The importance of off-axis beaming in jet models∗

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Summary. — Gamma-Ray Bursts (GRBs) are widely thought to originate from collimated jets of material moving at relativistic velocities. Emission from such a jet should be visible even when viewed from outside the angle of collimation. Using Monte Carlo population synthesis methods and including the effects of this off-axis beaming, we can compare various GRB jet models against the global properties of observed bursts. We explore whether or not the X-Ray Flashes (XRFs) seen by HETE-2 and BeppoSAX can be explained as classical GRBs viewed off-axis, and begin to address the more general question of the importance of off-axis beaming in current burst samples.

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1. – Introduction

The importance of collimated jets in GRBs was highlighted by the extremely large isotropic-equivalent energies of very bright events like GRB 990123 and by the observation of breaks in afterglow light-curves. [1] and [2] corrected the isotropic-equivalent energies by the beaming fraction obtained from afterglow light-curves and found that the values of $E_\gamma$ were clustered around $10^{51}$ ergs (although see [3]). Recent results from HETE-2 [4] have shown that XRFs [5,6], X-Ray–Rich GRBs and GRBs lie along a continuum of properties and that XRFs with known redshift extend the $E_{\text{peak}} \propto E_{\text{iso}}^{1/2}$ relation predicted by [7] and found by [8] to over 5 orders of magnitude in $E_{\text{iso}}$ [9].

Relativistic kinematics implies that even a “top-hat”-shaped jet will be visible when viewed outside the angle of collimation, $\theta_0$ [10]. [11,12] used this fact to construct a model where XRFs are simply classical GRBs viewed at an angle $\theta_v > \theta_0$. The authors showed that such a model could reproduce many of the observed characteristics of XRFs. [13] showed that in such a model, the distribution of both on- and off-axis observed bursts was roughly consistent with the $E_{\text{peak}} \propto E_{\text{iso}}^{1/2}$ relation.

In this paper, we explore further the possibility that the XRFs observed by \textit{HETE-2} and \textit{BeppeSAX} are primarily off-axis GRBs. Using and extending the population synthesis techniques presented by \cite{14} and \cite{15}, we present predictions for the global properties of bursts localized by \textit{HETE-2}. We show that it is difficult to account for the observed properties of XRFs by modelling them solely as regular GRBs viewed off-axis. However, since off-axis emission must exist solely on physical grounds, we seek to understand its properties of XRFs by modelling them solely as regular GRBs viewed off-axis. However, of bursts localized by \textit{HETE-2} presented by \cite{14} and \cite{15}, we present predictions for the global properties of bursts localized by \textit{HETE-2}. We revisit the model put forward in \cite{14}, since off-axis emission must exist solely on physical grounds, we seek to understand its relative importance in large burst populations. We note that rough constraints on the relative importance in large burst populations. We revisit the model put forward in \cite{14}, since off-axis emission must exist solely on physical grounds, we seek to understand its relative importance in large burst populations.

2. – Simulations

\cite{11-13} work with a fairly detailed model of the burst emission; for this work, we adopt a simpler model of off-axis beaming in GRB jets. We make no assumptions about the underlying physics generating the burst, and we make the approximation that the bulk of the emission comes directly from the edge of the jet closest to the viewing angle. We ignore all integrals over the surface of the jet and time-of-flight effects. Our model focuses on the kinematic transformations of two important burst quantities, $E_{\text{iso}}$ and $E_{\text{peak}}$, as a function of viewing angle.

Frequencies in the rest frame of the burst material will appear Doppler shifted by a factor, $\delta = \gamma (1 - \beta \cos \theta)$, where $\beta$ is the velocity of the bulk material and $\theta$ is the angle between the direction of motion and the source frame observer. The quantities $E_{\text{peak}}$ and $E_{\text{iso}}$ then transform as $E_{\text{peak}} \propto E_{\text{peak}} \delta^{-1}$ and $E_{\text{iso}} \propto E_{\text{iso}} \delta^{-3}$. For a burst viewed off-axis, these relations imply $E_{\text{peak}} \propto E_{\text{iso}}^{1/3}$. \cite{13} do not consider $E_{\text{iso}}$ to be fully bolometric and so derive a slightly different prescription for the off-axis relation.

We adopt an effective angular distribution of the emissivity, $\epsilon(\theta_v)$, which is uniform for $\theta_v < \theta_0$ and decreases for $\theta_v > \theta_0$:

\begin{equation}
\epsilon(\theta_v) = \frac{E_{\text{iso}}}{4\pi} = \begin{cases} A, & \text{if } \theta_v \leq \theta_0, \\
A \cdot (\delta/\delta_0)^{-3}, & \text{if } \theta_v > \theta_0,
\end{cases}
\end{equation}

where in this expression, $\delta = \gamma [1 - \beta \cos (\theta_v - \theta_0)]$, $\delta_0 = \gamma (1 - \beta)$ is the value of $\delta$ when $\theta_v = \theta_0$. $A$ is a normalization constant described below, $B = C_A \cdot (E_{\text{iso}}/10^{52} \text{ ergs})^{1/2}$, and $C_A$ is drawn from a narrow lognormal distribution. Hence, $E_{\text{peak}}$ obeys the $E_{\text{peak}} \propto E_{\text{iso}}^{1/3}$ relation inside the jet and $E_{\text{peak}} \propto E_{\text{iso}}^{1/3}$ outside the jet. We then define the “true” standard energy by integrating this emissivity over the entire sphere:

\begin{equation}
E_{\gamma}^{\text{true}} = 2 \cdot 2\pi \int_0^{\pi/2} \epsilon(\theta_v) \sin \theta_v \, d\theta_v = 4\pi A \left[ 1 - \cos \theta_0 + I(\gamma, \theta_0) \right].
\end{equation}

We define the $E_{\gamma}$ value inferred via the method of \cite{1} to be $E_{\gamma}^{\text{inf}} = 4\pi A (1 - \cos \theta_0)$. The presence of beaming implies $E_{\gamma}^{\text{inf}} \neq E_{\gamma}^{\text{true}}$. We fix our normalization constant, $A$, to match the original prescription used by \cite{1}, by drawing values for $E_{\gamma}^{\text{inf}}$ from a narrow lognormal distribution centered at $E_{\gamma}^{\text{inf}}$. We perform Monte Carlo simulations using the method presented in \cite{14}, and employing the detector thresholds from the WXM on \textit{HETE-2}. 

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E_{\gamma}^{\text{true}} = 2 \cdot 2\pi \int_0^{\pi/2} \epsilon(\theta_v) \sin \theta_v \, d\theta_v = 4\pi A \left[ 1 - \cos \theta_0 + I(\gamma, \theta_0) \right].
\end{equation}
Fig. 1. — Distribution of bursts detected on-axis (black) and off-axis (gray) in the \([\theta_0, \theta_v]\)-plane (top row), \([E_{\text{iso}}, E_{\text{peak}}]\)-plane (middle row) and \([E_{\text{iso}}^{\text{obs}}, S_\text{E}(2-400)]\)-plane (bottom row) for the models Y04 (left), VOAUJ1 (center) and VOAUJ2 (right). Bursts not detectable by the WXM are not shown.

3. — Results

Here we explore the relative importance of off-axis beaming for three variable opening-angle uniform jet models. The first (Y04) using the parameters from [13], assumes \(\gamma = 100\) and draws \(\theta_0\) values from a power-law distribution given by \(f_0 \, d\theta_0 \propto \theta_0^{-2} \, d\theta_0\), defined from 0.3 to 0.03 rad. The Y04 model attempts to explain classical GRBs in terms of the variation of jet opening angles, while XRFs are interpreted as bursts viewed off-axis. The other two models explain both GRBs and XRFs by a distribution of jet opening angles, following results presented in [14]. Here we add the presence of off-axis beaming to this picture, considering both \(\gamma = 100\) (VOAUJ1) and \(\gamma = 300\) (VOAUJ2).

As can be seen from the top row of fig. 1, the relative importance of off-axis events increases for models with a population of very small opening angles. This is mainly due to the fact that narrower jets with a constant \(E_\gamma\) will have larger \(E_{\text{iso}}\) values, and therefore such bursts viewed off-axis will also be brighter. More importantly, the middle and bottom rows show that the \(HETE-2\) XRFs are not easily explained as classical GRBs viewed off-axis. The two XRFs with known redshift lie along the \(E_{\text{peak}} \propto E_{\text{iso}}^{1/2}\) relation,
and furthermore the larger sample of HETE-2 XRFs without known redshifts do not fall in the region of the $[E_{\text{obs}}^{\text{peak}}, S_E]$-plane expected for this model; they lie at lower, rather than higher, $E_{\text{peak}}^{\text{obs}}$ values for a given $S_E$. Even given the model of the off-axis emission in [13], these HETE-2 XRFs are difficult to explain.

The other two models we consider generate XRFs that obey the $E_{\text{peak}} \propto E_{\text{iso}}^{1/2}$ relation by extending the range of possible jet opening-angles to cover five orders of magnitude (see [14] for details and discussion). Hence, XRFs that obey the $E_{\text{peak}} \propto E_{\text{iso}}^{1/2}$ relation are bursts that are seen on-axis, but have larger jet opening-angles. Nonetheless, these models generate a significant populations of off-axis events, although increasing $\gamma$ reduces the fraction of off-axis bursts in the observed sample.

4. – Discussion

Bursts with known redshift have been found to obey the $E_{\text{peak}} \propto E_{\text{iso}}^{1/2}$ relation, and a large population of off-axis bursts is not readily apparent in the observed datasets. [16] found that the $E_{\text{peak}} \propto E_{\text{iso}}^{1/2}$ relation holds internally within a large sample of bright BATSE bursts without redshift. It is unknown whether fainter bursts might deviate from this relation or what fraction of bursts are inconsistent with the $E_{\text{peak}} \propto E_{\text{iso}}^{1/2}$ relation. The result may be an indicator of the bulk $\gamma$ of the material.

In future work we will investigate the effect of possible correlations between $\theta_0$ and $\gamma$. If narrower jets have larger bulk $\gamma$ values, this could reduce the importance of off-axis beaming even further. Secondly, off-axis beaming will be important for non-uniform jets as well [17]. Gaussian and Fisher-shaped jets rely on the exponential falloff of the emissivity with viewing angle to match the wide spread of observed burst quantities [18,19]. If off-axis beaming is important, the exponential falloff will be dominated at some angle by the power-law falloff due to beaming, thereby broadening the emissivity distribution.

REFERENCES