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Wind speed and thermal gradient in a moderately stable PBL

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Summary. — In the inertial layer of a moderately stable BL ($0 < Ri < 0.2$) the M.O. similarity theory gives the profiles of wind speed U and thermal gradient $\Delta\vartheta$ as functions of the dimensionless height $\zeta = z/L$. Experimental works permit to evaluate ζ and the similarity functions Ψ_m and Ψ_h from Ri. Giving the correct values for z_0 and u_* if $\Delta\vartheta$ known it is possible to calculate $U(z)$ at any z contained in the inertial layer. This relationship is used to explain the clustering of values of U and $\Delta T/\Delta z$ measured at two stacks of a TWA power plant.

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1. – Introduction

The PBL is still the object of many studies, both theoretical and experimental, see, just for recent instances, Vickers and Mahrt [1], Patel *et al.* [2], Venkatram *et al.* [3]. In this work we will consider the vertical profiles of wind and thermal gradient in the inertial layer of a moderately stable PBL.

Following the Kolmogorov 1941 phenomenology [4], in the inertial layer between the integral scale l_0 and the dissipation scale η the mean energy dissipation rate ε is constant with l all along the "Richardson cascade". The length scale l is the length typical of eddies, but because eddies are largest at the top and smallest at the bottom of the inertial layer, ε can be considered constant with height z.

The dimensions of ε are L^2T^{-3}, the same as the flux of the turbulent kinetic energy (TKE), we will consider instead of ε. Figure 1, from a selected data set of Kansas Field Experiment, shows the behavior of TKE as a function of Ri. TKE goes near 0 for Ri similar to 0.25. For stronger stability, the downstream transfer of energy is not sufficient for an effective driving of the lower wind, and the lower layer is sporadically stirred by isolated eddies and low level jets [5], outside the field where the M.O. similarity theory is applicable. Our work will be limited to week to moderate stability ($0 < Ri < 0.20$).

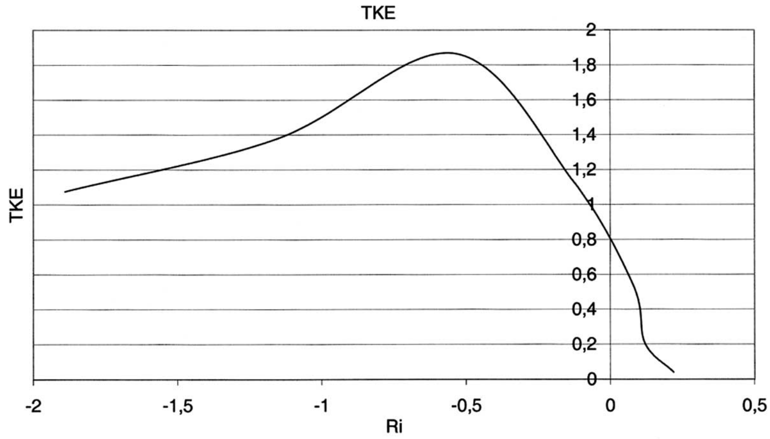

Fig. 1. – Turbulent kinetic energy *vs.* the Richardson number (from [6]).

2. – The wind speed profile

In a neutral boundary layer (NBL) the profile of the wind speed $U(z)$ is given by the well-known logarithmic profile law

$$U/u_* = (1/k)\ln(z/z_0)\,, \tag{2.1}$$

if the influences of geostrophic wind, surface roughness and PBL height are all contained in u_* through the surface drag τ_0.

For stability conditions other than—but not too far from—neutral, the empirical form of the M.O. similarity function is, with the habitual notations [6],

$$U/u_* = (1/k)[\ln(z/z_0) - \Psi_m(\zeta)]\,, \tag{2.2}$$

being $\zeta = z/L$.

Equation (2.2) differs from (2.1) for the similarity function $\Psi(\zeta)$, representing the deviation from the logarithmic law due to buoyancy forces.

The similarity functions like (2.2) were validated by experiments, mainly the Kansas Field Program, whose results suggested also the empirical form for $\Psi(\zeta)$. For $z/L > 0$ it is

$$\Psi_{\mathrm{m}}(\zeta) = \Psi_{\mathrm{h}}(\zeta) = -5\zeta\,,$$

and

$$\zeta = Ri/(1-5Ri) \qquad \text{for} \qquad 0 < Ri < 0.2\,.$$

A very recent work [7] gives values slightly different for $\Psi_{\mathrm{m}}(\zeta)$ and $\Psi_{\mathrm{h}}(\zeta)$, but we have conserved the value 5 for the sake of simplicity.

With these approximations, eq. (2.2) establishes a relationship between $U(z)$ and Ri:

$$U = (u_*/k)[\ln(z/z_0) - (-5Ri/(1-5Ri)]\,. \tag{2.3}$$

Being

$$Ri = (g/T_0)[(\Delta\vartheta/\Delta z)/(\Delta u/\Delta z)]^2 ,$$

we have

$$\Delta u = \Delta z Ri^{1/2}[(g/T_0)(\Delta\vartheta/\Delta z)]^{-1/2} . \tag{2.4}$$

Chosen a value for the potential temperature gradient and for wind speed at a reference height U_r, the wind speed $U(z)$ has to satisfy both eqs. (2.3) and (2.4), establishing a relationship between $U(z)$ and Ri.

3. – The potential temperature profile

With the same approximation of the preceding equation (2.3) the potential temperature profile has the form

$$(\vartheta - \vartheta_0) = (\vartheta_*/[\ln(z/z_0) - (-5Ri/(1 - 5Ri)] , \tag{3.1}$$

where ϑ_0 is the potential temperature extrapolated at $z = z_0$ and ϑ_* is the temperature scale $\vartheta_* = H_0/\rho c_p u_*$.

An analogue of eq. (2.4) is

$$\Delta\vartheta = \Delta z Ri(\Delta u/\Delta z)^2 . \tag{3.2}$$

Chosen a value for the wind speed gradient, the potential temperature gradient has to satisfy both eqs. (3.1) and (3.2), establishing a relationship between $\Delta\vartheta$ and Ri.

4. – A numerical experiment

If we choose for all the variables values typical of night-time stable conditions, it is possible to test the results from systems (2.3), (2.4) and (3.1), (3.2). In particular, we choose $u_* = 0.2\ \mathrm{ms}^{-1}$, $z_r = 10$ m, $z = 150$ m, $u_r = 1\ \mathrm{ms}^{-1}$, $\vartheta_* = 0.2$, $H_0 = -20\ \mathrm{Wm}^{-2}$. For z_0 the value 0.02 m was chosen, the exact value being of minor importance [8].

The results are best illustrated by figs. 2 and 3.

Figure 2 contains eqs. (2.2) and (2.3) and shows the dependence of U_{150} on Ri. The curve of eq. (2.2) shows a dependence almost linear with Ri, peaking only for values of Ri approaching the critical value of 0.20, when eq. (2.2) loses its significance. The parameter of the quadratic curves of eq. (2.3) is the potential temperature gradient, namely from the bottom 0.04, 0.08 and 0.12 Km^{-1}.

Figure 3 contains eqs. (3.1) and (3.2) and shows the dependence of ϑ_{150} on Ri. The curve of eq. (3.1) shows the same dependence on Ri as eq. (2.3). The parameter of the linear curves of eq. (3.2) is the wind speed at $z = 150$ m, namely from the bottom 4, 6 and 8 ms^{-1}.

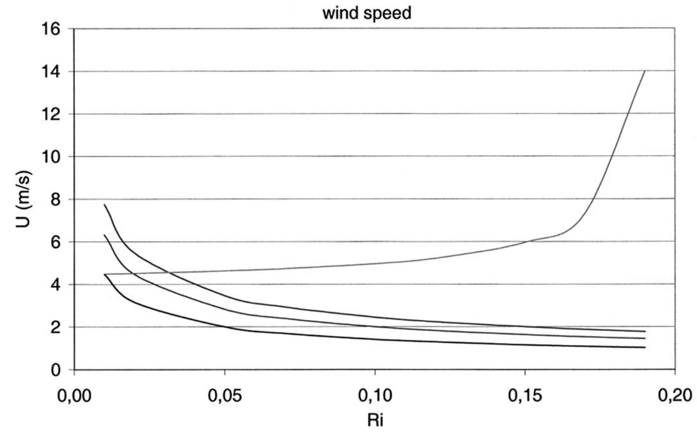

Fig. 2. – Wind speed U_{150} *vs.* Ri from eqs. (2.3) and (2.4). The parameter of curves (2.3) from the bottom is $\Delta\vartheta = 0.04, 0.08$ and $0.12\ \mathrm{Km}^{-1}$.

5. – Discussion

This work has many limitations. Apart from the conditions necessary for the validity of the M.O. similarity law, it must be $Ri < 0.20$ and $z < h$, h being the thickness of the stable boundary layer.

An outstanding aspect of eqs. (2.3) and (3.1) is, unless Ri is high, the dominance of the term $\ln(z/z_0)$ over $\Psi(\zeta)$; as a consequence u_z and $\Delta\vartheta$ cluster in a narrow range.

This work is no more than a mathematical exercise, but we can compare this last result with data of wind speed and potential temperature gradient taken at night at two stacks of a power plant of Tennessee Valley Authority [9], respectively 183 and 152 m tall. The wind speed of the 21 data from the 183 m stack range from 2.4 to 11.6 ms^{-1}, with an average of 6.3 ms^{-1}. The same values from the 49 cases of the 152 m stacks are,

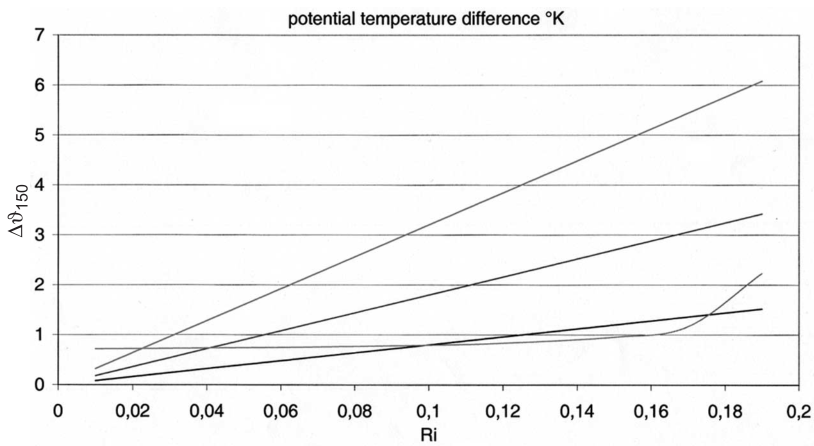

Fig. 3. – Temperature gradient $\Delta\vartheta_{150}$ *vs.* Ri from eqs. (3.1) and (3.2). The parameter of curves (3.1) from the bottom is $U = 4$, 6 and 8 ms^{-1}.

respectively, 1.5, 6.7 and 3.9 ms^{-1}, as can be expected. But the most interesting feature of the latter series is that out of 49 cases, 21 cluster at $u_{152} = 4.6\ \mathrm{ms}^{-1}$, and $\Delta\vartheta/\Delta z = 0.1\ \mathrm{Km}^{-1}$. Also 3 cases from the 183 m stack cluster at 4.8 ms^{-1}. In all the other cases the surface layers were either very near the neutrality ($\Delta\vartheta/\Delta z < 0.005\ \mathrm{Km}^{-1}$), or the tops of stacks were outside of the stable layer.

Those figures meet very well with the corresponding data on fig. 2, validating the rationale of the work.

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