of the results obtained and the building of new hypotheses is proposed.

Work sheet WS6 – Non-formal discussion concerning non-locality suggested by D. Mermin [31].

A1. Aside an initial presentation of the phenomenon studied (fig. 5) open questions on the outcomes of specific cases are asked.

A2. The PEC cycle is activated with the formulation of interpretative hypotheses. In particular the following hypothesis is discussed:

Can experimental outcomes be explained on the basis of "deterministic" instructions transported by the particles?

A3. All the possible instructions for the A1 are formulated, completing a table.

A4. The PEC cycle is completed by recognising the fact that the outcomes for the A2 case (fig. 5) are not compatible with the interpretative hypothesis formulated.

A5-A6-A7. The discussion of the outcome is proposed with open questions. The students are asked to draw their own conclusions on how to interpret the correlation between the measures carried out on the three particles and on the possibility of foreseeing the outcomes.

5. – Some examples

In order to contextualise some of the obtained results, here we present some specific cases as an example of the different conceptual paths followed by the students.

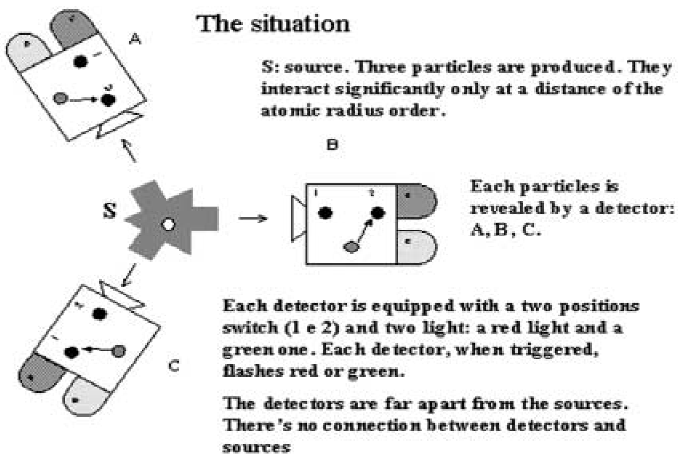

Fig. 5. – Drawing of the ideal experiment set up by Mermin to discuss non-locality [31]. The phenomenology: Case A1: Switches 122, 212, 212 → always odd N red flashes. Case A2: Switches 111 → always even N red flashes.

a) *Michele's case*, represents an example in which there is a conceptual revision, but there is not the overcoming of a first interpretational level, and of the cognitive contrast between classical and quantum ideas.

To interpret the 45° polarization he first chooses hypothesis A ($\Diamond\Diamond\Diamond\Diamond = \Delta\Delta + **$), that is the statistic mixture, and assigns a passive role to the Polaroid. He maintains that a photon possessing the property Δ can also possess the property $\Diamond$. In the WS2 Michele wrote:

> Question: A5. – Michele: *"a photon possesses at the same time the properties Δ and $\Diamond$. They are compatible properties."*
>
> Question A6. – A photon therefore possesses the property Δ.
> Can it therefore be established if it possesses and in what measure the property $\Diamond$? Explain
>
> Michele: *"It can be analysed with a polaroid at 45°: if it goes through in any case then it possesses the property $\Diamond$"*.
> Then he corrects himself: *"Ah! It is not passive"*.

To support his assertion, he emphasizes the fact that if the photon passes through a polaroid at 45°, this means that the photon previously had a property that allowed it to do so. Michele does not make a distinction between hypothesis A of the statistics mixture and hypothesis B of the pre-existing properties. The discussion between peer activates a conceptual revision on this aspect, but he does not master the phenomenological framework from the operative point of view, as demonstrated by the many answers not given in the EX2-3 work sheets. He therefore is not able to overcome the first level of conceptualisation or the cognitive crisis which has been activated. For the rest of the activity he tries to interpret the phenomenology, which has been explored, on the basis of *a priori* properties of systems, pre-existing the measuring process.

b) *Claudia's and Paola's cases* are characterised as examples in which conceptual revision is accompanied by the building of new concepts. They both start off with the same ideas as Michele's and show the same contradictory positions. But, unlike Michele, their operative knowledge of the phenomenology, as testified by the answers given in the Ex-1-2-3 work sheets, enables them to make a coherent analysis. They therefore overcome the first level of formalisation.

These sentences summarise some of Claudia's answers to the questions of the WS6:

Question A6. What can be understood from the correlation of results?

Claudia: *"The properties that I can attribute after the measurement with a set of switches would not be valid if I used a different switch setting.*
It is as if the particle knew how the switches have been set or that the instruments change the particles' properties.
I cannot attribute properties to the particles in a determinate way".

Question A7. How can the results be explained on the basis of QM?

Claudia: *"QM claims that properties of bodies are not independent of the way and moments in which they are measured".*

Claudia acquires the awareness that the association of a property to a quantum system cannot leave out of consideration the process in which it is measured. She appears to have understood the fundamental role of the measurement process in QM, and the fact that it is not possible to attribute *a priori* a "property" to quantum systems, independently of the measurement process with which these properties are determined.

Paola changes her mind, after comparing hypothesis and outcomes of the (ideal) experiments. She uses the concept of incompatibility with awareness to characterise the phenomenon discussed:

WS6 - Question A5: What can be said about the possibility to foresee the outcomes of the measurements? Explain

Paola: *"The outcomes of the measurements cannot be foreseen since different configurations of the same experiment have outcomes that are incompatible between each other"*

c) *Paolo's case* is proposed as an example of the capability to autonomously manage concepts and formalism.

Paolo has a good knowledge of the phenomenology. He immediately recognizes the active role played by the polaroid in the interaction with the photons. He also demonstrates a knowledge of the concept of incompatibility:

WS2 - Question A4.1

Paolo: *"Photons in a 45° position have only the property* $\Diamond$*, the polaroid has an active role"*

Question A5.1 Study of the hypothesis: : $\Diamond\Diamond\Diamond\Diamond = \Delta\Delta + **$?

Paolo: *"The hypothesis is not plausible. A photon cannot possess at the same time the properties* $\Diamond$ *and* Δ*. These properties are incompatible"*

The understanding of the concept of incompatibility allows him to develop the route taken by adopting, from the beginning, a way of thinking that is coherent with quantum models.

He manages formalism with autonomy according to the answers proposed by the WS4 and WS5 work sheets, using elements of formalism as working instruments for conceptual exploration.

6. – Elements shown by the monitoring

Here we give a summary of the elements shown by the monitoring of the discussion with the students. They are exemplified in the cases discussed above and are therefore presented in relation to them.

The link with the phenomenological context. Two students did not actively participate in the operative exploration of polarization and did not understand the phenomenological framework, as in Michele's case. They remained on a purely descriptive level, without managing to overcome the cognitive crisis triggered by the group discussion. All other students managed to overcome this first conceptual step, but they still showed the need to link their reasoning to a specific and well-known phenomenological context, like the one developed during the first experimental phase. The acquisition of the quantum conceptual framework depends, in a certain way, on the examined context. It is therefore necessary to adopt strategies for the generalisation of results, which imply the exploration of many contexts.

Concept of incompatibility. Initially the students find it difficult to recognize the incompatibility between vertical and 45° polarization properties, because they do not have knowledge of the concept of incompatibility in the context of classical probability. In the specific phenomenological field explored, most students acquired the notion of incompatibility managing to deal with it correctly, as demonstrated by the cases of Paola and Paolo. A coherent use of the concept of incompatibility can therefore be considered a good indicator of the comprehension of the concept of quantum state and of the principle of superposition.

The aspects linked to incompatibility are also those which mostly show the conflict between classical and quantum ideas. In fact, not always the comprehension of compatibility is accompanied by the acceptance of the consequences. Only in a quarter of cases the recognition of the fact that the existence of incompatible properties causes an impossibility to associate a trajectory to a particle, does not determine a cognitive crisis. In most cases it determines a fall-back on classical ideas.

Facing further critical situations, such as the ones in which non-locality of quantum phenomena manifests itself, helps to overcome the cognitive crisis in cases like the one in which phenomenology is managed and the challenge to coherently examine the hypotheses with regard to the experimental outcomes is "accepted".

Realistic interpretations. All the students, in different ways, showed a need to interpret the experiments' outcomes on the basis of deterministic/realistic hypotheses. In particular there is a strong need to formulate interpretations of the experiments' outcomes on the basis of the properties previously possessed by systems. The students show no difficulty in accepting the fact that a superposition of physical quantum states is different from a statistic mixture. They attempt in every way to interpret phenomena with reality elements, in the meaning defined by Einstein [36]. Quantum interpretation only occasionally appears among the possible ones, like in Paolo's case. More often, it acquires plausibility only after the interpretative hypotheses based on elements of reality have been taken to their extreme consequences. This happens in particular after having recognised the fact that a similar interpretation is contextual.

It must be said that, in the students, quantum interpretation, as well as generally emerging among other possible interpretations, often continues to live beside a deeply classical vision of phenomena. In some cases, like Michele's, it causes a cognitive crisis that is never overcome. The classic vision and the quantum vision are superimposed and not distinct. In various cases (more than half, as demonstrated by the cases of

Claudia and Paola) it produces a new conceptual vision. This vision however does not oust classic ideas with quantum ones, rather it juxtaposes them. Even students who, like Paolo, showed from the beginning a vision of phenomena more adherent to quantum ideas, in some cases made this juxtaposition.

Formalism. No matter how much the iconographic representation proposed to the students is a great work tool for building and exploring hypotheses, the students themselves feel the need to place the concepts explored and the results reached on mathematic bases.

From this experience it can be said that most students (at least three-quarters) manage with sufficient autonomy the formalism developed at a first level and within the specific contexts studied. It did not represent an obstacle to learning. On the contrary, it favoured it thanks to the simplicity with which it allows to represent states and observables and to give account of the explored phenomena. Even though nobody was able to deal with formalism in a convinced way, it was a useful work tool to give account of the phenomenological framework, and to clarify the concepts of superposition and of incompatible observables. This was also possible thanks to the use of the WS4 and WS5 work sheets, elaborated in order to guide first (through examples) and then gradually leave more autonomy to the students in the use of formalism.

7. – Conclusions

A pilot activity for a restricted group of students was carried out, discussing the nodal points of the didactical path on QM for secondary school, that we developed in previous researches. We use didactic materials developed for class experimentation and teacher formation.

The discussion with the students was centred on exploring interpretative hypotheses of simple phenomenological contexts (polarization) to build basic concepts of quantum mechanics. The students were given operative work sheets.

From the analysis of the outcomes the following results emerged.

- The use of iconographic representation activates the capacity to build hypotheses, it favours the passage from the descriptive level of the studied phenomenologies to the level of the formalisation of concepts.
- The use of PEC-type strategies favours the development of competences in the elaboration of hypotheses and in their conceptual revision, the passage from classic ideas to a quantum mechanics way of thinking, as well as from the idea of definite properties to ones that are definable only on the basis of measures, and finally from the idea of measurement as a collection of properties to an idea of measure as interaction.

Concerning the research questions asked, the following information emerged.

- The biggest learning difficulties concerning the concept of quantum state are linked to that of incompatibility. The problem is to abandon the deeply rooted (classic) idea of being able to foresee the outcome of a measurement on the basis of pre-existing properties of the measured system.
- The knowledge of the phenomenological context on which the discussion is held, acquired from the operative analysis of real experiments and from their description,

both qualitative and formalised in empirical laws, represents an element which cannot be renounced for the building of quantum interpretation. Software simulations help, but they are not enough for the whole building of such knowledge.

– The elaboration of hypotheses and the recognition of the consequences which coherently derive from them allow the overcoming of cognitive crisis on the comparison of previsions and the experiments' outcomes. The students show this competence by anchoring their reasoning to a specific phenomenological context, known from an operative point of view and not only with relation to the experiments' outcomes. The use of simulations can help in building the phenomenological framework of reference, but it cannot replace it.

– The end result is an acquisition of the conceptual framework which generally depends on the context examined. Therefore strategies for the generalisation of results, which imply the exploration of many contexts, must be adopted.

– The quantum ideas juxtapose themselves on the classic ones. There has not been a conceptual change, but rather an alternative use of different theoretical schemes. The problem that therefore arises is that of analysing coherently a certain phenomenology keeping within a single theoretic framework. This represents an element in favour of didactic approaches focalised on aspects that are characteristic of quantum theory, able to stimulate the development of theoretic thought, as well as of argumentative and generalisation abilities.

– Our specific proposal, dealing with the concept of incompatibility not only on the qualitative level, but also on the formal one, corresponds to a need of the students to dispose of interpretative tools, it also supplies tools for acquiring awareness on the meaning itself of incompatibility and its implications, and it develops a capacity of managing concepts and formalism itself.

In the experience carried out, the formalism developed on a first level and within specific contexts studied did not represent an obstacle to learning, but on the contrary it favoured it, for at least three-quarters of the students. The limitation to the real field and to systems with two conditions does not exhaust the foundation of quantum theory, but it certainly allows to analyze in depth the peculiar aspects of quantum concepts.

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