WORKING PAPER
DIPARTIMENTO DI ECONOMIA PUBBLICA

Working Paper n.132

William Addessi e Francesco Busato

Relative-preference shifts and the business cycle

Roma, Maggio 2010

Relative-preference shifts and the business cycle*

William Addessi† Francesco Busato‡

This Version: April 29, 2010

Abstract

This paper develops a two-sector dynamic general equilibrium model in which intertemporal fluctuations (and sectoral comovement) are driven by idiosyncratic shocks to relative preferences between consumption goods. This class of shocks may be interpreted as shifts in consumer tastes. When shifts in preferences occur, consumers associate a new and different level of satisfaction to the same basket of consumption goods according to the modified preferences. The paper shows that, if the initial composition of the consumption basket is sufficiently asymmetric, a shift in relative preferences produces a so strong "perception effect" capable of inducing *inter* and *intra* sectoral positive comovement of the main macroeconomic variables (i.e., output, consumption, investment, and employment). Furthermore, extending the theoretical framework to a multi-sector model and introducing a more flexible structure of the relative preference shock, we show that the parameter restrictions, necessary in order to observe sectoral comovement after a relative preference shock, are much less severe. In particular, the comovement between the most of the sectors emerges under general conditions, without requiring high asymmetry in the composition of the consumption basket and/or high aversion to risk. It is a welcome result that these findings are reached without introducing either aggregate technology shocks or input-output linkages, or shocks perturbing the relative preference between aggregate consumption and leisure.

Journal of Economic Literature Classification Numbers: F11, E320
Keywords: Demand Shocks, Two-sector Dynamic General Equilibrium Models

*We are grateful to J.P. Danthine, John Donaldson, Giorgio Rodano, and Mario Tirelli for many conversations. We wish also to thank the participants at Seminars and Workshops at University of Naples Parthenope, at annual meeting of the Association of Southern European Economic Theorists (2008) at European University Institute in Florence, and an anonymous referee of the Working Paper Series of the Department of Public Economics of "La Sapienza" for criticisms and suggestions on earlier drafts of this paper. Part of the research was completed at the Graduate School of Business of Columbia University, New York, while Francesco Busato was visiting the School. Of course all errors are ours.

†University of Rome "La Sapienza", Dept. of Public Economics; via del Castro Laurenziano 9, 00161 Rome (Italy), fax +39 06 4462040; email: addessi@dep.eco.uniroma1.it; tel. +39 320 0223939.

‡University of Naples "Parthenope" Dipartimento di Studi Economici; via Medina, 40, 80133 Napoli, Italy; email: busato@uniparthenope.it

1 Introduction

The comovement of economic activity across different sectors is one of the most important regularities of all business cycles (e.g., Lucas, 1977). Burns-Mitchell (1946) included inter-sectoral comovement in the definition of business cycles, and many empirical studies prove pro-cyclical behavior of cross-sector measures of employment, output, and investment (see Christiano and Fitzgerald, 1998 and Huffman and Wynne, 1999). Since it is difficult to identify reasonable aggregate disturbances capable of explaining historical business cycles, a vast literature investigates the transmission mechanisms from sectoral shocks to aggregate fluctuations. This approach has dealt with two main challenges: *i*) the explanation of how sectoral fluctuations spread over the entire economy; *ii*) the explanation of why sectors *move together*.

The multi-sector dynamic general equilibrium literature uses productivity shocks and technological linkages to explain sectoral comovement. In fact, the input-output structure grants that after an idiosyncratic productivity shock fluctuations in each sector take the same direction.[1] On the other side, the role of the demand has been highlighted by Cooper and Haltiwanger (1990). The authors suggest that the normality of demands for consumption goods is the channel through which sectoral shocks spread over the economy; meanwhile, the main mechanism of shocks' intertemporal transmission relies on the fact that only few sectors hold inventories. In this framework, an increase of inventories immediately reduces the production, and the income in sectors holding inventories. This reduces the demand for the goods produced in the other sectors and, consequently, the expected positive comovements of employment and output emerge.

Departing from the existing economic mechanisms implemented by the cited literature, this paper develops a framework without introducing exogenous changes to productivity and intra-sectoral technological linkages, and without relying on the "income effect" in the spirit of Cooper and Haltiwanger (1990).

In our paper, fluctuations are induced by exogenous shocks to the structure of preferences. In particular, shocks hit consumers' relative preference between consumption goods. The paper shows that this class of preference shock is capable of explaining the positive comovements of output, consumption, investment, and employment between sectors. It is important to highlight that this kind of shock affects only the relative preference between consumption goods and does not directly modify the preference structure between the composite consumption good and the leisure time. In this element, our mechanism differs from Wen (2006, 2007) or Bencivenga (2002) who investigate the effects of variations in the marginal rate of substitution between consumption and leisure.

The stylized economy is characterized as follows. Consider a two-sector economy, where, in each sector, a distinct output is produced by using labor services (freely mobile across sectors) and a sector-specific capital stock; the sector-specific output yields one type of consumption good and one type of investment good that can be used as capital only in the same sector. Such assumption excludes that sectoral comovement is induced by complementarity in the production process. Utility is defined over leisure and a consumption index that includes the consumption goods of both sectors. Next, assume that the consumption index is a Cobb-Douglas (homogeneous of degree 1) function over the consumption goods. Hence, the modeled preference shifts represent

[1] Selected references are Long and Plosser (1983), Murphy et al. (1989), Hornstein and Praschnik (1997), Huffman and Wynne (1999), Horvath (1998, 2000).

the one and only exogenous source of inter-temporal, inter-sectoral, and intra-sectoral dynamics of employment, consumption, output, and investment.

We interpret the dynamics focusing on different ways of perceiving the same consumption basket, according to the state of the relative preferences.

The paper is organized as follows. Section 2 details the benchmark economy. Section 3 presents the theoretical mechanism and selected numerical results. Section 4 discusses and extends model results, and Section 5 concludes. Finally, the Appendix includes all proofs and derivations.

2 A Two-Sector Model with Relative-Preference Shifts

This section presents the baseline dynamic equilibrium model with relative-preference shocks. Since there are no restrictions to trade, we solve the dynamic planning problem of a benevolent planner.

2.1 The Benchmark Economy.

The benchmark model is structured as a two-sector two-good economy, with endogenous labor supply choice. There exists a continuum of identical households of total measure one. The relative demand for goods are driven by autonomous changes in preferences of the representative household. Capital goods are sector specific while labor services can be reallocated across sectors without bearing any adjustment cost.

Preferences. Define a Cobb-Douglas consumption index in the following way:

$$C_t = c_{1,t}^{s_{1,t}} c_{2,t}^{s_{2,t}}, \tag{1}$$

where $c_{1,t}$ and $c_{2,t}$, respectively denote the consumption of good 1 and good 2 at time t; $s_{1,t}$ and $s_{2,t}$ denote the preference weights, following exogenous stochastic processes (defined below).[2] In this framework, a positive shock to $s_{1,t}$ changes the instantaneous structure of preferences in favor of c_1. In order to analyze the aggregate consequences of only relative preference shifts between consumption goods, we preserve the homotheticity of degree 1 of preferences and assume that $s_{1,t} + s_{2,t} = 1$, $\forall$ all $t = 1, 2, ...$ Then, it is sufficient to specify the characteristics of s_1 that follows an autoregressive process, $s_{1,t} = \rho s_{1,t-1} + (1-\rho)\, s_1 + \varepsilon_t$, where $0 \leq \rho \leq 1$ and s_1 indicates the steady state value. Quantity ε_t is a random variable normally distributed with zero mean and variance σ_ε^2. A relative-preference shock $\{\varepsilon_{1,t}\}_{t=1}^{\infty}$ is transitory, but because of the preference structure, it has persistent effects. Finally, it is important to note that C_t is not the aggregate consumption that is reported in national accounts at time t, is not a macroeconomic aggregate. C_t is an index that represents the structure of preferences, but we will return to this subject later.

Preferences over consumption index C_t and leisure ℓ_t are described by a state dependent felicity function $u(C(\mathbf{c}_t), \ell_t; \mathbf{s}_t) : \mathbb{R}_+^2 \times \mathcal{S}^2 \times [0,1]^2 \to \mathbb{R}$:

$$U(\mathbf{c}_t, \ell_t; \mathbf{s}_t) = \frac{(C_t)^{1-\gamma} - 1}{1-\gamma} + B\ell_t, \tag{2}$$

[2] Also Stockman and Tesar (1995) use the Cobb-Douglas aggregator for tradable consumption goods in a two country framework.

where γ measures the degree of risk aversion and is inversely proportional to the elasticity of intertemporal substitution; ℓ_t denotes leisure hours. In order to better understand the behavior of demands for consumption goods, we assume that the marginal utility of leisure is constant and equal to B.[3] Leisure hours are defined as:

$$\ell_t = 1 - n_{1,t} - n_{2,t}, \tag{3}$$

where $n_{1,t}$ and $n_{2,t}$ denote working hours in sector 1 and 2. The structure implies that available hours are normalized to 1 and labor services shift across sectors without adjustment costs.

Production Technologies. Each good is produced by physical capital and labor, using a sector-specific Cobb-Douglas technology:

$$y_{1,t} = \lambda_1 k_{1,t}^{\alpha_1} n_{1,t}^{1-\alpha_1} \text{ and } y_{2,t} = \lambda_2 k_{2,t}^{\alpha_2} n_{2,t}^{1-\alpha_2}, \tag{4}$$

where $y_{j,t}$, $k_{j,t}$, and λ_j denote, respectively, output, capital stock, and technology level in sector j, for $j = 1, 2$ hereafter. α_j measures the elasticity of output to capital in sector j. The production is not subject to exogenous technology changes (i.e., λ_j parameters are constant over time). As remarked in the introduction, this strongly differentiates our model from the traditional approach that focuses on the effects of idiosyncratic productivity shocks.

The allocation constraint is specific for each sector and is given by

$$c_{1,t} + i_{1,t} = y_{1,t} \quad \text{ and } \quad c_{2,t} + i_{2,t} = y_{2,t}, \tag{5}$$

where $i_{j,t}$ denotes the investment flows at time t.

In each sector, capital accumulation follows the standard formulation

$$k_{1,t+1} = (1 - \delta_1)k_{1,t} + i_{1,t} \text{ and } k_{2,t+1} = (1 - \delta_2)k_{2,t} + i_{2,t}, \tag{6}$$

where δ_j denotes the depreciation rates of capital stocks at time t. Eqs.(4) through (6) dictate that the capital stock used in sector j is produced entirely in sector j. This hypothesis makes capital goods fixed across sectors and then rules out input-output transmission mechanisms. So, it is possible to isolate the way preferences drive intersectoral comovements with no influences of production processes.

Model's Solution and Equilibrium Characterization. Planner maximizes the expected present discounted value of the return function $\mathcal{V}_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(\mathbf{c}_t, \ell_t; \mathbf{s}_t)$, where β $(0 < \beta < 1)$ is a subjective discount factor, subject to the allocation constraints (eq.(5)), the capital accumulation constraints (eq.(6)), and the total-hour constraint (eq.(3)). The state of the economy at time t is represented by a vector $\boldsymbol{\chi}_t = \langle k_{1,t}, k_{2,t}, s_{1,t}, s_{2,t} \rangle$. Controls for the problem are consumption flows $\mathbf{c}$, investment flows $\mathbf{i}$, and the labor services $\mathbf{n}$. Introducing dynamic multipliers $\phi_{1,t}$ and $\phi_{2,t}$, forming the Langrangean $\mathcal{L}_0$ yields:

[3] In a following section we show that linearity in leisure is not a necessary condition. This assumption simplifies the explanation of the mechanism underlying the relative-preference shifts in consumption goods.

$$\begin{aligned}\max_{\{c_{j,t},n_{j,t},k_{j,t+1}\}_{j=1}^{2}} \mathcal{L}_0 &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \frac{\left(c_{1,t}^{s_{1,t}} c_{2,t}^{s_{2,t}}\right)^{1-\gamma} - 1}{1-\gamma} + B\left(1 - n_{1,t} - n_{2,t}\right) + \\ &\quad + \phi_{1,t}\left[\lambda_1 k_{1,t}^{\alpha_1} n_{1,t}^{1-\alpha_1} - c_{1,t} + \left(1-\delta_1\right) k_{1,t} - k_{1,t+1}\right] + \\ &\quad + \phi_{2,t}\left[\lambda_2 k_{2,t}^{\alpha_2} n_{2,t}^{1-\alpha_2} - c_{2,t} + \left(1-\delta_2\right) k_{2,t} - k_{2,t+1}\right] \Bigg\}, \qquad (7)\end{aligned}$$

where $\mathbb{E}_0$ is the conditional expectation operator on time 0 information. First order conditions with respect to *j-th* consumption flow and working hours (FOC($c_{j,t}$), FOC($n_{j,t}$)) read:

$$\begin{aligned} c_{1,t} &: \; s_{1,t} c_{1,t}^{s_{1,t}-1} c_{2,t}^{s_{2,t}} \left(c_{1,t}^{s_{1,t}} c_{2,t}^{s_{2,t}}\right)^{-\gamma} = \phi_{1,t} \\ c_{2,t} &: \; s_{2,t} c_{1,t}^{s_{1,t}} c_{2,t}^{s_{2,t}-1} \left(c_{1,t}^{s_{1,t}} c_{2,t}^{s_{2,t}}\right)^{-\gamma} = \phi_{2,t} \qquad (8)\end{aligned}$$

$$\begin{aligned} n_{1,t} &: \; B = \phi_{1,t}\left(1-\alpha_1\right) \lambda_1 k_{1,t}^{\alpha_1} n_{1,t}^{-\alpha_1} \\ n_{2,t} &: \; B = \phi_{2,t}\left(1-\alpha_2\right) \lambda_2 k_{2,t}^{\alpha_2} n_{2,t}^{-\alpha_2}, \qquad (9)\end{aligned}$$

where $\left(1-\alpha_j\right)\lambda_j k_{j,t}^{\alpha_j} n_{j,t}^{-\alpha_j} = w_{j,t}$ is the marginal productivity of labor in sector j.

Combining the previous equations, the optimality conditions for sectoral consumptions and working hours can be rewritten as:

$$\begin{aligned} s_{1,t}\frac{C_t^{1-\gamma}}{c_{1,t}} w_{1,t} &= B \\ s_{2,t}\frac{C_t^{1-\gamma}}{c_{2,t}} w_{2,t} &= B. \qquad (10)\end{aligned}$$

Optimality conditions (eq.(10)) indicate the standard equality between the weighted marginal utility of consumption ($s_{j,t}\frac{C_t^{1-\gamma}}{c_{1,t}}$) and the weighted marginal utility of leisure ($\frac{B}{w_{j,t}}$).

Optimal investment dynamics are determined by the following two Euler Equations:

$$\begin{aligned} \beta\mathbb{E}_t\left[\frac{s_{1,t+1}\frac{C_{t+1}^{1-\gamma}}{c_{1,t+1}}}{s_{1,t}\frac{C_t^{1-\gamma}}{c_{1,t}}}\left(\alpha_1\lambda_1 k_{1,t+1}^{\alpha_1-1} n_{1,t+1}^{1-\alpha_1} + \left(1-\delta_1\right)\right)\right] &= 1 \\ \beta\mathbb{E}_t\left[\frac{s_{2,t+1}\frac{C_{t+1}^{1-\gamma}}{c_{2,t+1}}}{s_{2,t}\frac{C_t^{1-\gamma}}{c_{2,t}}}\left(\alpha_2\lambda_2 k_{2,t+1}^{\alpha_2-1} n_{2,t+1}^{1-\alpha_2} + \left(1-\delta_2\right)\right)\right] &= 1 \qquad (11)\end{aligned}$$

where $\mathbb{E}_t$ denotes the expectation operator, conditional on information available at time t. Notice that the pricing kernel $\Pi_{j,t} = \frac{C_{t+1}^{1-\gamma}}{C_t^{1-\gamma}} \frac{s_{j,t+1} c_{j,t}}{s_{j,t} c_{j,t+1}}$ depends on the $j-th$ preference parameters, the level of consumption of $j-th$ good and the level of consumption index.

The system of the optimal conditions and resource constraints determines the deterministic steady state; then, the log-linearization of the model around the steady state describes the dynamics.[4] In the next section we illustrate the parameterization of the model and then we show the simulation results.

2.2 Parameterization.

The system of equations that defines the dynamic equilibrium of the model depends on a set of twelve parameters. Six pertain to technology (the capital share α_j, the capital stock quarterly depreciation rate δ_j, and the level of technology λ_j in both sectors), while the other six pertain to consumer's preferences (the subjective discount factor β, the relative risk aversion coefficient γ, the marginal utility of leisure B, the relative preference for good 1, s_1, and for good 2, s_2, and the autoregressive coefficient of the preference process ρ).[5]

The sectors are characterized by the same technology, to make sure that all differences between the equilibrium values of the sectoral variables derive from consumers' preferences. This assumption makes it easy to associate the parameterization of the relative preferences between consumption goods to the composition of the initial consumption basket. In fact, under the symmetric hypothesis concerning the supply side, it emerges that $c_1 \gtreqless c_2 \Leftrightarrow s_1 \gtreqless s_2$. Assuming differences in the supply side would complicate the exposition of the mechanisms with no significant added value in the understanding of the role of preferences. The model is parameterized for the U.S. economy based on the post-war period, apart of relative preference parameters that are set to develop the theoretic investigation. The parameterization is detailed below:

Technology parameters $(\delta_j, \alpha_j, \lambda_j)$ are set to commonly used values in the Real Business Cycle (RBC) literature. In particular, the paper considers a symmetric economy by the supply side so, $\delta_1 = \delta_2 = 0.025$, $\alpha_1 = \alpha_2 = 0.36$, and $\lambda_1 = \lambda_2 = 1$.

Consumer's preference $(\beta, \gamma, B, s_j, \rho)$: the quarterly subjective discount factor β is set to correspond to an annual real interest rate of 4%; it yields $\beta = 0.99$. The relative risk aversion γ is equal to 5. The relative preference for good 1, s_1, varies in the range $0 < s_1 < 1$. The autoregressive coefficient of the preference process ρ is 0.99. The marginal utility of leisure, B is endogenously calibrated to generate $n_1 + n_2 = 0.3$.[6]

3 Results

3.1 Structure of the simulations

This section investigates how the stylized economy responds to an increase in the relative preference for good 1. Anticipating a result, the initial composition of the consumption basket is a

[4] Appendix (A) and (C) reports some steps to determine the steady state values and the dynamic equations of the model following Uhlig (1999).

[5] Recall that the preference parameter for good 2, s_2, is set equal to $1 - s_1$.

[6] See Appendix (B).

key element for determining the sign of inter-sectoral comovements; for this reason, the model is simulated for different composition of the consumption basket in steady state.

The simulations show how the dynamics change according to the relative weight of each consumption good in the consumption basket. In order to explain the emerging results, we will introduce a "perception effect", which describes how consumers' satisfaction, for a given consumption choice, changes according to the state of preferences. Subsequently, we investigate the role of selected parameters by some sensitivity analyses.

3.2 Baseline simulations

The paper considers three different pre-shock scenarios, in detail: a fully symmetric consumption basket, when $\frac{s_1}{s_2} = \frac{c_1}{c_2} = 1$ ($s_1 = 0.5$); two asymmetric consumption baskets in which $\frac{s_1}{s_2} = \frac{c_1}{c_2} = \frac{1}{9}$ ($s_1 = 0.1$) and $\frac{s_1}{s_2} = \frac{c_1}{c_2} = 9$ ($s_1 = 0.9$).

Technically, we run three sets of simulations maintaining the baseline calibration with the exception of the steady state value of s_1. We set s_1 equal to 0.1, 0.5, and 0.9, and report the impulse response functions of the sectoral variables in Figure 1 with blue, green, and red lines, respectively. To ease the comparison between the consequences of the different settings, we impose that the preference shocks have always the same magnitude, specifically, 1% of 0.5. It follows that immediately after the shock s_1 moves from 0.1 to 0.105 (blue line case), from 0.5 to 0.505 (green line case), and from 0.9 to 0.905 (red line case).

[FIGURE 1]

Figure 1, as the figures in the following sections, describes the impulse response functions of labor services (n_1, n_2), consumption (c_1, c_2), investment (i_1, i_2) and output (y_1, y_2) of both sectors; C is the consumption basket as defined in eq.(1) and N is the total employment ($N = n_1 + n_2$). The last two boxes refer to the ratio between the marginal utility of each consumption good and the marginal utility of leisure ($p_1 = \frac{U_{c_1}(\mathbf{c}_t, \ell_t; \mathbf{s}_t)}{U_l(\mathbf{c}_t, \ell_t; \mathbf{s}_t)}$, $p_2 = \frac{u_{c_2}(\mathbf{c}_t, \ell_t; \mathbf{s}_t)}{u_l(\mathbf{c}_t, \ell_t; \mathbf{s}_t)}$), that we can consider the prices of each good when the marginal utility of leisure is the numeraire. The figure shows the first 80 quarterly percentage deviations from a scenario where all innovations are set to zero.

The green lines (with $s_1 = s_2 = 0.5$) in Figure 1 show an economy where input factors (n_2 and i_2) are withdrawn from the production of good 2 (y_2 and c_2 decrease) and are allocated to sector 1. Intra-sectoral comovements between consumption, investment, employment, and output are positive in both sectors, but inter-sectoral comovements are negative. In fact, the sector characterized by an increase in preference (sector 1) goes through an expansive phase while the other sector goes through a recessive phase. Prices follow the direction of the preference weights.

The blue lines (with $s_1 = 0.1$, $s_2 = 0.9$) show an economy with both sectors in expansion. Both inter-sectoral and intra-sectoral comovements are positive. Prices comove and notice that p_2 increases while s_2 falls.

The red lines (with $s_1 = 0.9$, $s_2 = 0.1$), similarly to the previous case, show an economy characterized by positive inter-sectoral and positive intra-sectoral comovements, but the dynamics are completely reversed. The stylized economy experiences a recession in both sectors and in all variables (consumption, investment, employment, and output). Prices comove with different dynamics for p_1 and s_1.

3.3 The source of inter-sectoral comovement

Roughly speaking, the representative agent chooses between the consumption of two different goods and leisure. Under the preference structure reported in eq.(1) and eq.(2), the marginal rate of substitution between sector-specific consumption and leisure depends on the consumption of both goods. In fact, the influence of c_j on consumer's utility (i.e., the marginal utility of c_j) is given by the effect of c_j on C and by the effect of C on the utility function. If preferences shift, both effects vary but, as we are going to demonstrate, only the latter can induce positive inter-sectoral comovement.

In the previous three simulations (reported in Figure 1), after the positive shock to s_1 the *ceteris paribus* effect of c_1 on C increases while the effect of c_2 on C decreases.[7] This produces a sort of "substitution effect" that reduces the consumption of good 2 and increases the consumption of good 1. So, the way C_{c_j} change after a preference shift does not sustain positive inter-sectoral comovements. On the contrary, the *ceteris paribus* change of C affects the optimal choice in both sectors in the same direction. In fact, if C decreases (increases) then the marginal impact of C on the utility function increases (decreases) and this pushes up (down) the marginal utility of both goods. We call this change the "perception effect". It does not affect the marginal rate of substitution between c_1 and c_2 but it affects the marginal rate of substitution between each consumption good and leisure.[8]

The blue line case describes a context where the perception effect increases the marginal utility of both consumption goods. The reverse occurs in the red line case: the marginal utility of consumption basket has fallen so that also sector 1 experiences a recession phase. In both cases positive inter-sectoral comovements emerge driven by sectoral (and not aggregate) preference shocks. In the opposite, the green lines report dynamics where the perception effect is absent, then only the substitution effect matters.

Table 1 resumes the possible scenarios after a positive shock to s_1.

TABLE 1 Possible dynamics of consumption after a positive shock to s_1.

Before the shock	Perception	Substitution	Final Result
$c_1 < c_2$	$c_1 \uparrow$ $c_2 \uparrow$	$c_1 \uparrow$ $c_2 \downarrow$	$c_1 \uparrow$ $c_2 \uparrow\downarrow$
$c_1 = c_2$	$c_1 \leftrightarrow$ $c_2 \leftrightarrow$	$c_1 \uparrow$ $c_2 \downarrow$	$c_1 \uparrow$ $c_2 \downarrow$
$c_1 > c_2$	$c_1 \downarrow$ $c_2 \downarrow$	$c_1 \uparrow$ $c_2 \downarrow$	$c_1 \uparrow\downarrow$ $c_2 \downarrow$

Now, let's deepen the argument more analytically. The starting point is the optimal condition ruling the choice between consumption and leisure in each sector,

[7] In Appendix (D) and (E) we prove that $C_{c_1 s_1} > 0$ and that $C_{c_2 s_1} < 0$.

[8] The preference shock generates a sort of "real wealth effect", if it is assumed that real wealth can be measured by the level of utility that consumer can reach. Indeed, after the shock the level of satisfaction varies because the consumer associates different satisfaction to the same goods. So, the level of satisfaction changes even if the consumption is fixed. But, it is necessary to specify that, even if we adopt the previous definition, this mechanism would be quite different from that reported in microeconomic manuals. In fact, the first element to change is not the budget constraint but the indifference curve.

$$U_{c_j} w_j = U_C C_{c_j} w_j = B. \tag{12}$$

Eq.(12) imposes that in equilibrium the marginal utility of consumption of good j, U_{c_j}, weighted with the marginal productivity of labor in sector j, w_j, has to be equal to the marginal utility of leisure, B.[9] The marginal utility of c_j can be decomposed in the product between the first derivative of the utility function with respect to the consumption basket, U_C (i.e., the marginal utility of C), and the first derivative of such index with respect to the single consumption good, C_{c_j}. The key question concerns what happens to U_C after an increase in s_1 (as in our simulations). In order to provide an answer, it is necessary to focus on the signs of two derivatives. The first is the sign of the derivative of the marginal utility U_C with respect to the consumption basket (i.e., the second derivative of the utility function with respect to the consumption basket). This sign is negative (in fact, $U_{CC} = -\gamma C^{-\gamma-1} < 0$ for $\gamma \geq 0$).

The second sign concerns the partial derivative of the consumption basket with respect to the exogenous shock, C_{s_1}.[10] The sign of this derivative depends on the ratio between the consumption goods composing the consumption basket, in fact $C_{s_1} = c_1^{s_1} c_2^{1-s_1} \ln(\frac{c_1}{c_2})$.[11] In the blue line version (reported in Figure 1) $s_1 = 0.1$ so $\frac{c_1}{c_2} < 1$, and then $C_{s_1} < 0$. If $C \downarrow$ then $U_C \uparrow$.[12] In this case, the direct effect of a positive shock to s_1 reduces C and then increases U_C. Notice that according to the optimal conditions (eq.(12)) the product between $C_{c_j} w_j$ has to fall in both sectors. This can occur by both an increase in c_j (the derivative of C_{c_j} with respect to c_j is negative, in fact $C_{c_1 c_1} = s_1 (s_1 - 1) c_1^{s_1-2} c_2^{1-s_1} < 0$, $C_{c_2 c_2} = -s_1 (1 - s_1) c_1^{s_1} c_2^{-s_1-1} < 0$) and an increase in employment n_j (to reduce w_j). The reverse occurs in the case with $s_1 = 0.9$ (red lines in Figure 1).[13]

This mechanism contributes in explaining the positive inter-sectoral comovements reported in the cases with blue and red lines, and why the economic booms (dooms) occur after a preference shock when s_1 is set low (high).

Finally, the green lines represent a perfectly symmetric economy: $s_1 = 0.5$, $\frac{c_1}{c_2} = 1$, $C_{s_1} = 0$. The direct effect of preference shifts on U_C is null and, then, the dynamics of the stylized economy are driven by only substitution effects between sectoral goods.

In this section we have not described all the forces at work, because we are mainly interested in the mechanisms that induce positive inter-sectoral comovements. So, as evidenced in Table 1 (by the use of arrows), the initial composition of the consumption basket represents a sort of

[9] Assuming B constant, the dynamic equation of eq.(12) can be expressed in the following way: $\tilde{s}_{j,t} - \widetilde{c}_{j,t} + (1-\gamma)\widetilde{C}_t + \alpha_j \widetilde{k}_{j,t} - \alpha_j \widetilde{n}_{j,t} = 0$ where the tilde indicates the growth rate. This way of representing the dynamics characterizing eq.(12) can be helpful to follow the mechanism described in this section.

[10] We are interested in the direct effect of s_1 on C taking the rest as given. So we are not considering the indirect effect generated by variations in consumption composition.

[11] Recall that in our model the supply sides of each sector are perfectly symmetric; it follows that the relative dimensions of steady state values of the sectoral variables depend only on consumer's preferences. Then $c_1 \gtreqless c_2$ iff $s_1 \gtreqless s_2$.

[12] Notice that assuming a CES function for the consumption index, the economic mechanism does not change significantly. In fact, if $C = (s_1 c_1^q + (1-s_1) c_2^q)^{\frac{1}{q}}$; then $\frac{\partial C}{\partial s_1} = \frac{1}{q} (s_1 c_1^q + (1-s_1) c_2^q)^{\frac{1}{q}-1} (c_1^q - c_2^q)$, consequently the sign depends again on the relative dimensions between the consumption goods.

[13] It is noteworthy to remark that $C_{c_1 s_1} > 0$ while $C_{c_2 s_1} < 0$. Consequently (not considering the behavior of the marginal productivity of labor), if $C_{s_1} < 0$ an increase in c_1 is surely needed in order to produce a fall of C_{c_1}, while the same is not always true for c_2.

necessary, but not sufficient, condition to observe positive inter-sectoral comovements in response to relative-preference shifts.

Before extending the analysis to the role of some selected parameters a further clarification of our results is needed. As previously pointed out C is not the aggregate consumption. To build aggregate macroeconomic variables we should define the aggregation technique and in order to test our model we should introduce a "more realistic" structure with linkages in the supply-side, but this is beyond our purpose. Our aim is to show a new possible source of inter-sectoral comovement, so it is sufficient to focus on sectoral variables. Nevertheless it is worth noting that in the cases with inter-sectoral comovement (blue and red lines in Figure 1), sectoral real variables and prices move in the same direction. It suggests that emerging sectoral dynamics (expansion or recession) should characterize in the same way the paths of aggregate variables. Then, results reported in Figure 1 should be consistent with positive correlation between aggregate variables.

3.4 And what about investment choice?

The behavior of investments depends on the persistence of the shock. So, it is useful to run another set of simulations with a lower value of the autoregressive coefficient: $\rho = 0.92$. Results are represented in Figure 2. Hereafter, we focus our explanations on the case represented by blue lines, that is $s_1 = 0.1$, but the same mechanisms work in the other cases (with different results).

[FIGURE 2]

Some evidence emerge clearly. First, the impulse responses are less persistent. Second, the intra-sectoral comovements change. Particularly, by reducing the persistence of the preference shock the responses of sectoral investments change direction. Such mechanisms are the same as discussed by Wen (2006, 2007) in one sector model: in the absence of increasing returns to scale (as in this case), not persistent changes in the preference for consumption crowds out investment. It follows that only persistent increases in consumption demand can sustain investment by prompting a further increase in labor supply.[14] Finally, it has to be noticed that the persistence of the preferences does not affect the sign of inter-sectoral comovements, because it does not affect the marginal rates of substitution between consumption goods and leisure.

3.5 The role of relative risk aversion coefficient over consumption flows

Roughly speaking, the relative risk aversion coefficient determines how much the marginal utility of consumption varies after a change of the consumption basket. Then, it is reasonable to assume that this parameter may be really relevant in the present framework. In order to illustrate the effects of a change in γ, we substitute $\gamma = 5$ with $\gamma = 1.5$ and simulate the usual preference shock. The resulting dynamics are reported in Figure 3.

[FIGURE 3]

[14] Just as in Wen (2006), the impulse responses show that the response of c_j is relatively higher with respect to that of n_j and y_j when the ρ is low.

The figure shows that with low values of γ the positive comovements between sectors vanish. The reason is that the significance of the perception effect has strongly decreased. By the dynamic equation of the first derivative of the utility function with respect to the consumption basket ($\widetilde{U}_C = -\gamma \widetilde{C}$) (see also the dynamic equation reported in note (9)), it is clear that γ is a scale factor of the effect of C on the marginal utility of consumption. So if γ is low, variations in C poorly affect the relative preference between consumption goods and leisure and thus inter-sectoral comovements are infrequent.

3.6 The role of the inter-temporal elasticity of substitution over leisure

Along the previous simulations we assumed that the marginal utility of leisure was constant. This assumption allowed us to observe how the impact of the substitution effect and of the perception effect changes without the influence of variations in the marginal utility of leisure.

Now, let's remove this hypothesis and solve the model assuming the following utility function: $u(\mathbf{c}_t, \ell_t; \mathbf{s}_t) = \frac{(C_t)^{1-\gamma}-1}{1-\gamma} + \frac{1}{v} B(1 - n_{1,t} - n_{2,t})^v$, where $(1 - v)$ controls the degree of the risk aversion and is inversely proportional to the elasticity of inter-temporal substitution in leisure. We set $v = -1$ and report the impulse response functions in Figure 4.

[FIGURE 4]

Results indicate that positive inter-sectoral comovements are less frequent when v decreases. This parameter does not directly modify the perception effect or the substitution effect between consumption goods. The different dynamics emerge because of the behavior of the marginal utility of leisure that, when $v < 1$, is positively related to the labor supply. In fact, the higher labor supply in sector 1 increases the marginal utility of leisure, and then reduces the incentive to increase the labor supply also in sector 2. Under this parameterization, the perception effect is still able to generate positive comovements between sectoral consumption and employment, but it is not high enough to support investment in sector 2. Total labor supply is less reactive.

4 Discussion and extension

From the previous analysis, an important implication is that a relative preference shift is able to induce sectoral comovement more likely when the asymmetry of the composition of the consumption index and the risk aversion are high. Indeed, previous simulations have been run under a quite "comfortable" parameterization in order to report clear impulse response functions in the figures. That requires to discuss the robustness and the relevance of the proposed mechanism.

Let's start reporting the binding constraints concerning the setting of s_1 and γ in order to observe inter and intra sectoral comovement. When $\gamma = 5$, it is necessary $s_1 \leq 0.179$ (upturn) ($\frac{c_1}{c_2} \leq 0.218$) or $s_1 \geq 1 - 0.179$ (downturn) ($\frac{c_1}{c_2} \geq 4.587$). Otherwise, keeping $s_1 = 0.1$, or $s_1 = 0.9$, it is necessary $\gamma \geq 2.1$, that falls to 1.56 if $s_1 = 0.05$, or $s_1 = 0.95$.

The linkage between the setting of the risk aversion and the sectoral relative preference emerge from simulations and is consistent with the suggested economic intuition (the former measures the effect of a change in C on the marginal utility of consumption, the latter determines the magnitude of the variation of C after the preference shock). The point is to evaluate if such constraint lets the sketched mechanism be relevant. In order to address this issue we consider proper to reduce, and take constant, the value of the risk aversion (next we run other simulations with $\gamma = 2$) and focuses on the interpretation of the asymmetry in the consumption index. We propose two arguments to support the idea that the constraints on the parameterization setting do not set aside the relevance of this contribution.

The first argument concerns the identification of the kinds of goods. Since we consider a general equilibrium framework, goods 1 and goods 2 represent all the available goods in the economy. It follows that their relative size can be extremely high, extremely low, or next to one according to what they represent. For instance, recent kinds of goods may represent a very small share of the economy and be subject to positive preference shocks.

The second argument concerns the range of the comovement that has to be explained. As well documented by Hornstein (2000), *almost all*, but *not all*, industries of the economy comove. That implies that in presence of many kinds of goods it is important to identify a mechanism able to explain the comovement between the most of them. Then, we extend our model to analyze how the proposed mechanism works in an economy with m different goods and we show that a relative preference shock can induce comovement between a lot of sectors without imposing relevant constraints on parameters. Let's set a more general definition of the consumption index:

$$C_t = \prod_{j=1}^{m} c_j^{s_{j,t}}, \tag{1b}$$

where $\sum_{j=1}^{m} s_{j,t} = 1, \forall t$. A shift ε_t in the relative preferences may change the preference structure in the following way: $s_{j,t} = s_{j,t-1} + \mu_j \varepsilon_t$, $\forall j$ with $\sum_{j=1}^{m} \mu_j = 0$. In this case, the partial derivative of the consumption index with respect to the preference shift is:[15]

$$C_\varepsilon = C \sum_{j=1}^{m} \mu_j \ln c_j. \tag{13}$$

Notice that, as in the two-sector case, the way the preference shock affects the composite consumption index influences the consumption-leisure choice in the same direction in each sector. That opens endless possibilities according to the composition of C and how the shock hits the preference for the different goods. In this generalized case, the perception effect fades out only if $\sum_{j=1}^{m} \mu_j \ln c_j = 0$.

Let's focus on the case in which the preference shock concerns only two kinds of goods. For example, assume $\mu_j = 0$ for $j \neq 1, 2$ and $\mu_1 = -\mu_2 = 1$. From eq.(13) it follows:

$$C_\varepsilon = C \ln \left(\frac{c_1}{c_2} \right). \tag{14}$$

Notice that this time c_1 and c_2 do not represent the whole economy. Eq.(14) permits to anticipate an important result that will be confirmed by the next simulations: Under the hypotheses

[15] Hereafter we drop time reference.

characterizing this stylized economy, a preference shift between two kinds of goods generate comovement between all the other goods. That implies that, even if the perception effect is not sufficiently high to induce comovement between the sectors directly affected by the shock (sectors 1 and 2), the preference shift pushes the other $(m-2)$ sectors in the same direction (that changes according to the sign of $\ln\left(\frac{c_1}{c_2}\right)$).

As example we solve a four-good model with the same characteristics of the two-good model analyzed in the previous sections. The only changes concern the value of the relative risk aversion, $\gamma = 2$ (previously, $\gamma = 5$) and the value of the persistence coefficient of the preference shock, $\rho = 0.95$ that is consistent with the empirical findings of Foster et al. (2008) (previously, $\rho = 0.99$). We run two sets of simulations. In the first set we consider a preference shift between two sectors, mainly to show that if the aim is to explain comovement between most of (but not all) the economic sectors, then the proposed mechanism is not significantly constrained by the parameterization of the preferences. In the second set of simulations we analyze a preference shift between three sectors. Specifically, we assume that the increase in the preference for a sector may happen at expense of two sectors. It emerges that splitting the shock between more than two sectors, the proposed mechanism is able to generate comovement between sectors whose preferences move in opposite directions with less binding restrictions on the parameter setting.

The first set of simulations of the impulse response functions is reported in Figure 5, while Table 2 resumes the setting of the preferences and reports the sectoral outcomes. The common hypothesis is that $\mu_1 = -\mu_4 = 1$ and $\mu_2 = \mu_3 = 0$. The analyzed cases differ in the parameterization of the relative preferences. In case (I) the sectors directly affected by the shock have the same weight in the consumption index, then there is no perception effect. The sector with the positive shock grows, the sector with the negative shock falls, while the others remain stable. In case (II) there is high asymmetry among the weights of the sectors directly affected by the shock. That produces a high push towards comovement and all the sectors moves (up) in the same direction, including the sector with the decreasing preference. Case (III) and case (IV) confirm that the dynamics of the sectors, whose relative preferences do not change, depend on the perception effect. Indeed, it is worth noting that, given s_1 and s_4, the values of s_2 and s_3 do not affect the dynamics of sector 2 and sector 3 and that the dynamics of these sectors in case (III) are exactly the inverse of those in case (IV), since parameters have been chosen to produce the same perception effect but with the opposite sign.

The last set of simulations (see Table 3 and Figure 6) assumes that $\mu_1 = -\mu_4 - \mu_3 = 1$ and $\mu_2 = 0$. Case (V) and case (VI) are characterized by the same structure of the relative preferences but different values of μ_3 and μ_4. The perception effect is strong and pushes towards an increase in consumption in both cases: In case (V) all the sectors experience an upturn, while in case (VI) sector 4 experiences a downturn because it is the most negatively affected by the shock.[16] Case (VII) takes to similar results of case (VI) but with less asymmetry between the size of the sectors. That indicates that the theoretical mechanism presented in this paper can explain comovement also between sectors whose relative preferences go in opposite directions (sector 1 and sector 4) not only with extreme parameter setting. Finally, case (VIII) represents a special combination of parameter values that produces an almost null perception effect. That induces dynamics similar to case (I) (the sector(s) with stable relative preferences do(es) not move).

[16] Consistently with the parameterization, Figure (6) shows that sector 1 and sector 2 have the same dynamics in case (V) and in case (VI), while sector 3 experiences higher increase in case (VI) than in case (V).

TABLE 2 Sectoral dynamics after a shock that affects positively s_1 and negatively s_4.

	(I)	(II)	(III)	(IV)
s_1	0.25	0.05	0.2	0.4
s_2	0.4	0.05	0.3	0.2
s_3	0.1	0.05	0.1	0.2
s_4	0.25	0.85	0.4	0.2
μ_1	1	1	1	1
μ_2	0	0	0	0
μ_3	0	0	0	0
μ_4	-1	-1	-1	-1
sectors in upturn	1	1, 2, 3, 4	1, 2, 3	1
sectors in downturn	4	$\varnothing$	4	2, 3, 4
stable sectors	2, 3	$\varnothing$	$\varnothing$	$\varnothing$

TABLE 3 Sectoral dynamics after a shock that affects positively s_1 and negatively s_3 and s_4.

	(V)	(VI)	(VII)	(VIII)
s_1	0.05	0.05	0.2	0.25
s_2	0.05	0.05	0.05	0.3
s_3	0.45	0.45	0.4	0.3
s_4	0.45	0.45	0.35	0.15
μ_1	1	1	1	1
μ_2	0	0	0	0
μ_3	-0.5	-0.1	-0.9	-0.75
μ_4	-0.5	-0.9	-0.1	-0.25
sectors in upturn	1, 2, 3, 4	1, 2, 3	1, 2, 4	1
sectors in downturn	$\varnothing$	4	3	3, 4
stable sectors	$\varnothing$	$\varnothing$	$\varnothing$	2

5 Conclusions

The main intent of this paper is to suggest a new source of sectoral comovements during business cycles. In this model relative preference shocks between consumption goods, which may be interpreted as shocks to households' tastes, are the only mechanism generating fluctuation. The results indicate that in order to induce inter and intra sectoral comovement in a stylized economy with only two kinds of goods, constraints on the parameter setting are necessary (especially on risk aversion and relative preferences) but, at least in our opinion, not so binding to make the theoretical point irrelevant. In the last section, we have shown that extending the analysis to a multi-sector model and introducing a more flexible structure of the propagation of the preference shock, constraints on the parameter setting become much less severe. Consistently with the empirical evidence reported in Hornstein (2000), the model generates comovement between almost all, but not all, industries of the economy under a vast range of parameterizations. In this case, both the values of the relative preferences and the spread of the preference shock determine the direction of the comovement and the number of sectors involved.

In fact, the (whole) economy tends to expansions (recession) when preferences shift towards goods that represent a minor (major) share in the consumption basket. This is due to the fact that households obtain less satisfaction by the consumption basket chosen before the shock (perception effect). Consequently, they increase the labor supply in order to restore the optimal condition between leisure and each kind of consumption goods.

Moreover, this model suggests clear indications concerning the role of some parameters. Main results are briefly reported. First, the persistence of the preference shift strongly affects the responses of investments. This is due to the fact that ρ determines the duration of the perception effect and, consequently, it affects the inter-temporal optimal path of consumption. Second, the coefficient of risk aversion γ, determines the relevance of the perception effect in the trade-off between consumption and leisure in each sector. Finally, if the marginal utility of leisure is increasing in labor supply, the perception effect has to be higher in order to generate positive inter-sectoral comovements.

The sketched mechanism is quite new in economic literature. In fact, multi-sectoral models generally tend to explain positive comovements of economic sectors relying on input-output structure that transmits sectoral shocks over the entire economy.

It should be emphasized that this model differs from models such Bencivenga (1992) and Wen (2006, 2007) which consider direct variations in the relative preference between consumption and leisure, independently of the composition of consumption basket. This can be due to the fact that leisure time is employed in other activities (as homework production, see Benhabib et al., 1991), or it can be induced by alternative phases of the "urge to consume" (see Wen, 2006) that modifies the importance of consumption. On the contrary, this model focuses on two other elements: the starting composition of consumption basket and the shifts in relative preference between consumption goods. The last element is studied in Phelan and Trejos (2000), but they use such element in order to explain aggregate fluctuations with negative comovement in sectoral employment.

Particularly interesting implications and extensions of the model concern the analysis of advertising and innovation. Both elements can produce externalities which strongly resemble to that analyzed in this paper. This opens other research fields that lie outside this model.

References

[1] Benhabib J., Rogerson R. and Wright R. (1991): "Homework in Macroeconomics: Household Production and Aggregate Fluctuations", Journal of Political Economy, 99, 6, 1166-1187.

[2] Bencivenga, V. (1992): "An Econometric Study of Hours and Output Variation with Preference Shocks", International Economic Review, 33 (2), 449-71.

[3] Burns, A. and W.C. Mitchell (1946): "Measuring business cycles, Studies in Business Cycles, n. 2", New York National Bureau of Economic Research.

[4] Christiano, L. and T. Fitzgerald (1998): "The business cycle: It's still a puzzle", Economic Perspectives, Federal Reserve Bank of Chicago, issue Q IV, pages 56-83.

[5] Cooper, R. and J. Haltiwanger (1990): "Inventories and the Propagation of Sectoral Shocks", American Economic Review, 80(1), 170-190.

[6] Foster, L., Haltiwanger J., and C. Syverson (2008): "Reallocation, Firm Turnover, and Efficiency: Selection on Productivity or Profitability?", American Economic Review, 98(1), 394-425.

[7] Hornstein, A. and J. Praschnik (1997): "Intermediate inputs and sectoral comovement in the business cycle", Journal of Monetary Economics, 40, 573-595.

[8] Hornstein A. (2000): "The Business Cycle and Industry Comovement", Federal Reserve Bank of Richmond Economic Quarterly, 86 (1), 27-48.

[9] Horvath, M. (1998): "Cyclicality and Sectoral Linkages: Aggregate Fluctuations from Independent Sectoral Shocks", Review of Economic Dynamics, 1, 781-808.

[10] Horvath, M. (2000): "Sectoral shocks and aggregate fluctuations", Journal of Monetary Economics, 45, 69-106.

[11] Huffman, G.W. and M. Wynne (1999): "The Role of Intratemporal Adjustment Costs in a Multisector Economy", Journal of Monetary Economics, 43, 2, 317-50.

[12] Long, J. B. Jr and C. Plosser (1983): "Real Business Cycles, Journal of Political Economy", 91 (1), 39-69.

[13] Lucas R.E. Jr. (1977): "Understanding Business Cycles", Journal of Monetary Economics, 5 (0), 7-29.

[14] Murphy, K.M., Shleifer, A., Vishny R.W. (1989): "Building blocks of market clearing business cycle models", NBER Macroeconomics Annual 1989, MIT Press.

[15] Phelan C., and A. Trejos (2000): "The aggregate effects of sectoral reallocations", Journal of Monetary Economics, 5 (0), 7-29.

[16] Stockman, A.C. and L. L. Tesar (1995): "Tastes and Technology in a Two-Country Model of the Business Cycle: Explaining International Comovements", American Economic Review, 85 (1), 168-85.

[17] Uhlig H. (1999): "A Toolkit for Analysing Nonlinear Dynamic Stochastic Models Easily", in R. Marimon and A. Scott (eds) Computational Methods for the Study of Dynamic Economies, (Oxford University Press; Oxford).

[18] Wen, Y. (2006): "Demand Shocks and Economic Fluctuations", Economic Letters, March 2006, 90(3), 373-83.

[19] Wen, Y. (2007): "By force of Demand: Explaining International Comovements", Journal of Economic Dynamics and Control, January 2007, 31(1), 1-23.

Appendix

A. Steady State

From Euler equations (11):

$$\frac{k_1}{n_1} = \left(\frac{\alpha_1 \lambda_1}{\frac{1}{\beta} - 1 + \delta_1}\right)^{\frac{1}{1-\alpha_1}}$$

$$\frac{k_2}{n_2} = \left(\frac{\alpha_2 \lambda_2}{\frac{1}{\beta} - 1 + \delta_2}\right)^{\frac{1}{1-\alpha_2}}$$

Combining the first-order conditions for consumption and labor:

$$c_1 = \left(s_1 TT^{s_2(1-\gamma)} \frac{(1-\alpha_1)\,\lambda_1}{B} \left(\frac{k_1}{n_1}\right)^{\alpha_1}\right)^{\frac{1}{\gamma(s_1+s_2)+1-s_1-s_2}}$$

$$c_2 = TTc_1$$

$$C_t = c_1^{s_1} c_2^{s_2}$$

where $TT = \frac{s_2}{s_1} \frac{(1-\alpha_2)\lambda_2 k_2^{\alpha_2} n_2^{-\alpha_2}}{(1-\alpha_1)\lambda_1 k_1^{\alpha_1} n_1^{-\alpha_1}}$. The feasibility constraint implies:

$$k_1 = \left(\lambda_1 \left(\frac{k_1}{n_1}\right)^{\alpha_1 - 1} - \delta_1\right)^{-1} c_{1,t}$$

$$k_2 = \left(\lambda_2 \left(\frac{k_2}{n_2}\right)^{\alpha_2 - 1} - \delta_2\right)^{-1} c_{2,t}$$

Finally using the capital accumulation process:

$$i_1 = \delta_1 k_1$$

$$i_2 = \delta_2 k_2$$

$$n_1 = \left(\frac{k_1}{n_1}\right)^{-1} k_1$$

$$n_2 = \left(\frac{k_2}{n_2}\right)^{-1} k_2$$

B. Calibration of B

We assume that:

$$n_1 + n_2 = N = 0.3$$

Using eq.(8) and eq.(9) it is possible to express working time in the following way:

$$N = \left(\left(\frac{k_1}{n_1}\right)^{-1} \left(\lambda_1 \left(\frac{k_1}{n_1}\right)^{\alpha_1 - 1} - \delta_1\right)^{-1} + \left(\frac{k_2}{n_2}\right)^{-1} \left(\lambda_2 \left(\frac{k_2}{n_2}\right)^{\alpha_2 - 1} - \delta_2\right)^{-1} TT \right) c_1$$

Substituting the steady state value of c_1 and finding the value of B according to the parametrization of n_j :

$$B = \left(\begin{array}{c} \left(\left(\frac{k_1}{n_1}\right)^{-1} \left(\lambda_1 \left(\frac{k_1}{n_1}\right)^{\alpha_1 - 1} - \delta_1\right)^{-1} + \left(\frac{k_2}{n_2}\right)^{-1} \left(\lambda_2 \left(\frac{k_2}{n_2}\right)^{\alpha_2 - 1} - \delta_2\right)^{-1} TT \right)^{\gamma} \times \\ \times \left(s_1 TT^{s_2(1-\gamma)} \left(1 - \alpha_1\right) \lambda_1 \left(\frac{k_1}{n_1}\right)^{\alpha_1} \right) N^{-\gamma} \end{array} \right)$$

C. Log-Linearization

1. $\tilde{s}_{1,t} - \widetilde{c}_{1,t} + (1-\gamma)\widetilde{C}_t + \alpha_1 \widetilde{k}_{1,t} - \alpha_1 \widetilde{n}_{1,t} = 0$
2. $\tilde{s}_{2,t} - \widetilde{c}_{2,t} + (1-\gamma)\widetilde{C}_t + \alpha_2 \widetilde{k}_{2,t} - \alpha_2 \widetilde{n}_{2,t} = 0$
3. $y_1 \widetilde{y}_{1,t} = c_1 \widetilde{c}_{1,t} + i_1 \widetilde{i}_{1,t}$
4. $y_2 \widetilde{y}_{2,t} = c_2 \widetilde{c}_{2,t} + i_2 \widetilde{i}_{2,t}$
5. $k_1 \widetilde{k}_{1,t+1} = (1-\delta_1) k_1 \widetilde{k}_{1,t} + i_1 \widetilde{i}_{1,t}$
6. $k_2 \widetilde{k}_{2,t+1} = (1-\delta_2) k_2 \widetilde{k}_{2,t} + i_2 \widetilde{i}_{2,t}$
7. $\widetilde{y}_{1,t} = \alpha_1 \widetilde{k}_{1,t} + (1-\alpha_1)\widetilde{n}_{1,t}$
8. $\widetilde{y}_{2,t} = \alpha_2 \widetilde{k}_{2,t} + (1-\alpha_2)\widetilde{n}_{2,t}$
9. $r_1 \widetilde{r}_{1,t} = \alpha_1 (\alpha_1 - 1) k_1^{\alpha_1 - 1} n_1^{1-\alpha_1} \widetilde{k}_{1,t} + \alpha_1 (1-\alpha_1) k_1^{\alpha_1 - 1} n_1^{1-\alpha_1} \widetilde{n}_{1,t}$
10. $r_2 \widetilde{r}_{2,t} = \alpha_2 (\alpha_2 - 1) k_2^{\alpha_2 - 1} n_2^{1-\alpha_2} \widetilde{k}_{2,t} + \alpha_2 (1-\alpha_2) k_2^{\alpha_2 - 1} n_2^{1-\alpha_2} \widetilde{n}_{2,t}$
11. $\widetilde{C}_t = s_1 \widetilde{c}_{1,t} + s_2 \widetilde{c}_{2,t} + s_1 \ln(c_1) \widetilde{s}_{1,t} + s_2 \ln(c_2) \widetilde{s}_{2,t}$
12. $\widetilde{N}_t = \frac{n_1}{N} \widetilde{n}_{1,t} + \frac{n_2}{N} \widetilde{n}_{2,t}$
13. $s_2 \widetilde{s}_{2,t} = -s_1 \widetilde{s}_{1,t}$
14. $\widetilde{s}_{1,t} - \widetilde{s}_{2,t} - \widetilde{c}_{1,t} + \widetilde{c}_{2,t} + \alpha_1 \widetilde{k}_{1,t} + (1-\alpha_1)\widetilde{n}_{1,t} = 0$
15. $\widetilde{s}_{2,t} - \widetilde{s}_{1,t} - \widetilde{c}_{2,t} + \widetilde{c}_{1,t} + \alpha_2 \widetilde{k}_{2,t} + (1-\alpha_2)\widetilde{n}_{2,t} = 0$

16. $-\widetilde{p}_{1,t}+\widetilde{s}_{1,t}+(1-\gamma)\,\widetilde{C}_t-\widetilde{c}_{1,t}-(1-v)\,\frac{N}{1-N}\widetilde{N}_t=0$

17. $-\widetilde{p}_{2,t}+\widetilde{s}_{2,t}+(1-\gamma)\,\widetilde{C}_t-\widetilde{c}_{2,t}-(1-v)\,\frac{N}{1-N}\widetilde{N}_t=0$

Forward equations:

18. $1=\widetilde{s}_{1,t+1}-\widetilde{s}_{1,t}+\widetilde{c}_{1,t}-\widetilde{c}_{1,t+1}+(1-\gamma)\,\widetilde{C}_{t+1}-(1-\gamma)\,\widetilde{C}_t+\widetilde{r}_{1,t+1}$

19. $1=\widetilde{s}_{2,t+1}-\widetilde{s}_{2,t}+\widetilde{c}_{2,t}-\widetilde{c}_{2,t+1}+(1-\gamma)\,\widetilde{C}_{t+1}-(1-\gamma)\,\widetilde{C}_t+\widetilde{r}_{2,t+1}$

D.

Proof. We now proof that $\frac{\partial^2 C}{\partial c_1 \partial s_1}>0$.From (1):

$$\frac{\partial C}{\partial c_1}=s_1 c_1^{s_1-1}c_2^{1-s_1},0<s_1<1$$

and the corresponding steady state equation is:

$$\frac{c_1}{c_2}=\frac{s_1}{1-s_1}$$

The derivative of $\frac{\partial C}{\partial c_1}$ with respect to $s_1$is given by:

$$\frac{\partial^2 C}{\partial c_1 \partial s_1}=c_1^{s_1-1}c_2^{1-s_1}\left(1+s_1\ln\left(\frac{c_1}{c_2}\right)\right)$$

and then

$$1+s_1\ln\left(\frac{c_1}{c_2}\right)>0$$

from which:

$$e>\left(\frac{c_2}{c_1}\right)^{s_1}$$

substituting the steady state values of c_1 and c_2, leads to:

$$e^{\frac{1}{s_1}}s_1+s_1-1>0$$

as $e^t>t$, for $t>1$, the following always holds:

$$\frac{e^t}{t}+\frac{1}{t}-1>0$$

■

E.

Proof. We now proof that $\frac{\partial^2 C}{\partial c_2 \partial s_1} < 0$. From (1)

$$\frac{\partial C}{\partial c_2} = (1 - s_1)\, c_1^{s_1} c_2^{-s_1}, 0 < s_1 < 1$$

and in steady state:

$$\frac{c_1}{c_2} = \frac{s_1}{1 - s_1}$$

The derivative of $\frac{\partial C}{\partial c_2}$ with respect to s_1 is:

$$\frac{\partial^2 C}{\partial c_2 \partial s_1} = -c_1^{s_1} c_2^{-s_1} \left(1 - (1 - s_1) \ln\left(\frac{c_1}{c_2}\right)\right)$$

and it proves that

$$1 - (1 - s_1) \ln\left(\frac{c_1}{c_2}\right) > 0$$

it follows that:

$$e > \left(\frac{c_1}{c_2}\right)^{1 - s_1}$$

substituting the steady state values of c_1 and c_2 leads to:

$$e^{\frac{1}{1 - s_1}} (1 - s_1) - s_1 > 0$$

Setting $t = \frac{1}{1 - s_1}$, leads again to:

$$\frac{e^t}{t} + \frac{1}{t} - 1 > 0$$

■

Figure 1: IRFs benchmark version

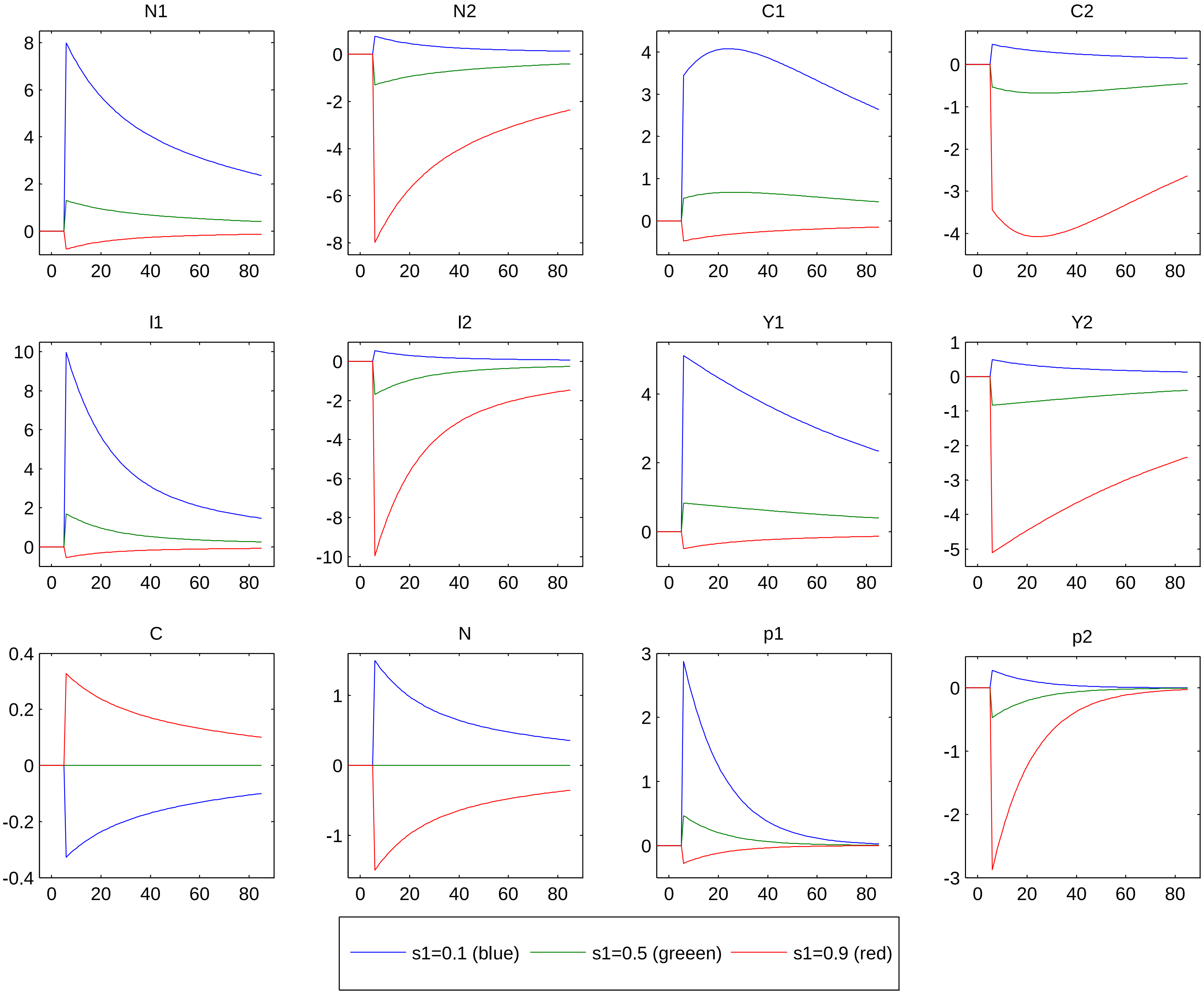
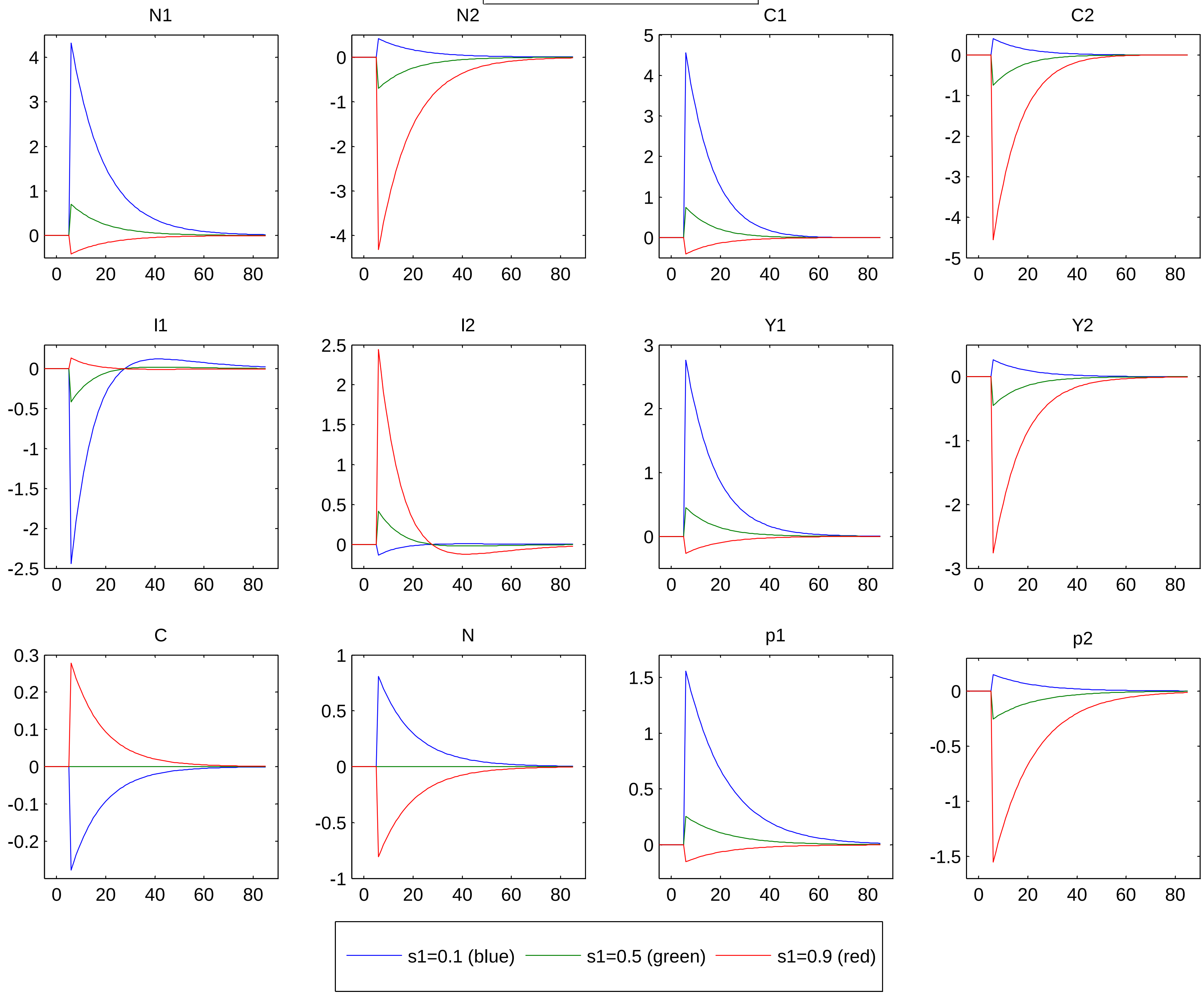


Figure 2: IRFs Low persistence



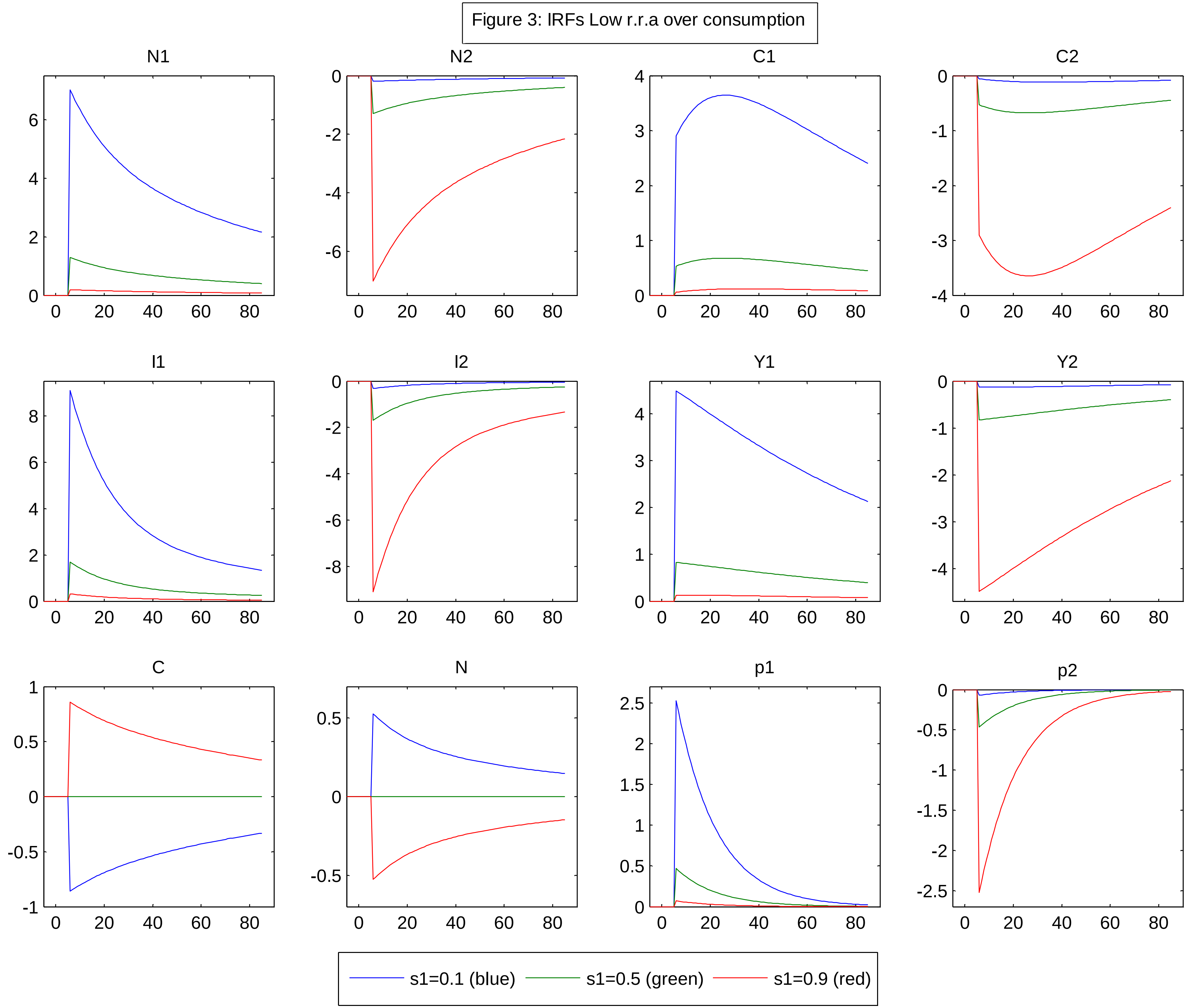


Figure 3: IRFs Low r.r.a over consumption
N1
N2
C1
C2
I1
I2
Y1
Y2
C
N
p1
p2
s1=0.1 (blue)
s1=0.5 (green)
s1=0.9 (red)

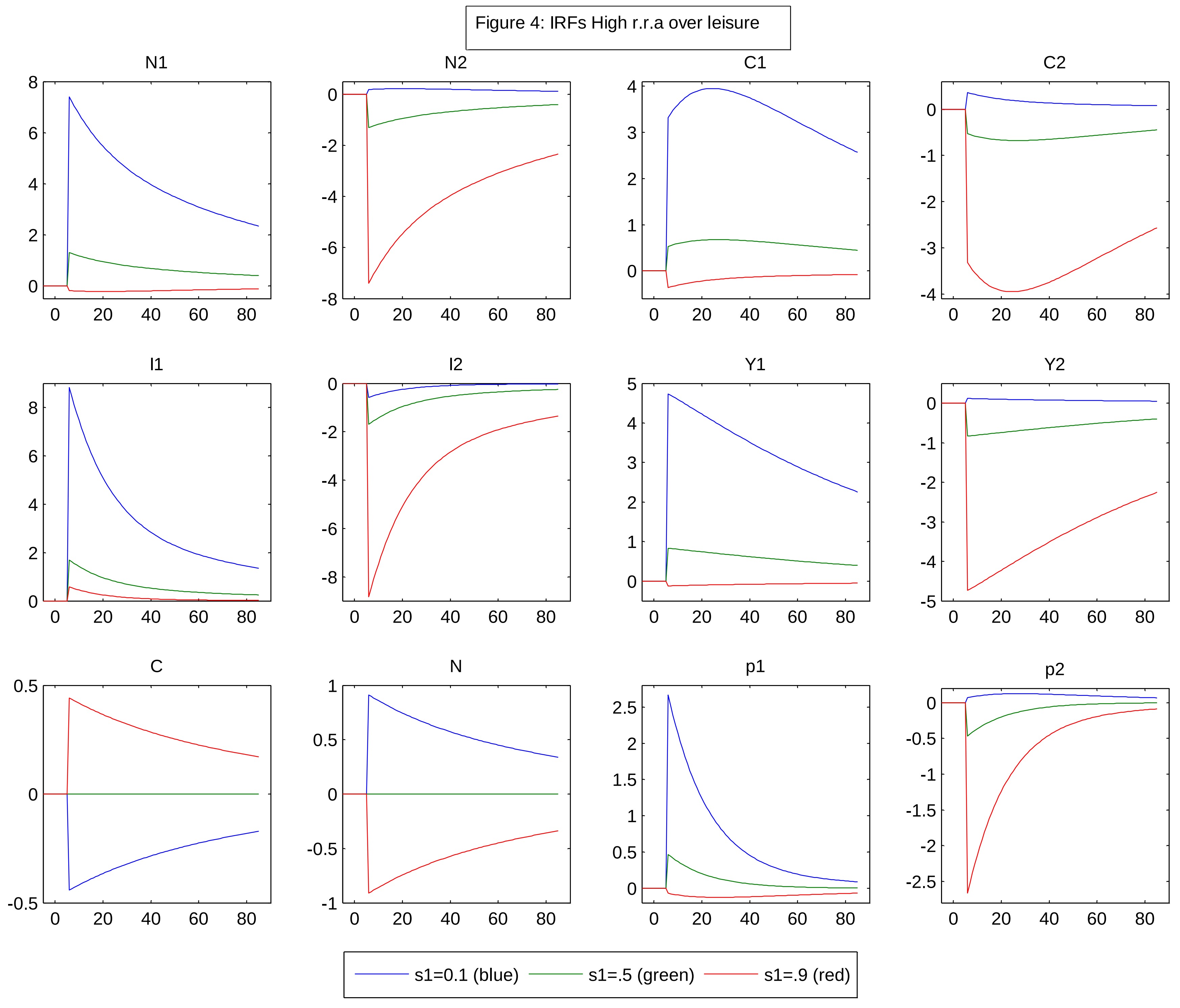


Figure 4: IRFs High r.r.a over leisure
N1
N2
C1
C2
I1
I2
Y1
Y2
C
N
p1
p2
s1=0.1 (blue)
s1=.5 (green)
s1=.9 (red)

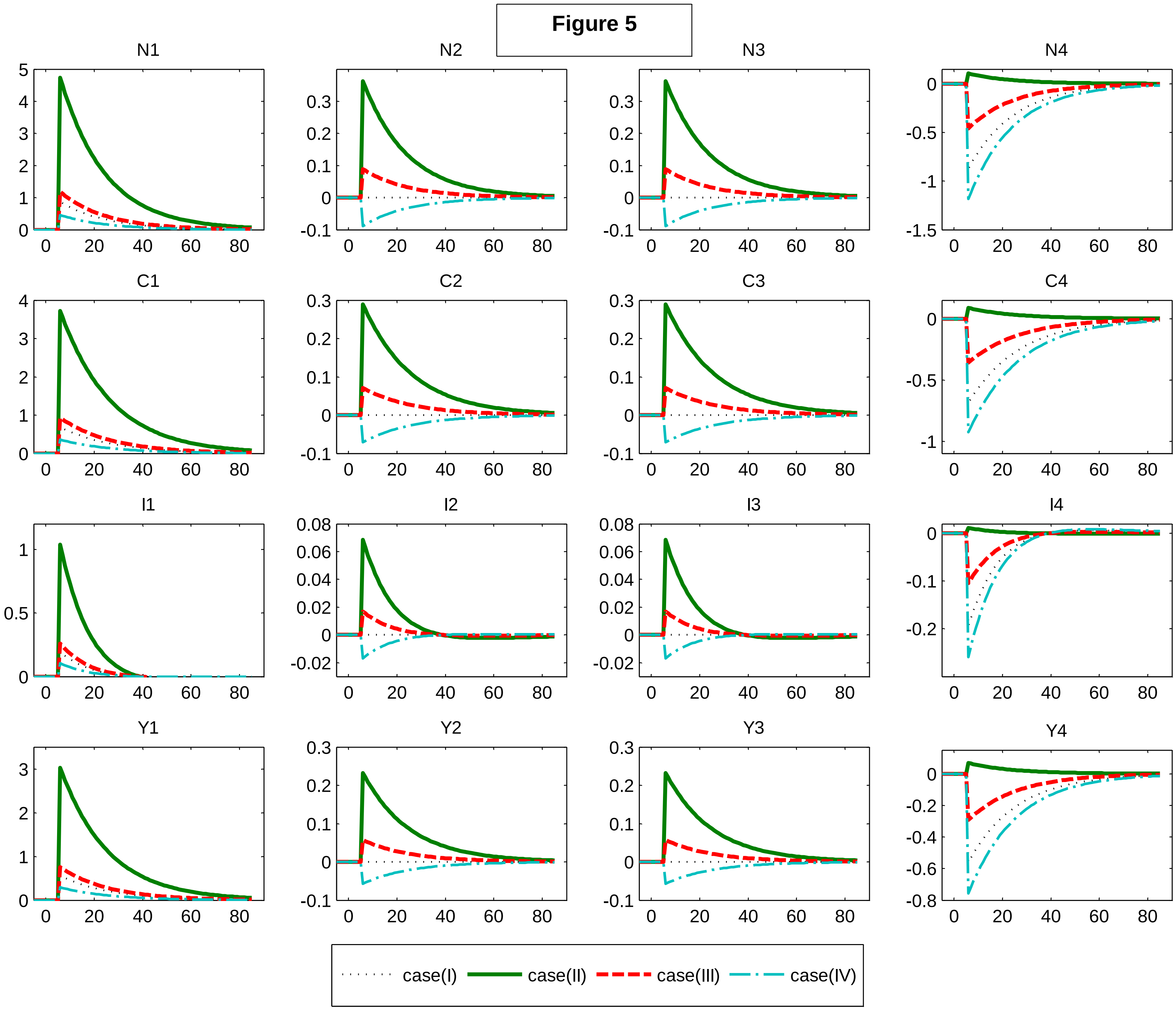
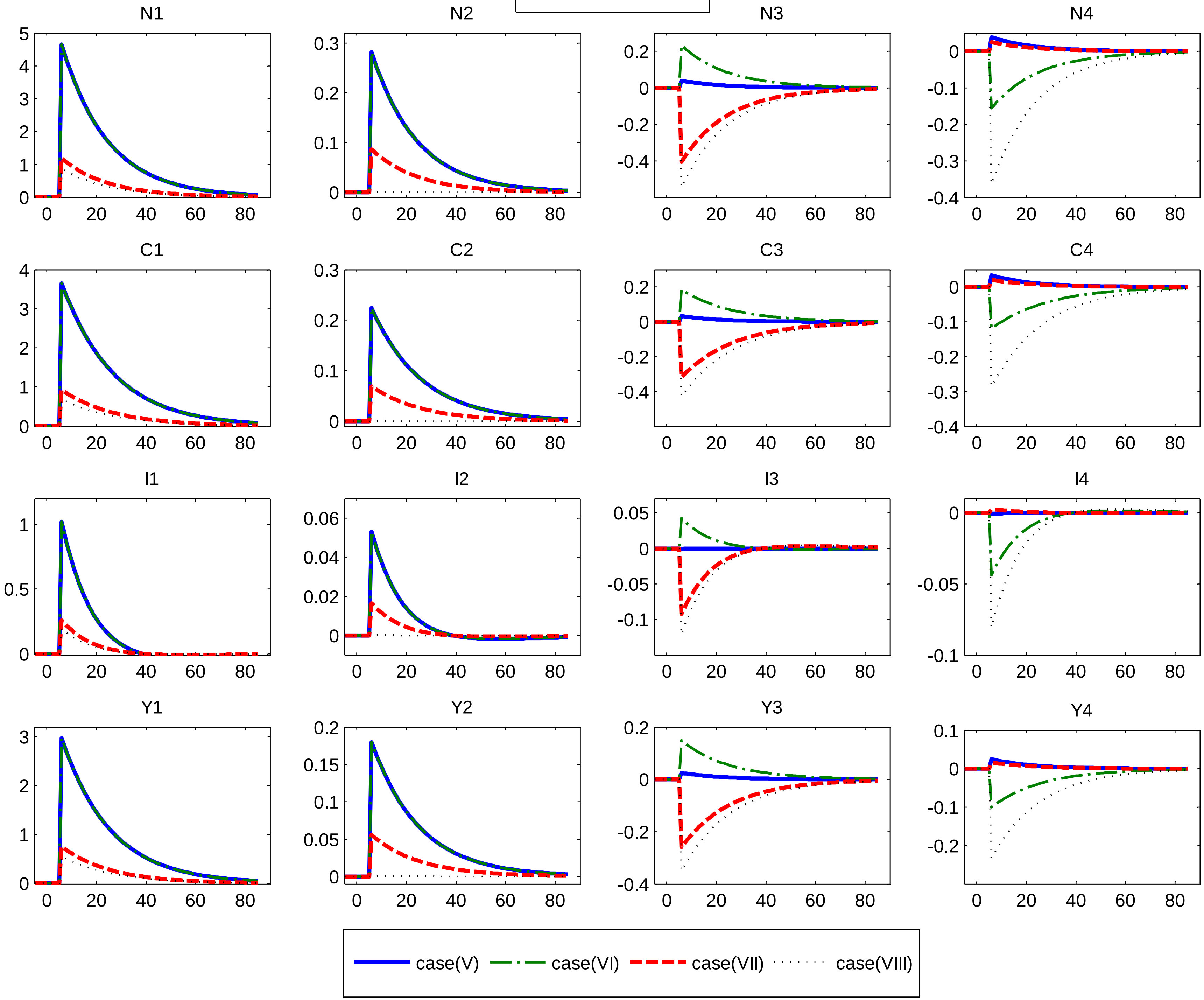


Figure 5
N1
N2
N3
N4
C1
C2
C3
C4
I1
I2
I3
I4
Y1
Y2
Y3
Y4
case(I)
case(II)
case(III)
case(IV)

Figure 6


N1
N2
N3
N4
C1
C2
C3
C4
I1
I2
I3
I4
Y1
Y2
Y3
Y4
case(V)
case(VI)
case(VII)
case(VIII)

Pubblicato in proprio
Dipartimento di Economia Pubblica
Facoltà di Economia
Università degli Studi di Roma “La Sapienza”
Via del Castro Laurenziano 9 – 00161 Roma

ISSN 1974-2940

Working Paper del Dipartimento di Economia Pubblica
Università degli studi di Roma "La Sapienza"
Via del Castro Laurenziano 9 – 00161 Roma