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A maximum entropy profile for the mesosphere(*)

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Summary. — Following a recent work by Verkley and Gerkema, we obtain a pressure-temperature profile for the mesosphere which reproduces rather well the *U.S. Standard Atmosphere*, 1976 data and the COSPAR *International Reference Atmosphere*, 1972 as well. This profile results from the maximization of entropy in a column of air in hydrostatic equilibrium, maintaining fixed the mass, the enthalpy and the vertical integral of the potential temperature.

PACS `92.60.-e` – Meteorology.
PACS `94.10.Fa` – Atmospheric composition (atomic or molecular), chemical reactions and processes.

1. – Introduction

In a recent paper, Verkley and Gerkema [1] henceforth denoted by VG, have proposed a first-principles variational method to generate a p-T profile for the troposphere. In this method one considers a column of dry air with unit horizontal area in hydrostatic equilibrium, bounded by two fixed values of the pressure, and one maximizes its entropy with the condition that the enthalpy and the vertical integral of the potential temperature are kept fixed. As the top and bottom pressures, p_2 and p_1, of the column are specified, the total mass of air is also fixed. Thus, denoting by

$$S = \frac{c_p}{g}\int_{p_2}^{p_1} \ln\theta \, \mathrm{d}p\,, \tag{1}$$

$$M = \frac{1}{g}\int_{p_2}^{p_1} \mathrm{d}p = \frac{1}{g}\left(p_1 - p_2\right)\,, \tag{2}$$

(*) The authors of this paper have agreed to not receive the proofs for correction.

$$H = \frac{c_p}{g} \int_{p_2}^{p_1} T \mathrm{d}p\,, \tag{3}$$

$$L = \frac{c_p}{g} \int_{p_2}^{p_1} \theta \, \mathrm{d}p\,, \tag{4}$$

the entropy, the mass, the enthalpy and the vertical integral of the potential temperature respectively. With c_p being the specific heat at constant pressure, g the gravitational potential, T the temperature, and θ the potential temperature given by

$$\theta = T \left(\frac{p_\mathrm{r}}{p} \right)^{\kappa}, \tag{5}$$

where p_r is a reference pressure, and $\kappa = R/c_p$ with R the gas constant. As said above, VG imposed the maximization of S, for M, H, and L fixed

$$\delta S + \lambda \delta H + \mu \delta L = 0, \tag{6}$$

with λ and μ being Lagrange multipliers. From (6) they obtained

$$\frac{1}{T} + \lambda + \mu \left(\frac{p_\mathrm{r}}{p} \right)^{\kappa} = 0\,. \tag{7}$$

Introducing $\alpha = \mu/\lambda$, and denoting the temperature at p_r by T_r, we have

$$T(p) = T_\mathrm{r} \frac{1 + \alpha}{1 + \alpha \left(p_\mathrm{r}/p \right)^{\kappa}}\,. \tag{8}$$

This profile lies between the isothermal profile ($\alpha \to 0$) and the isentropic profile ($\alpha \to \infty$). Following VG, α is determined by requiring that H/L that derives from (8) is equal to its empirical value. Afterwards, T_r is fixed by requiring that H (or L) is equal to its empirical value as well. The thus obtained theoretical p-T profile fits well the *U.S. Standard Atmosphere*, 1976 data for the troposphere [2].

In this paper we present how this method works for the mesosphere. This will be developed in sect. **2**. In sect. **3**, we write our conclusions.

2. – Application to the mesosphere

In the mesosphere, pressure and density are considerably smaller than in the troposphere and in the stratosphere, however, the similarity between the temperature distributions in troposphere and mesosphere suggests the existence of somewhat similar processes. Absorption of solar radiation in the region of the stratopause provides the energy source, and resulting motions with accompanying adiabatic expansion and cooling may account for the temperature distribution in this layer. T also drops with height: this lapse is maintained by weak vertical air motions. In the mesosphere there is a slight reduction in g with respect to its value near the surface. The composition of the dry air in the mesosphere, from a height $z = 50$ km up to say $z = 80$ km is the same as in the troposphere, *i.e.* R and c_p have the same value. Beyond $z = 80$ km, however, the

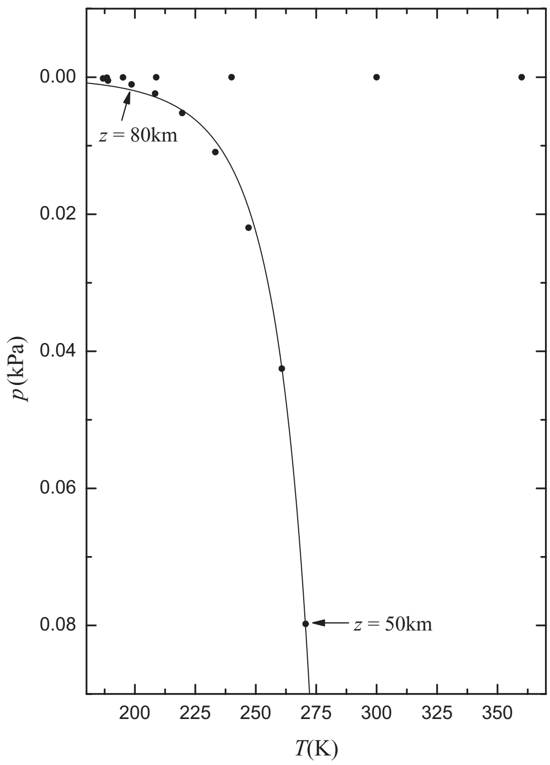

Fig. 1. – Pressure (kPa) *vs.* temperature (K) for the mesosphere. The *U.S. Standard Atmosphere*, 1976 [2] is represented by dots, and the solid line is the theoretical profile with $\alpha = 0.23223$. The arrows indicate the interval of height considered for the air column in the work. The height interval between consecutive dots is 5 km.

composition of air starts differing and for this reason we will extend our calculations only up to $z = 80$ km.

In figs. 1 and 2 we have plotted, for the mesosphere, the maximum-entropy theoretical profiles and the data given by the *U.S. Standard Atmosphere*, 1976 [2] (fig. 1), and the COSPAR *International Reference Atmosphere*, 1972 (CIRA) [3] (fig. 2). These data have been taken from Fleagle and Businger [4]. In both cases, we have worked with the column of air between 50 and 80 km, which amounts to seven (p, T) coordinates. The two pairs of coordinates corresponding to $z = 85$ km and $z = 90$ km have been discarded for the reason mentioned above. The values of the α parameters resulting from the method explained in VG, are $\alpha = 0.23223$ ($H/L = 0.75215$, $T_r = 270.53$ K, $p_r = 0.07978$ kPa) for fig. 1 and $\alpha = 0.26735$ ($H/L = 0.75332$, $T_r = 272.17$ K, $p_r = 0.08241$ kPa) for fig. 2. The two adjustments are slightly different but both are fairly good.

3. – Conclusions

It is needless to emphasize that the task of adjusting a theoretical p-T profile to the mesosphere is harder than a similar job for the troposphere. This is because, to begin with, the width of the troposphere (in middle latitudes) is about 10 km, while the

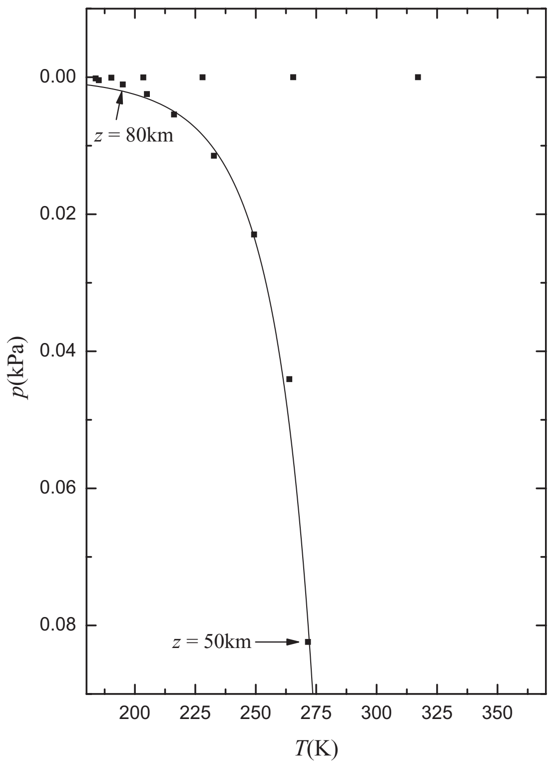

Fig. 2. – The same as fig. 1, but here the squares represent the COSPAR *International Reference Atmosphere*, 1972 [3] and the theoretical profile is obtained with $\alpha = 0.26735$.

width of the mesosphere is greater than 30 km. And a second reason is that, as in the mesosphere the air is much less dense, the accuracy of the data is worse. For this reason, we consider that our results are more than acceptable and strengthen the interest for this first-principles profile. Our next purpose is to try to fit the low-level structure of some planetary atmospheres using this method.

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