Measuring the density of the Universe by means of Newton's gravitational constant G

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Abstract:

Questions regarding the measure of Universe's density is of great interest today. Several suggestions have been presented but the results suffers from our lack of knowledge of the fraction of dark matter present in and around galaxies.

1. Introduction

Methods to measure the actual density of the Universe are basically two:
The geometrical method and **The computational method** **.**

The geometrical method is based on the converging/diverging parallel lines. If the Universe is closed (density parameter $\Omega_0 > 1$) the parallel lines converge and the observed density of distant galaxies appears less than that expected by extrapolating the local density of galaxies backwards in time. Oppositely if the Universe is open (density parameter $\Omega_0 < 1$) , the parallel lines diverge and observed density of distant galaxies appears greater than expected.

In the **computational method** it is necessary to sample a representative space region of the universe that is larger than the scale on which the Universe becomes sufficiently homogeneous and measure the masses of objects within the volume. The ratio of mass to volume gives the density **ρ** .
If the space region is the whole Universe, the volume is:

$$V = (4 * pi * R^3) / 3$$

where:

R is the radius of a sphere and it is equivalent at the distance of the most far object.

This distance will be determined using the Hubble Law :

$$R = v_r / H_o$$

where:

v_r is the recessional velocity, typically expressed in km/s.
H_0 is Hubble's constant or the ratio of velocity to distance in the expansion of the Universe at the time of observation.
The "o" on H_o means the current value, since the Hubble "constant" changes with time (but it is the same everywhere in the Universe at a given time).

But since $v_r = c * z_{max}$ *:*

$$R = c * z_{max} / H_o$$

where:

z_{max} is the maximum redshift at the R distance.
c is the light velocity.

Then:

$$V = (4 * pi / 3) * (c * z_{max} / H_o)^3$$

The mass of the objects in the selected space region is determined using the virial theorem or one of its variants, which states that:

$$v^2 = G * M / r$$

For hot X-ray gas in clusters of galaxies:

v is the typical thermal velocity and is determined by the temperature of the gas.
r is an effective radius and is determined using an angular size *theta* and the distance $D = c * z / H_o$.
so: $r = theta * D = theta * c * z / H_o$.
M is the mass,.

G is Newton's gravitational constant.

Therefore the mass is given by:

$$M = r * v^2 / G = theta * c * z * v^2 / (G\ H_o)$$

Then:

$$\rho = M / V = (theta * c * z * v^2 / (G * H_o)) / ((4 * pi / 3) * (c * z_{max} / H_o)^3)$$

2. Calculation of the universe's density by means of Newton's gravitational constant G and Universe age.

The gravitational constant is a physical constant that is difficult to measure with high accuracy.

George T. Gillies (1997), "The Newtonian gravitational constant: recent measurements and related studies", *Reports on Progress in Physics* **60** (2): 151–225

The 2014 CODATA-recommended value of the gravitational constant is:

$$\mathbf{G} = 6{,}67408 * 10^{-11}\ \mathbf{m^3 / (sec^2 * Kg)}$$

"The 2014 CODATA Recommended Values of the Fundamental Physical Constants" (**Last update:** June 25 2015).
Available: http://physics.nist.gov/constants

This constant is necessary to quantify gravitational force between two bodies and presumably it must ride on the global Universe's mass allocated in the full Universe's volume at a definite moment.
Investigating its dimensions we notice:

$$\mathbf{G} = \mathbf{m^3 / (sec^2 * Kg)}.$$

We make this hypothesis :

G = Universe's volume / (Universe's age ^{2} * Universe's mass)
G = V / (t^2 * M)

So we have:

M / V = 1 / (G * t^2) = ρ = Universe's density.

In physical cosmology, the age of the universe is the time elapsed since the Big Bang.
The best measurement of the age of the universe is 13.798±0.037 billion years (13.798±0.037×10^9 years or 4.354±0.012×10^{17} seconds) within the Lambda-CDM concordance model.

Planck Collaboration (2015). "Planck 2015 results. XIII. Cosmological parameters"

The age of the Universe can be estimated by means of other methods too.
There are at least 3 ways:

The age of the chemical elements that gives a value of 14.5+2.8/-2.2 Gyr.
(Nicolas Dauphas *Nature* **435**, 1203-1205 (30 June 2005))

The age of the oldest star clusters that gives a value of 14.1 +/- 2.5 Gyr
(Shinya Wanajo Astrophys.J. 577 (2002) 853-865)

The age of the oldest white dwarf stars that give a value of 12.8 +/- 1.1 Gyr.
(Harvey B. Richer: Astrophys.J.574:L155-L158,2002)

Replacing known values:

$$\rho = 1 / (G * t^2) = 1 / ((6.67408*10^{-11}) * (4.354 * 10^{17})^2)$$

$$\rho = 1 / ((6.67408*10^{-11}) * (18.957316*10^{34}))$$

$\rho = 1 / (126.52264356928 * 10^{23}) = 0.0079 * 10^{-23}$ **Kg/m^3** $= 7.9 * 10^{-26}$ **Kg/m^3**

$\rho = 7.9 * 10^{-29}$ **g/cm^3**

It is important to note that the achieved ρ is the *total* **mass**/energy density of the Universe. In other words, it is the sum of a number of different components including both normal (baryonic) and **dark matter** as well as the **dark energy** .

3. Discussion and future work

The achieved result gives a density value of about one order of magnitude greater than the estimated values obtained by other methods. So may be justified since we do not know accurately all the matter and the energy existent in the whole universe.
It's notable that we do not have obtained a less value that would have invalidated the assumptions we have made.

It is customary to express the density as a fraction of the density required for the critical condition with the density parameter Ω_0.
The density parameter Ω_0 is given by:

$$\Omega_0 = \rho / \rho_c$$

where (ρ) is the *actual* density of the Universe and (ρ_c) is the critical density (the average density of matter required for the Universe to *just* halt its expansion, but only after an infinite time).

This relation determines the overall geometry of the universe.
When the ratio is *exactly* equal to 1, the geometry of the universe is flat (Euclidean) and contains enough matter to halt the expansion but not enough to recollapse it. If Ω_0 is less than 1, the Universe is open and will continue to expand forever. If Ω_0 is greater than 1, the Universe is closed and will halt its expansion and recollapse.

The *current* critical density is approximately $(8.62\pm0.12)\times10^{-27}$ kg/m^3 $= 0.862\times10^{-29}$ g/cm^3
Planck Collaboration (2015). “Planck 2015 results. XIII. Cosmological parameters”

Using the value of $\rho = 7.9 * 10^{-29}$ to calculate the density parameter Ω_0 we have:

$\Omega_0 = \rho / \rho_c = 7.9 * 10^{-29} / 0.862 \times 10^{-29} = 9.16$

With the value of $\Omega_0 = 9.16$ the Universe is closed.

References:

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Zombeck's Handbook of Space Astronomy and Astrophysics.
Chapter 1, Page 11.
Chapter 10, Page 249.