

Magnetic fireball: The acceleration efficiency of hydromagnetic outflows in GRB sources^(*)

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Summary. — Gamma-ray bursts (GRBs) have been inferred to arise in highly collimated, ultrarelativistic jets that emanate from the vicinity of a solar-mass compact object. Electromagnetic stresses are the most plausible candidate for extracting rotational energy at the source and converting it into outflow kinetic energy. Two questions that need to be answered in order for this process to be well understood are: what determines the terminal Lorentz factor of the flow? What is the asymptotic value of the Poynting-to-matter energy flux ratio? We discuss the general characteristics of the relativistic magnetohydrodynamic (MHD) solutions that, together with previously obtained exact results, help to shed light on these questions.

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1. – Introduction

According to the internal/external shock scenario the prompt GRB as well as the afterglow energy is stored, just before the emission, as kinetic energy of a relativistically moving outflow (see [1] for an alternative scenario in which the emitted energy comes directly from a Poynting flux). In this picture, the Lorentz factor of the outflow should be of the order of a few hundreds to avoid the compactness problem, and the baryonic mass of the order of $\mathcal{E}/\gamma c^2 \sim 10^{-6} M_{\odot}$. A question arises on how this mass was accelerated to such high velocities. One possible answer is related to a thermal fireball produced by viscous dissipation inside an accretion disk and subsequent escape of neutrinos. However, in this picture the photospheric emission in the outflow would have been detectable [2], in contrast with the observations.

A plausible alternative is the case when the outflow near the disk is Poynting-dominated and the Lorentz force transfers the energy from the electromagnetic field to the outflowing

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matter. Furthermore, if the outflow is largely Poynting flux-dominated, then the implied lower radiative luminosity near the origin could alleviate the baryon contamination problem. Another important aspect of magnetic driving is that in the case of neutron rich outflows, contrary to the purely hydrodynamic flow, the neutrons decouple at a Lorentz factor that is over an order of magnitude smaller than the γ_∞ for the protons, (see [3] and implications by [4]).

Assuming that all the ejected energy ($\mathcal{E} \sim 10^{51}$ ergs) is coming from a disk with surface $\sim 10^{13}$ cm² that lasts for ~ 10 s, we find that a magnetic field of the order of 10^{14} G is required (this value is somewhat larger when the energy is extracted from a black hole via the Blandford and Znajek mechanism, because in that case the area is smaller). Since typical rotation periods ($\sim 10^{-4} - 10^{-3}$ s) are much shorter than typical burst durations, steady-state is a safe assumption. From a different viewpoint, the ultrarelativistic velocities make different parts of the outflow causally disconnected, allowing the study of each part of the flow using steady-state MHD (frozen-pulse approximation, see [5]).

In the following sections we analyze the MHD equations and give analytical estimates for the asymptotic Lorentz factor, the Poynting-to-matter energy flux ratio, and how these depend on the conditions near the disk.

2. – Hydromagnetic equations

The system of equations of special relativistic, steady, ideal MHD, consist of the Maxwell equations $0 = \nabla \cdot \mathbf{B} = \nabla \times \mathbf{E} = \nabla \times \mathbf{B} - 4\pi \mathbf{J}/c = \nabla \cdot \mathbf{E} - 4\pi J^0/c$, Ohm's law $\mathbf{E} = \mathbf{B} \times \mathbf{V}/c$, the continuity $\nabla \cdot (\rho_0 \gamma \mathbf{V}) = 0$, and force-balance $-\gamma \rho_0 (\mathbf{V} \cdot \nabla) (\xi \gamma \mathbf{V}) - \nabla P + (J^0 \mathbf{E} + \mathbf{J} \times \mathbf{B})/c = 0$ equations. Here \mathbf{V} is the velocity of the outflow, γ the associated Lorentz factor, (\mathbf{E}, \mathbf{B}) the electromagnetic field as measured in the central object's frame, $J^0/c, \mathbf{J}$ the charge and current density respectively, ρ_0 the gas rest-mass density in the comoving frame, and $\xi c^2 = c^2 + 4P/\rho_0$ the relativistic specific enthalpy.

Assuming axisymmetry ($\partial/\partial\phi = 0$, in cylindrical $[z, \varpi, \phi]$ coordinates), five conserved quantities along the flow exist. If $A = (1/2\pi) \int \mathbf{B}_p \cdot d\mathbf{S}$ is the poloidal magnetic flux function (with the inverse relation giving the poloidal magnetic field $\mathbf{B}_p = \nabla A \times \hat{\phi}/\varpi$), these are (*e.g.*, [5])⁽¹⁾: the field angular velocity

$$(1) \quad \frac{V_\phi}{\varpi} - \frac{V_p}{\varpi} \frac{B_\phi}{B_p} = \frac{c}{\varpi} \frac{E}{B_p} = \Omega(A),$$

Michel's magnetization parameter $\sigma_M(A)$ which is related with the magnetic-to-mass flux ratio $B_p/\gamma\rho_0 V_p = (4\pi c^3/A\Omega^2)\sigma_M$, the total specific angular momentum $\xi\gamma\varpi V_\phi - (\sigma_M c^3/A\Omega^2)\varpi B_\phi = L(A)$, the entropy $P/\rho_0^{4/3} = Q(A)$, and the total energy-to-mass flux ratio

$$(2) \quad \xi\gamma c^2 - \frac{\sigma_M c^3}{A\Omega} \varpi B_\phi = \mu(A) c^2.$$

The left-hand side of eq. (2) consists of the matter energy-to-mass flux ratio $\xi\gamma c^2$, and the Poynting-to-mass flux ratio $(\mu - \xi\gamma) c^2$. Note that since the B_ϕ component is negative (required in order to have $V_\phi < c$ beyond the light cylinder; see eq. (1)) the contribution of the electromagnetic field in eq. (2) is positive.

⁽¹⁾ The subscripts p/ϕ denote poloidal/azimuthal components.

Using the above expressions of the five integrals of motion we may express all the flow quantities as functions of γ and A . The two equations that give the remaining unknowns γ and A are the Bernoulli and transfield force-balance equations (*e.g.*, Appendix A in [5]).

3. – The asymptotic Lorentz factor

Equation (2) implies that the maximum value of the Lorentz factor is the constant of motion μ . It also shows the two acceleration mechanisms causing an increase of γ : (1) thermal, corresponding to a decreasing specific enthalpy ξc^2 (thermal fireball phase), and (2) magnetic, corresponding to a decline in the poloidal current $I = (c/2)\varpi B_\phi$. The former works up to the point where ξ gets its minimum value ($= 1$), while the latter is the result of the Lorentz force and its final outcome depends on the value of the asymptotic current I_∞ .

We may re-write eq. (2) as $\xi\gamma/\mu = 1 - \sigma_M c^3 \varpi |B_\phi| / \mu A \Omega = 1 - 2\sigma_M |I| / \mu A \Omega$. Downstream of the classical fast magnetosonic surface (where the poloidal flow speed equals the phase speed of the fastest magnetosonic waves propagating along the flow) the magnetic field is mainly toroidal and, for a highly relativistic poloidal flow ($V_\phi \ll V_p$), eq. (1) gives $B_\phi \approx -\varpi B_p \Omega / c$. As a result, the poloidal current is $|I| \approx B_p \varpi^2 \Omega / 2$ and the matter-to-total energy flux depends on the key function $B_p \varpi^2 / A$:

$$(3) \quad \frac{\xi\gamma}{\mu} \approx 1 - \frac{\sigma_M}{\mu} \frac{B_p \varpi^2}{A}.$$

For an initially Poynting-dominated flow the matter part of the energy flux is negligible, $\xi_i \gamma_i / \mu \ll 1$ ⁽²⁾. This continues to be the case in the neighborhood of the classical fast magnetosonic surface (*e.g.*, [6]). Suppose that the value of the function $B_p \varpi^2 / A$ near the classical fast surface is $(B_p \varpi^2 / A)_f$. Since $\xi\gamma \ll \mu$ at this point, eq. (3) implies that the constant of motion $\sigma_M / \mu \approx 1 / (B_p \varpi^2 / A)_f$.

Denoting with $\delta\ell_\perp$ the distance between two neighboring poloidal field lines A and $A + \delta A$, magnetic flux conservation implies $2\pi\delta A = B_p(2\pi\varpi\delta\ell_\perp)$, or, $B_p \varpi^2 = (\varpi / \delta\ell_\perp) \delta A$. Thus, a decreasing $B_p \varpi^2$ —and hence, from eq. (3), an accelerating flow—corresponds to poloidal field lines expanding in a way such that their distance $\delta\ell_\perp$ increases faster than ϖ . How fast the field lines expand is determined by the transfield force balance equation; thus, this equation indirectly determines the flow acceleration. Since the available solid angle for expansion of the field lines is finite, there is a minimum value of the $B_p \varpi^2 / A$ function. The field lines asymptotically have a shape $z \approx z_0(A) + \varpi / \tan \vartheta(A)$, where $\vartheta(A)$ is their opening angle [7]. Differentiating the latter equation we get a decreasing function $B_p \varpi^2 / A = (A\vartheta' / \sin \vartheta - A z'_0 \sin \vartheta / \varpi)^{-1}$, reaching a minimum value $\sin \vartheta / A\vartheta'$ at $\varpi \gg z'_0 \sin^2 \vartheta / \vartheta'$. Since the factor $\sin \vartheta / A\vartheta'$ is ~ 1 , the minimum value of the $B_p \varpi^2 / A$ function is ~ 1 , corresponding to

$$(4) \quad \frac{\gamma_\infty}{\mu} \approx 1 - \frac{\sigma_M}{\mu} \left(\frac{B_p \varpi^2}{A} \right)_\infty \approx 1 - \frac{(B_p \varpi^2)_\infty}{(B_p \varpi^2)_f} \sim 1 - \frac{\sigma_M}{\mu} \sim 1 - \frac{1}{(B_p \varpi^2 / A)_f}.$$

Equivalently, the asymptotic Lorentz factor is $\gamma_\infty \sim \mu - \sigma_M$, and the asymptotic Poynting-to-mass flux ratio is $\sim \sigma_M c^2$.

⁽²⁾ The subscript i denotes values at the origin of the flow.

Another interesting connection with the boundary conditions near the source can be found by using the relation between the poloidal current and the function $B_p\varpi^2/A$: $|I| = B_p\varpi^2\Omega/2$. Thus, $(B_p\varpi^2/A)_f \approx 2|I|_f/A\Omega$, and since $|I|$ remains practically a constant of motion inside the force-free subfast regime, $(B_p\varpi^2/A)_f \approx 2|I|_i/A\Omega$, and $\mu/\sigma_M \approx 2|I|_i/A\Omega$. Hence, equation (4) implies a direct connection of the acceleration efficiency and the asymptotic Lorentz factor to the ejection characteristics

$$(5) \quad \frac{\gamma_\infty}{\mu} \approx 1 - \frac{A\Omega}{2|I|_i} \left(\frac{B_p\varpi^2}{A} \right)_\infty \sim 1 - \frac{A\Omega}{2|I|_i}.$$

4. – Methods to obtain solutions

Although the above analysis gives an analytical relation between the conditions near the origin of the flow and the asymptotic flow speed, we need to solve the system of the remaining equations (Bernoulli and transfield force-balance) in order to find the $B_p\varpi^2/A$ function and thus how fast the Lorentz factor reaches its asymptotic value. In the following we review known methods to obtain solutions.

4.1. Numerical methods. – The Bernoulli is an algebraic equation for γ . After substituting its solution (in terms of A and its derivatives) in the transfield force-balance equation, we get a second order partial differential equation for the magnetic flux function A . Its solution determines the field-streamline shape on the poloidal plane. Due to the fact that this equation is of mixed type, *i.e.*, changes from elliptic to hyperbolic, it is beyond the capability of existing numerical codes to solve this highly nonlinear problem, and no solution has been obtained so far.

An alternative numerical approach is to solve the time-dependent problem (hyperbolic in time). However, all existing codes fail to simulate relativistic magnetohydrodynamic flows for more than a few rotational periods. On top of that, it is not easy at present to construct a solution ranging from scales of the order of the light cylinder distance up to the asymptotic regime.

A promising combination of the two above methods is followed by [8], who solves the inner problem using time-dependent evolution (avoiding the elliptic to hyperbolic transitions), and the outer problem using steady-state equations. The code is not yet capable of solving the problem at large distances, though.

4.2. The force-free assumption. – In the force-free limit inertial terms are ignored compared to the electromagnetic field terms. This assumption, however, brakes down in the superfast regime, where the flow becomes hyperbolic and the back reaction of the matter to the field cannot be neglected. Since at the classical fast surface the flow is still Poynting-dominated the force-free method cannot be used for examining the efficiency of the magnetic acceleration.

4.3. The prescribed field line shape assumption. – If one assumes a known magnetic flux distribution, *i.e.*, a known function $B_p\varpi^2/A = \varpi|\nabla A|/A$, then it is trivial to solve the Bernoulli equation for the flow speed⁽³⁾ (*e.g.*, [9,10]). Thus, when we use this method, practically we implicitly give the function γ . However, these solutions do not satisfy the transfield force-balance equation; thus, they are not fully self-consistent.

⁽³⁾ In the superfast regime, this equation reduces to the much simpler eq. (3).

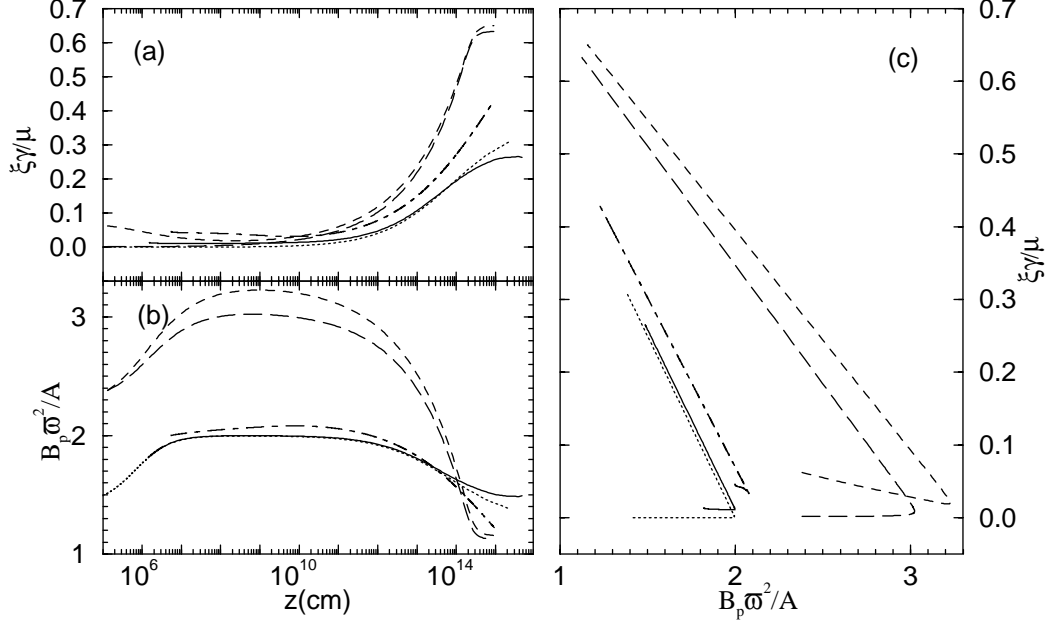


Fig. 1. – Acceleration efficiency in (a) and the function $B_p \varpi^2 / A$ in (b) vs. distance. (c): Efficiency vs. $B_p \varpi^2 / A$. In all plots *solid*, *dotted*, *dot-dashed*, *dashed*, and *long-dashed* lines correspond to the solutions a, b, d from [5] and a, b from [14], respectively. The value of σ_M / μ in these solutions is 0.49, 0.50, 0.47, 0.30, and 0.33. The classical fast point is located near the turning point in (c) and for the superfast part the results are in a very good agreement with eq. (3).

4.4. The monopole approximation. – The assumption that the poloidal magnetic field is quasi-monopolar is just a subcase of the prescribed field line case. This assumption, with the help of eq. (3), is equivalent to the assumption that γ is constant. In fact, a tiny acceleration is possible in the subfast regime, leading to $\gamma_\infty \sim \mu^{1/3}$ [11]. This solution gave the erroneous impression to the community that relativistic MHD is in general unable to give high acceleration efficiencies. However, the solution corresponds to a special case of boundary conditions, and most importantly, it does not satisfy the transfield force-balance equation.

Concluding, in order to solve the efficiency problem, one has to solve simultaneously the Bernoulli and transfield force-balance equations.

4.5. The r self-similar special relativistic model. – The only known exact relativistic MHD solution is the r self-similar special relativistic model, found independently by [12] and [13] in the cold limit, and further generalized by [5] including thermal and radiation effects. It corresponds to boundary conditions in a conical surface ($\theta = \theta_i$ in spherical coordinates $[r, \theta, \phi]$) of the form $B_r = C_1 r^{F-2}$, $B_\phi = -C_2 r^{F-2}$, $V_r = C_3$, $V_\theta = -C_4$, $V_\phi = C_5$, $\rho_0 = C_6 r^{2(F-2)}$, $P = C_7 r^{2(F-2)}$, with constant C_1, \dots, C_7 . The parameter of the model F controls the initial current distribution ($-\varpi B_\phi = C_2 \sin \theta_i r^{F-1}$ is an increasing or decreasing function of r for $F > 1$ or $F < 1$, respectively; see [5] for details). Despite the assumed form of the boundary conditions, the assumption that gravity is negligible,

and the absence of intrinsic scale, r self-similar remain the only self-consistent relativistic MHD solutions.

Figure 1 shows the function $B_p \varpi^2/A$ and the acceleration efficiency for various r self-similar solutions with application to GRB outflows. These exact solutions agree with the previously outlined general analysis, and in particular: 1) the asymptotic value of the function $B_p \varpi^2/A$ is close to 1; 2) the asymptotic value of $\xi\gamma/\mu$ is close to $1 - \sigma_M/\mu$; 3) in the superfast part of fig 1c the relation between $\xi\gamma/\mu$ and $B_p \varpi^2/A$ is linear, and almost identical with eq. (3).

5. – Discussion

Summarizing, it is important to note that in order to solve for the acceleration it is absolutely necessary to solve for the poloidal field line shape as well. The Bernoulli and transfield force-balance equations are interrelated and we cannot solve them separately, especially in the super-classical-fast regime. Models that assume quasi-monopolar magnetic field (Michel’s solution included), equivalently *assume* that the magnetic acceleration is inefficient. For other line shapes various acceleration laws have been found, however, further work is needed in order to check the validity of the prescribed field line models, in terms of satisfying the transfield force-balance equation.

In principle, magnetic fields provide a viable mechanism to accelerate GRB outflows. The acceleration efficiency is found to be of the order of $\sim 50\%$ using self-similar solutions. It is found analytically (and confirmed by the exact solutions) that the efficiency depends on the current distribution on the surface of the disk and its relation to the angular velocity of the field Ω and the magnetic flux A . Equivalently, the efficiency depends on the important function $B_p \varpi^2/A$, which is the solution of the transfield force-balance equation. As the field lines expand and become close to uniformly distributed asymptotically ($B_p \varpi^2 \sim A$), the acceleration efficiency depends on how bunched the lines are near the origin, *i.e.*, how large is the quantity $B_p \varpi^2/A$ at the classical fast surface.

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