

The beaming factor and other open issues in GRB Jets^(*)

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Summary. — I review several central open questions concerning GRB Jets: i) I discuss a new estimate of the beaming correction for the rate of GRBs $\sim 75 \pm 20$; ii) I discuss the universal structured jet (USJ) model and conclude that while jets might be structured they are less likely to be universal; iii) I discuss recent observations of a sideways expansion of a GRB afterglow and compare these with current hydrodynamics simulations of jet evolution; iv) I discuss the implications of resented outliers to the energy-angle relation.

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1. – Introduction

The realization that the relativistic outflow in GRBs is in the form of jets has direct implication to their energy budget and their rates. This has, in turn, further implications on the nature of the inner engines. For example prior to the understanding that GRBs are beamed, events such as GRB 990123 with an isotropic equivalent energy of more than 10^{54} erg were hard to explain. With beaming the energy output of this event is a “mere” 10^{51} erg and it is compatible with a simple stellar mass progenitor.

Evidence of jetted GRBs arises from observations [1, 2] of the predicted achromatic breaks in the afterglow light curves [3, 4]. Further support is given by the comparison of long-term radio calorimetry with the energy of the prompt emission [5]. The time of the jet break provides an estimate of the jet angle [4]:

$$(1) \quad \theta = 0.16 \text{ rad} (n/E_{k,iso,52})^{1/8} t_{b,days}^{3/8} = 0.07 \text{ rad} (n/E_{k,\theta,52})^{1/6} t_{b,days}^{1/2}$$

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where $t_{b,days}$ is the break time in days and $E_{k,iso,52}$ is the “isotropic equivalent” kinetic energy in units of 10^{52} ergs, while $E_{k,\theta,52}$ is the real kinetic energy in the jet, *i.e.*: $E_{k,\theta,52} = (\theta^2/2)E_{k,iso,52}$.

Frail *et al.* [6] and Panaitescu and Kumar [7] have estimated the opening angles θ for several GRBs with known redshifts. They find that the total gamma-ray energy release, when corrected using a beaming factor, f_b ,

$$(2) \quad E_\gamma = f_b E_{\gamma,iso} \equiv \frac{\theta^2}{2} E_{\gamma,iso},$$

is clustered. This energy-angle relation is commonly called the Frail relation. A precursor of this discovery was found already in 1999 by Sari, Piran and Halprn [4] who found that the two brightest bursts known at that time were beamed. Bloom, Frail and Kulkarni [8] confirmed this clustering around $\sim 1.3 \times 10^{51}$ erg on a larger sample. This result is remarkable as it involves two seemingly unrelated quantities, θ that is determined from the break in the afterglow light curve and $E_{\gamma,iso}$ which is a property of the prompt emission. The fact that the product of these two unrelated quantities is a constant is, in my mind, an indication that our overall model is correct.

There are two leading models for the jet structure and for the interpretation of the jet break and the beaming angle. According to the original uniform jet model (UJ) the energy per solid angle is roughly constant within some finite opening angle, θ , and it sharply drops outside θ . Within the UJ model, the observed break corresponds to the jet opening angle, θ (see eq. (1)). The Frail relation implies, here, that the total energy released in GRBs is constant and that the differences in the isotropic equivalent energies arise from variations in the opening angles.

According to the alternative universal structured jet (USJ) model [9-11] all GRB jets are intrinsically identical. The energy per solid angle varies as a function of the angle from the jet axis. The jet break corresponds to the viewing angle of the observer and the Frail relation imposes a specific distribution of energy per unit angle in the jet:

$$(3) \quad \mathcal{E}(\theta) = \begin{cases} E_0/(\pi\theta^2), & \text{for } \theta > \theta_c, \\ E_0/(\pi\theta_c^2), & \text{for } \theta < \theta_c, \end{cases}$$

where E_0 is a constant and θ_c is the core angle of the jet [10]. While the UJ model contains a free function, the luminosity function or the corresponding opening angle distribution, the USJ model is completely determined by the Frail relation. Its apparent luminosity function has the form $\Phi(L) \propto L^{-2}$ [12]. We can use this lack of freedom within the USJ model to test it.

The question which model is the correct one is still an open one. There are many other open questions concerning the structure of the jets and their evolution. In this paper I review some of these open questions. I refer the reader to a recent review [13] for a more extended overview on GRBs in general and on GRB jets in particular.

2. – The beaming and the rate of GRBs

The overall GRB rate depends clearly on the amount that GRBs are beamed. Within the UJ model this has been measured traditionally in terms of the beaming correction factor, f_b^{-1} , which is defined as the ratio of total number of bursts to the observed ones. To estimate the overall GRB rate we need the average beaming correction $\langle f_b^{-1} \rangle$ such

that $n_{true} = \langle f_b^{-1} \rangle n_{obs}$. The average is performed over the observed distribution. Taking into account the fact that for every observed burst there are f_b^{-1} that are not observed, Frail *et al.*, [6] estimated the average beaming correction⁽¹⁾

$$(4) \quad \langle f_b^{-1} \rangle_{F01} = \frac{1}{N} \sum_i \frac{2}{\theta_i^2} \simeq 520 \pm 85,$$

where the sum is over the observed distribution.

However, this calculation overestimates the actual beaming correction. In the *intrinsic* luminosity distribution there are many low luminosity bursts that have large opening angles. These bursts dominate the rate estimate. However, they are rather weak and can be observed only to small distances. Hence they are under-represented in the observed distribution [14]. This effect can be taken into account in the following way. For a given burst with a luminosity L we define the volume from which such a burst can be detected:

$$(5) \quad V_L \equiv \int_0^{z(L)} dz (dV/dz) R_{GRB}(z) / [R_{GRB}(0)(1+z)],$$

where $R_{GRB}(z)$ is the comoving rate of GRBs and $z_m(L)$ is the maximal redshift from which a burst with a luminosity L can be detected. Similarly $V_\infty = V_{L=\infty}$ is the whole effective volume of the observable universe. The *intrinsic* beaming correction can be written as

$$(6) \quad \langle f_b^{-1} \rangle \equiv \frac{\sum_i (2\theta_i^{-2})(V_\infty/V_{L_i})}{\sum_i (V_\infty/V_{L_i})}.$$

This estimate is, of course, somewhat model dependent as it requires an assumption on R_{GRB} . Guetta, Piran and Waxman [14] find that for several models in which GRBs follow the SFR $\langle f_b^{-1} \rangle_{int} = 75 \pm 25$, about a factor of 8 smaller than the previous estimate of 520 ± 85 that did not take this effect into account.

Following [14] we can also estimate the beaming correction for the rate of GRBs within the USJ model in the following way. The total flux of GRBs per year (or per any other unit of time) is an observed quantity obtained by summing over the observed distribution. A comparison of this total flux with the total energy emitted by a single burst can tell us directly the total number of bursts. Integrating over the energy distribution (eq. (3)) we obtain the total energy that a burst with a USJ emits:

$$(7) \quad E_{USJ} = 2 \left[\int_0^{\theta_c} (E_0/\theta_c^2)^2 \theta d\theta + \int_{\theta_c}^{\theta_{max}} E_0 \theta^{-1} d\theta \right] = E_0 [1 + 2 \log(\theta_{max}/\theta_c)],$$

where θ_{max} is the maximal angle to which the jet extends. This immediately implies that the ratio of UJs (emitting each E_0) to USJs (emitting each $E_0[1 + 2 \log(\theta_{max}/\theta_c)]$)

⁽¹⁾ Frail *et al.*, [6] estimate the observed beaming factor distribution as $p_{obs}(f_b) = (f_b/f_0)^{\alpha+1}$ and $(f_b/f_0)^{\beta+1}$ for $f_b \leq f_0$ respectively. They find that α is poorly constrained by the data while $\beta = -2.77_{-0.3}^{+0.24}$ and $\log(f_0) = -2.91_{-0.06}^{+0.07}$ and obtain $\langle f_b^{-1} \rangle_{F01}$ by integrating over this distribution.

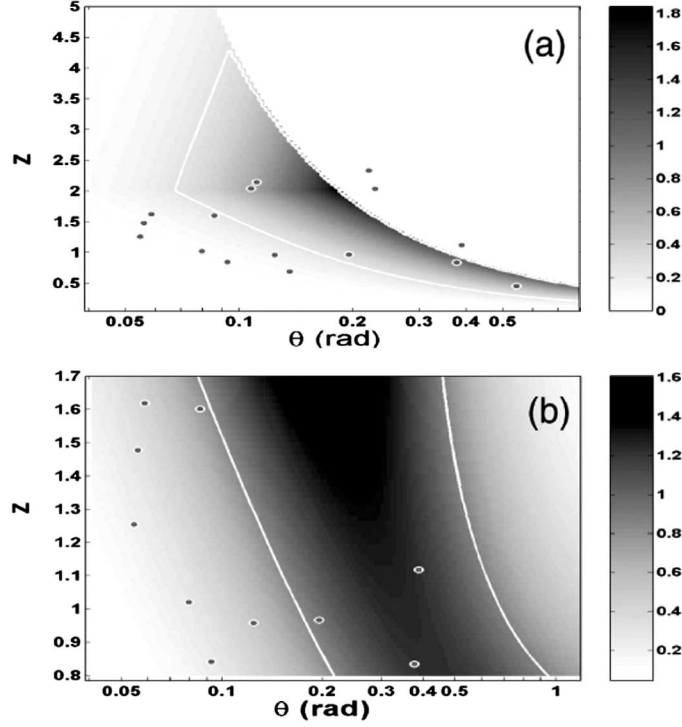


Fig. 1. – (a): The 2D distribution density, $dn(z, \ln \theta)/dz d \ln \theta$, of the GRB rate as a function of z and $\ln \theta$ in the USJ model. The white contour lines confine the minimal area that contains 1σ of the total probability. The circles denote 16 bursts with known z and θ [8]. (b): A limited redshift range, $0.8 < z < 1.7$ (containing 10 out of the 16 data points) in which both redshift selections effects and the sensitivity to the unknown GRB rate are minimized. From [15].

required to explain the observed flux is

$$(8) \quad N_{\text{UJ}}/N_{\text{USJ}} = [1 + 2 \log(\theta_{\text{max}}/\theta_c)].$$

The upper and lower limits of this integral are uncertain but the logarithmic dependence implies that the factor cannot be smaller than 2 or much larger than 5. This implies that the number of USJs required to produce the observed flux is about factor of 4 below the corresponding number of UJs. Hence, the average beaming correction for USJs is $\sim 20 \pm 10$.

3. – Universal structured jets

One of the intriguing open questions concerning GRB jets is their angular distribution. The two leading models are the UJ and USJ discussed earlier. The differences between USJ and UJ have crucial implications to the question of the nature of GRBs' inner engines and their progenitors. First, the universality of the USJ requires that more or less the same process operates with the same parameters within different GRBs. Second, a USJ carries roughly five times more energy than a UJ. This implies, for a USJ, an

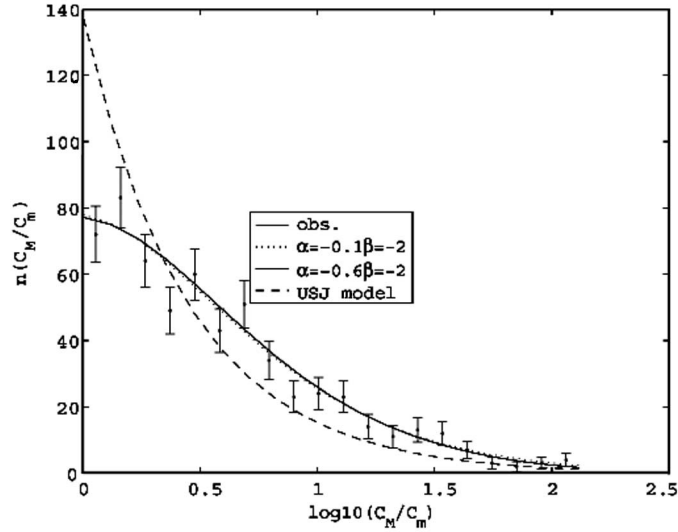


Fig. 2. – The predicted differential C_{\max}/C_{\min} distribution for the USJ model and the observed distribution taken from the BATSE catalog. The inconsistency at the low peak flux range is apparent. Predicted differential distributions for different parameterizations of the luminosity function within the UJ model are also shown. From [14].

energy budget of $\sim 10^{52}$ erg. The rate of USJs is, correspondingly, smaller by a factor of five. It is therefore, important to ask whether there are observations that can distinguish between the two models.

Perna, Sari and Frail [12] calculated the expected distribution of the observed opening angles within the USJ model assuming that GRBs follow the SFR. Following them we define the number of bursts in the interval $(\theta, \theta + d\theta)$ and $(z, z + dz)$ as

$$(9) \quad \frac{dn}{d\theta dz} d\theta dz = \sin\theta \frac{R_{GRB}(z)}{(1+z)} \frac{dV(z)}{dz} \mathcal{T}(\theta, z),$$

where, $V(z)$ is the comoving volume element and $\mathcal{T}(\theta, z)$ depends on the distribution of GRB duration (see [12, 15] for details). Perna *et al.*, [12] integrated over the redshift distribution and found, with reasonable assumptions on \mathcal{T} a remarkable agreement between the expected angular distribution $dn(\theta)/d\theta \equiv \int (dn/d\theta dz) dz$ and the observed angular distribution, lending a strong support to the USJ model. However, Nakar, Granot and Guetta [15] compared of the two dimensional distribution $dn/d\theta dz$ with the observed one. They found (see fig. 1) that the two distribution disagree strongly. The observed points are very far from the location of the peak of the expected distributions. Some are even in a non-allowed region. This implies that the agreement between the observed angle distribution and the one predicted by the USJ model was just a coincidence and should not be taken as supporting this model.

A similar discrepancy arose when Guetta *et al.*, [14] compared the observed count (C_{\max}/C_{\min}) distribution for the BATSE long bursts sample with the one expected from the USJ model. Guetta *et al.*, [14] found that the USJ model predicts a significant excess of weak bursts as compared with the observed distribution see (fig. 2).

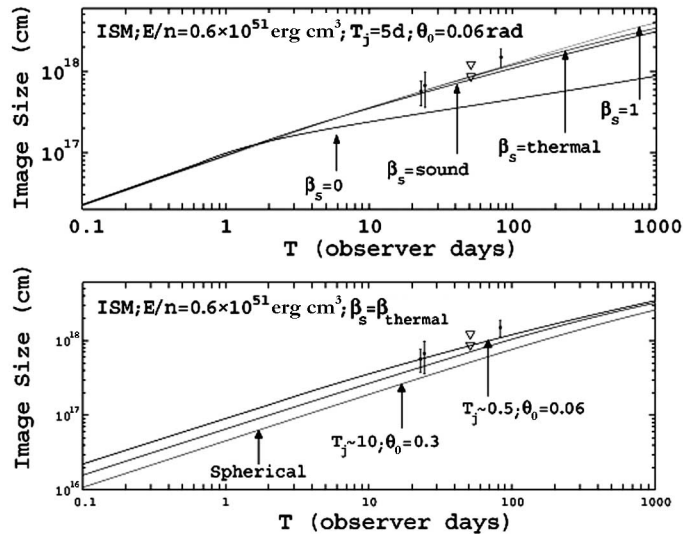


Fig. 3. – *Upper panel:* R_{\perp} as a function of T_d for different sideways expansion models in ISM. The energy to external density ratio $E/n = 0.6 \cdot 10^{51} \text{ erg cm}^3$ and $\theta_0 = 0.06 \text{ rad}$. *Lower panel:* R_{\perp} as a function of T_d for different opening angles in ISM, with a constant $E/n = 0.6 \cdot 10^{51} \text{ erg cm}^3$. T_j is in days, $\beta_s = \beta_{\text{thermal}}$. From [18].

4. – Jet evolution

Selection effects and uncertainties mean that these discrepancies are insufficient to rule out the USJ model. Still these discrepancies certainly imply that the agreement found for the angular distribution alone cannot be used, as hoped, to demonstrate the validity of this model. If it will turn out that these conclusions hold with additional data, we will have to reconsider the validity of the USJ. Both discrepancies could be removed if we consider Structured Jets that are not universal. That is if we allow an angular dependence of the flow parameters (such as Γ and/or E) but we do not require that all jets are similar. Namely we could replace USJ with SJ. While such a solution is viable it clearly takes away some of the simplicity and the elegance of the USJ mode.

An important question that determines the observed light curve of the afterglow is the sideways expansion of the jet after $\Gamma \sim \theta^{-1}$, where Γ is the Lorentz factor. The observations of the radio afterglow of GRB 030329 provided a unique opportunity to test this issue. Taylor *et al.*, [16,17] have measured the size of the radio afterglow of GRB 030329 between 20 and 300 days after the burst. Oren, Nakar and Piran [18] (see also [19]) compared the observed sizes with several schematic models of spherical and jetted propagation. The remarkable result is that the apparent size of the afterglow is rather insensitive to the details of the model. This robustness is a good indication for the validity of the model, as it is difficult to force it to have different values and the values obtained fit the observations. On the other hand it is a drawback when one wishes to use the size to determine the parameters of the outflow. It is insensitive to these parameters.

Oren *et al.*, [18] find that the image of a spherically expanding fireball is largest with expansion as $t^{0.6}$, while the size of a sideways expanding jet (at $v \approx c$) increases as $t^{0.5}$. As can be seen in fig. 3 there is practically no difference between an expansion at the

speed of light or at the sound speed, $c\sqrt{3}$. On the other hand the size of a non-expanding jet increases only as $t^{0.25}$. A comparison of these models to the observations (see fig. 3) shows that the non-expanding jet model is inconsistent with the observations. While an addition of the faster expansion during the Newtonian phase [19] can somewhat alleviate the problem, it is not clear that this can be done with reasonable parameters [18].

Based on simple analytic model Sari Piran and Halpern [4] estimated that the relativistic jet will expand sideways almost at the speed of light. On the other hand Panaitescu and Meszaros [20] estimated that the jet will expand only at the sound speed, $c/\sqrt{3}$. Both are consistent with the observations. More recently Kumar and Granot [21] integrated the hydrodynamic equations over the radial direction and obtained one dimensional simplified hydrodynamics equations. Solving these equations they find no or little sideways expansion. Similar results were obtained in a two dimensional numerical integration of the full hydrodynamic equations [22, 23]. Remarkably this does not influence much the observed light curve (see [22]).

We are left with a puzzle why do the numerical simulation indicate little or no expansion while the observations suggest a rather rapid sideways expansion. One may guess that the current computations are not refined enough to follow the jet evolution (see [24] for a detailed discussion of the problematics of these computations). We probably have to wait for a higher resolution codes to resolve this problem. Another possibility is that the size of the radio afterglow does not trace well the size of the expanding jet. Detailed emission computations have to be carried out to determine this possibility.

5. – The implication of outliers to the frail relation and a speculation

We begun stressing the importance of the Frail (energy-angle) relation. However, Berger *et al.*, [25] pointed out that in addition to the well known weak GRB 980425 there are three other outliers to this relation GRBs 980326, 980519 and 030329. Intriguingly enough all outliers are weaker relative to the common value of $\sim 10^{51}$ erg. I would like to suggest a simple explanation to this phenomenon. It seems that the common value indicates indeed the available energy reservoir. Under optimal conditions a significant fraction of this energy is released as γ -rays. An intriguing question, incidentally, is how is this conversion so efficient? These efficient cases produce the brightest and easiest to detect bursts. The less efficient case are less powerful (in γ -rays) and hence are easily missed. GRB 980425 would not have been discovered if it was not so close. Another outlier, GRB 030329, is also nearby at $z = 0.168$. The redshift of the other two outliers is unknown. Thus, I conclude with a speculation that as time progresses and more energies, redshifts and jet angles will become available it will turn out that the Frail relationship is satisfied as an inequality with $\sim 10^{51}$ erg being the upper limit to the γ -ray energy.

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