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Monin-Obukhov similarity theory consistent stability functions for the Prandtl turbulence closure model of the stable atmospheric boundary layer

M. Zampieri(*)
ISAC-CNR - Via Gobetti 101, 40129 Bologna, Italy

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Summary. — A new derivation of the stability functions for the Prandtl-Blackadar atmospheric boundary layer model from the Monin-Obukhov similarity theory is proposed by invoking the consistency between the surface layer and the boundary layer turbulent fluxes. The new stability functions are compared to the Louis stability functions and validated, against an ensemble of large eddy simulations, in the context of a single column model simulation of the stably stratified, shear driven, boundary layer. The results of the new stability functions are superior to those of the Louis model and are suggested for the boundary layer parameterization in the atmospheric numerical models.

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1. – Introduction

The Planetary Boundary Layer (PBL) is the region of the atmosphere, close to the surface, where the turbulent fluxes are important to determine the dynamics of the system. As the surface is approached, the surface layer, in which the turbulent fluxes become approximately constant, can be identified. The turbulent fluxes are not explicitly resolved by the numerical models and they have to be parameterized.

The turbulent fluxes at the surface are often treated with different closures with respect to the fluxes in the PBL above the surface layer. In the surface layer, the Monin-Obukhov similarity theory (MOST [1]) is widely used. It is a first-order model, that allows to compute the momentum and heat turbulent fluxes from the corresponding wind and temperature profiles.

(*) Present affiliation: ISAC-CNR, Str. Prov. Lecce-Monteroni km 1,200 - Polo Scientifico dell'Università, 73100 Lecce, Italy. E-mail: M.Zampieri@isac.cnr.it

In the PBL, higher-order models, that implement one or more prognostic equations for the turbulent quantities, can be considered in order to give a better representation of non-local effects of turbulence due to horizontal inhomogeneities [2] or to convection in the unstable boundary layer [3-7].

In the stable boundary layer modelling, to which this study is devoted, turbulence can be considered, to a good approximation, local in nature [8,9]. Thus the simple first-order Prandt-Blackadar closure model [10, 11], that employs a local flux-gradient relation for the turbulent fluxes, can be applied. This model is very popular because it is simple to implement and it gives reliable results. On the other hand, it needs stability functions in order to take into account the effect of the static stability on turbulence.

These stability functions can be obtained analytically, but after some assumptions [12, 13]. In the Louis model [14], these functions are obtained from comparison with the MOST, carrying the advantage of being based on experimental measurements. The Louis model can be applied, for the sake of consistency, both in the surface layer and in the PBL.

However, the Louis model carries some deficiencies. In fact, in the surface layer, it is less accurate than the MOST. This is mostly due to the fact that the Louis model is based on the bulk Richardson number, that is computed from the wind and temperature differences between a screen level and the surface. It takes into account only approssimatively the details of the surface layer phenomenology such as the logarithmic wind and temperature profiles. On the other hand, the MOST is formulated in terms of the stability parameter (z/L) that, depending on the turbulent fluxes, is a well-defined quantity.

Moreover, the Louis stability functions, in order to prevent the overestimation of the nocturnal surface cooling, are prescribed with an artificial enhancement of the turbulent fluxes. This is not consistent with the free upper level flow [15-17], thus Louis stability functions are not of general validity.

Finally, new experimental results can be applied directly in the context of the MOST, and not in the Louis model. For these reasons the MOST can be preferred, in the surface layer, with respect to the Louis model.

On the other hand, the MOST cannot be applied in the PBL because the assumption of constant fluxes on which it is based is no more justified. Thus, other models, like the Prandtl-Blackadar model, also known as mixing length model, have to be considered. However, a consistency problem may arise if the surface fluxes are computed by the MOST and the PBL fluxes are computed by the mixing length model.

In this paper a possible solution is presented. In fact, new stability functions for the mixing length model are derived, from the MOST, by prescribing the consistency between the two models.

The new stability functions are implemented in a mixing length model and the results of a simulation of the stably stratified PBL are discussed and compared to those of the Louis model and of an ensemble of Large Eddy Simulation (LES) models.

The paper is organized as follows: in sect. **2** the Monin-Obukhov similarity theory is summarized; in sect. **3** the Prandtl-Blackadar model with Louis stability functions is presented; in sect. **4** the derivation of the new, MOST-consistent, stability functions for the Prandtl-Blackadar model is shown; in sect. **5** the results of the simulation using the new stability functions are presented and validated; in sect. **6** the derivation and the results are discussed; in sect. **7** the conclusions are drawn.

The former two sections present no new theory. They summarize the models on which this study is based, but with a common formalism, for the reader's utility.

2. – The Monin-Obukhov similarity theory

In the surface layer, the vertical turbulent fluxes of the horizontal mean wind component U and the mean potential temperature Θ can be computed as MOST [1, 9]:

$$\overline{wu}|_0 = -u_*^2 \qquad \text{and} \tag{1}$$

$$\overline{w\theta}|_0 = -u_*\theta_*, \tag{2}$$

respectively, where u_* is the friction velocity and θ_* the temperature scale, given by

$$u_* = \frac{\kappa \Delta U}{\ln\left(\frac{z}{z_{0m}}\right)} f_{*m}, \tag{3}$$

$$\theta_* = \frac{\kappa \Delta \Theta}{\ln\left(\frac{z}{z_{0h}}\right)} f_{*h}, \tag{4}$$

where $\Delta U = U(z)$, $\Delta\Theta = \Theta(z) - \Theta(z_{0h})$, z_{0m} and z_{0h} are the roughness lengths for momentum and heat, respectively, and

$$f_{*m}^{-1}\left(\frac{z}{z_{0m}}, \frac{z}{L}\right) = 1 - \frac{\Psi_m\left(\frac{z}{L}\right) - \Psi_m\left(\frac{z_{0m}}{L}\right)}{\ln(z) - \ln(z_{0m})}, \tag{5}$$

$$f_{*h}^{-1}\left(\frac{z}{z_{0h}}, \frac{z}{L}\right) = 1 - \frac{\Psi_h\left(\frac{z}{L}\right) - \Psi_h\left(\frac{z_{0h}}{L}\right)}{\ln(z) - \ln(z_{0h})} \tag{6}$$

play the role of the stability functions in the context of the similarity theory.

In (5) and (6), $L = u_*^2/\kappa\frac{g}{T}\theta_*$ is the Obukhov length, z/L is the stability parameter and the closure functions, derived from the observations, are

$$-\Psi_m = a\frac{z}{L} + b\left(\frac{z}{L} - \frac{c}{d}\right)\exp\left[-d\frac{z}{L}\right] + \frac{bc}{d}, \tag{7}$$

$$-\Psi_h = \left(1 + \frac{2}{3}\frac{az}{L}\right)^{3/2} + b\left(\frac{z}{L} - \frac{c}{d}\right)\exp\left[-d\frac{z}{L}\right] + \frac{bc}{d} - 1 \tag{8}$$

with $a = 1$, $b = 0.667$, $c = 5$, $d = 35$ [18], that have the advantage to be reliable for very strong stratification ($z/L \sim 10$) and are used in the operational context.

In the atmospheric models the MOST equations can be evaluated at the height of the lower computational level, that is assumed to lie within the surface layer.

3. – The Prandtl-Blackadar-Louis PBL model

In the first-order models, the vertical turbulent fluxes of momentum and heat are given by a downgradient formulation:

$$-\overline{wu} = K_m\frac{\partial U}{\partial z}, \tag{9}$$

$$-\overline{wv} = K_m\frac{\partial V}{\partial z}, \tag{10}$$

$$-\overline{w\theta} = K_h\frac{\partial \Theta}{\partial z}, \tag{11}$$

respectively, where K_m and K_h are the eddy viscosity and diffusivity, that can be parameterized in terms of length scale (mixing length) and velocity scale [10, 9, 19]:

$$K_m = \ell_m^2 S \qquad \text{and} \tag{12}$$
$$K_h = \ell_h \ell_m S, \tag{13}$$

where $S = \sqrt{\left(\frac{\partial U}{\partial z}\right)^2 + \left(\frac{\partial V}{\partial z}\right)^2}$ and ℓ_m and ℓ_h are the mixing lengths for momentum and heat, respectively.

The length scale formulation for the neutral case (no stratification) was given by Blackadar [11]:

$$\frac{1}{\ell_n} = \frac{1}{\kappa z} + \frac{1}{\ell_\infty}\,, \tag{14}$$

where the asymptotic value ℓ_∞ is treated here as a closure parameter ($\ell_\infty = 75$ m). In this study the model is not much sensitive to this parameter because, in the stable case, a major role is played by the stability functions.

It is possible to define the mixing length in stratified environments as a correction of the neutral mixing length multiplying it by some functions of the Richardson number

$$\ell_m(Ri) = \ell_n f_m(Ri) \qquad \text{and} \tag{15}$$
$$\ell_h(Ri) = \ell_n f_h(Ri), \tag{16}$$

where the gradient Richardson number is defined as the ratio of the squared buoyancy (Brunt-Väisälä) frequency $N^2 = \frac{g}{\Theta}\frac{\partial \Theta}{\partial z}$ [9] and the squared shear frequency $Ri = N^2/S^2$.

Louis applied to the PBL the same functions that he proposed for the surface layer:

$$f_m^2 = \frac{1}{1 + d_m Ri(1 + e_m Ri)^{-1/2}}\,, \qquad \text{and} \tag{17}$$
$$f_m f_h = \frac{1}{1 + d_h Ri(1 - e_h Ri)^{-1/2}}\,, \tag{18}$$

that are used here with a revised set of closure coefficients $d_m = 10$, $d_h = 8.5$, $e_m = 1$ and $e_h = (d_m/d_h)^2$ [17].

The Louis PBL model is not consistent with the MOST, thus a new model is needed, and it will be proposed in the next section.

4. – The new stability functions

The MOST is formally consistent with the mixing length theory. However, a basic difference between the two approaches consists in the fact that, while the Richardson number is a local variable, the Obukhov length is, in a sense, non-local, in fact, by definition, it is representative of the whole surface layer.

A link can be found between the MOST and the mixing length model in discretized form, since, in this case, Ri is computed by means of finite differences and can be considered a bulk quantity like L.

The new method consists in the application of the constraint of continuity between the PBL and the surface layer fluxes, which is legitimate because all the fields are assumed to be smooth.

As the surface is approached, this constraint can be mathematically expressed as

$$\lim_{z \to z'} \overline{wu}(z) = -u_*^2 \qquad \text{and} \tag{19}$$

$$\lim_{z \to z'} \overline{w\theta}(z) = -u_* \theta_* \,, \tag{20}$$

where $z_0 < z' < z$ and z is some reference height within the surface layer. To derive the stability functions for the PBL, the same value of the surface roughness length is used for both momentum and temperature ($z_0 = z_{0m} = z_{0h}$). In fact, the use of different values conceptually reflects the differences in the transport mechanisms for heat and momentum in the surface layer [20], that do not exist in the PBL.

The l.h.s. of eqs. (19) and (20) are evaluated with the mixing length model as given by eqs. (12) and (13), where the vertical derivatives in eqs. (9), (11) and the Richardson number are computed with the standard finite differences method. The r.h.s. is evaluated with the MOST, as it is given by eqs. (3) and (4). Thus the result of eqs. (19) and (20) consists in the application of the two models in the same layer $\Delta z = z - z_0$ close to the surface. Thus (19) and (20) become

$$\ell_n^2(z') f_m^2(Ri) \left| \frac{\Delta U}{\Delta z} \right| \frac{\Delta U}{\Delta z} = \frac{\kappa^2 \Delta U^2}{\ln\left(\frac{z}{z_0}\right)^2} f_{*m}^2\left(\frac{z}{L}\right) \qquad \text{and} \tag{21}$$

$$\ell_n^2(z') f_m(Ri) f_h(Ri) \left| \frac{\Delta U}{\Delta z} \right| \frac{\Delta \Theta}{\Delta z} = \frac{\kappa^2 \Delta U \Delta \Theta}{\ln\left(\frac{z}{z_0}\right)^2} f_{*m}\left(\frac{z}{L}\right) f_{*h}\left(\frac{z}{L}\right), \tag{22}$$

respectively, where the mixing length is evaluated at the height $z' = \Delta z$. In fact, from (14), it is $\lim_{z \to z'} \ell_n(z) = \kappa \Delta z$. Substituting this in eq. (21) and using the fact that the wind speed gradients are positive in the surface layer, after some algebraic simplifications, one is left with

$$f_m(Ri) = \ln\left(\frac{z}{z_0}\right)^{-1} f_{*m}\left(\frac{z}{L}\right), \tag{23}$$

while, doing the same with (22) together with (23), one obtains

$$f_h(Ri) = \ln\left(\frac{z}{z_0}\right)^{-1} f_{*h}\left(\frac{z}{L}\right). \tag{24}$$

Equations (23) and (24) must hold, in particular, for $Ri = 0$ and $z/L = 0$, that is true if and only if $\ln(z/z_0) = 1$. For a specified ratio z/z_0, Ri and z/L are linked by a bijective function. Thus eqs. (23) and (24) become

$$f_m(Ri) = f_{*m}\left(\frac{z}{L}\right) \tag{25}$$

and

$$f_h(Ri) = f_{*h}\left(\frac{z}{L}\right), \tag{26}$$

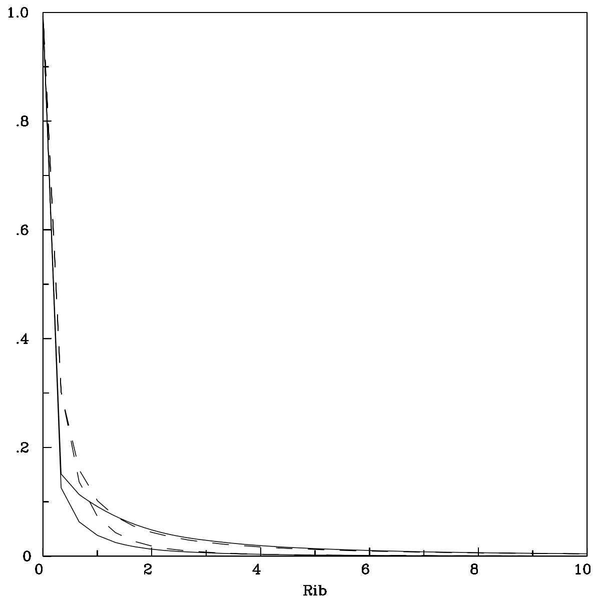

Fig. 1. – Momentum (higher curves) and potential temperature (lower curves) stability functions computed by the Monin-Obukhov similarity theory (solid line) and using analytic formulas (27) and (28) (dashed line) in the stable case.

allowing the derivation of the wanted stability functions from the similarity functions and closing the physical problem.

Now the desired stability functions can be obtained by curve fitting, for which it is suggested to proceed through the following steps:

- try to identify a possible functional relation for the Ri-dependent stability functions $f_m(Ri, a_m, b_m, ...)$ and $f_h(Ri, a_h, b_h, ...)$ in which a number of closure parameters to be determined appears;
- select a number of values of the stability parameter (z/L) that is equal to the number of closure parameters;
- for each z/L, and for the ratio z/z_0 that satisfies eqs. (25) and (26) compute the corresponding Ri;
- evaluate the identities (25) and (26) for each of these values, obtaining a system of equations that can be inverted to find the closure parameters, if the proposed functional dependence is simple enough.

This procedure is repeated many times with different functional relations. The best fitting is found with

$$f_m^{-1} = 1 + 5Ri + Ri^2 + a_m Ri^{1.5}, \tag{27}$$

$$f_h^{-1} = 1 + 5Ri + 3Ri^3 + a_h Ri^2, \tag{28}$$

that use only one closure parameter.

In the l.h.s. of eqs. (27) and (28), the first two terms are introduced to match the same asymptotic behaviour of the Beljaars-Holtslag functions for $Ri \to 0$ and $Ri \to \infty$. The last terms are introduced to improve the fitting at intermediate regimes.

The computation of the closure parameters is repeated with different z/L values till the best overall fitting is reached. At the end the closure parameters are found to be $a_m = 2.79$, $a_h = 4.71$ for $z/L = 2$, completing the procedure.

Figure 1 shows the resulting functions, that agree reasonably well with those obtained with the MOST. In particular they are consistent, by construction, in the limits $Ri \to 0$ and $Ri \to \infty$ and for a wide stability range. They mismatch for intermediate Richardson numbers ($Ri \sim 1$).

Since it is hard to evaluate the importance of this departure *a priori*, the new stability functions are implemented in a mixing length model and tested, in the next section, through the validation of the simulation results.

5. – Implementation and results

The Reynolds averaged equations for the horizontally homogeneous dry PBL are

$$\frac{\partial U(z)}{\partial t} = f(V(z) - V_g) - \frac{\partial \overline{wu}(z)}{\partial z}, \tag{29}$$

$$\frac{\partial V(z)}{\partial t} = f(U_g - U(z)) - \frac{\partial \overline{wv}(z)}{\partial z}, \tag{30}$$

$$\frac{\partial \Theta(z)}{\partial t} = -\frac{\partial \overline{w\theta}(z)}{\partial z}, \tag{31}$$

where f is the Coriolis parameter and U_g and V_g are the components of the geostrophic wind. Equations (29)-(31) have been implemented in a single column model consisting of a vertical domain of 10 km discretized in 100 unequally spaced levels, the higher resolution being in proximity of the surface (5 m). The mean wind and potential temperature are defined at these levels, while the turbulent fluxes are defined at staggered levels; the geostrophic forcing is splitted from the vertical diffusion of U, V and Θ, that is computed with an implicit scheme. The surface boundary conditions are $(U, V) = (0, 0)$ for the velocity and a prescribed time-dependent temperature. At the upper boundary, $(U, V) = (U_g, V_g)$ and $\Theta =$ const are specified. The time step is 100 s.

To test and compare the mixing length PBL model with the Louis and the new stability functions a weakly stratified, shear driven, boundary layer is simulated. In both cases, the surface layer fluxes are computed with the MOST.

The experiment is based on a Large Eddy Simulation (LES) by Kosovic and Curry [21]. With respect to the observations, the LES simulations have the advantage of providing a complete control on all variables. In this case study, the effects of radiation are taken into account only by the surface temperature prescription to isolate the effects of turbulence on the dynamics of the mean variables for a better interpretation of the results. This LES simulation inspired a recent PBL models and LES intercomparison study [22, 23]. The range of LES results from this study, where also Kosovic and Curry's was present, is taken here as reference for the model validation.

The initial potential temperature profile is constant in the lower 100 m ($\Theta = 265$ K) and increases with a constant lapse rate of 0.01 K m^{-1} above. The model is forced with a constant geostrophic wind ($U_g = 8$ m s^{-1}) that is also used as initial condition for the mean wind. The surface temperature is prescribed to decrease constantly at a rate of

0.25 K h^{-1}. The Coriolis parameter is $f = 1.39 \times 10^{-4}$ s^{-1}, the surface roughness is $z_{0m} = z_{0h} = 0.1$ m, and the surface pressure is $p_s = 1013.2$ hPa.

The simulated boundary layer evolves toward a quasi-steady state. Since the first stages of the simulation an Ekman-like solution appears, being characterized by a supergeostrophic wind that produces inertial oscillations as soon as the inversion is generated and the boundary layer decouples from the free flow.

In fig. 2 the results of the simulations after 9 hours of integration are shown. From the resulting potential temperature profile (fig. 2a) it is possible to see that the two models produce almost the same results in the lower boundary layer, below 200 m, overestimating the temperature, with respect to the LES, by less than 1 K. They overestimate the PBL growth and underestimate the strength of the inversion at its top with respect to the LES. However, the results obtained with the new stability functions are much closer to those of the LES. In fact, the Louis model produces a boundary layer that is approximatively twice deeper than the reference one. On the other hand, the new stability functions result in an intermediate PBL height and a better representation of the capping inversion. The heat flux profile, showed in fig. 2b, is overestimated by the Louis model by the 25% relative to the maximum reference value, while it is overestimated by less than 10% by the new model.

In fig. 2c and 2d the results for the mean wind profiles are shown. The results are consistent with those of temperature. In fact, the overestimation of the supergeostrophic wind height follows the overestimation of the PBL height, while the intensity of the supergeostrophic wind is correlated to the strength of the inversion. Thus, also for wind, the new model produces better results. On the other hand, differences in the simulated momentum flux are less important than those for the heat flux. In fact, the two models produce almost the same surface momentum flux (fig. 2e) and the two profiles would collapse if they were rescaled with the PBL depth.

The most evident differences are between the mixing length profiles, shown in fig. 2f, because they are more closely related to the closure functions. The new model produces a smaller mixing length, that goes to zero above the PBL, where the flow is truly free. The Louis model produces a similar profile, but with larger mixing length values, the maximum value being about the 50% more than that of the new model. The Louis mixing length never goes to zero neither above the PBL, contrarily, a minimum value of about 6 m is reached at 600 m height and is constant above (not shown). Thus the Louis model produces a non-zero turbulent mixing also above the PBL where, in this case study, turbulence should not be present.

The simulated Richardson number (not shown) is close to zero at the surface, where turbulence is mainly generated by the wind shear, and increases with height where buoyancy becomes more important. The value of $Ri = 1$ is produced at 150 m height by the Louis model, at about 230 m height by the new model. Therefore, the discrepancy of the new stability functions respect to the similarity functions at $Ri = 1$ results in the enhancement of the turbulent fluxes in the middle boundary layer respect to those that could be obtained with a perfect fit.

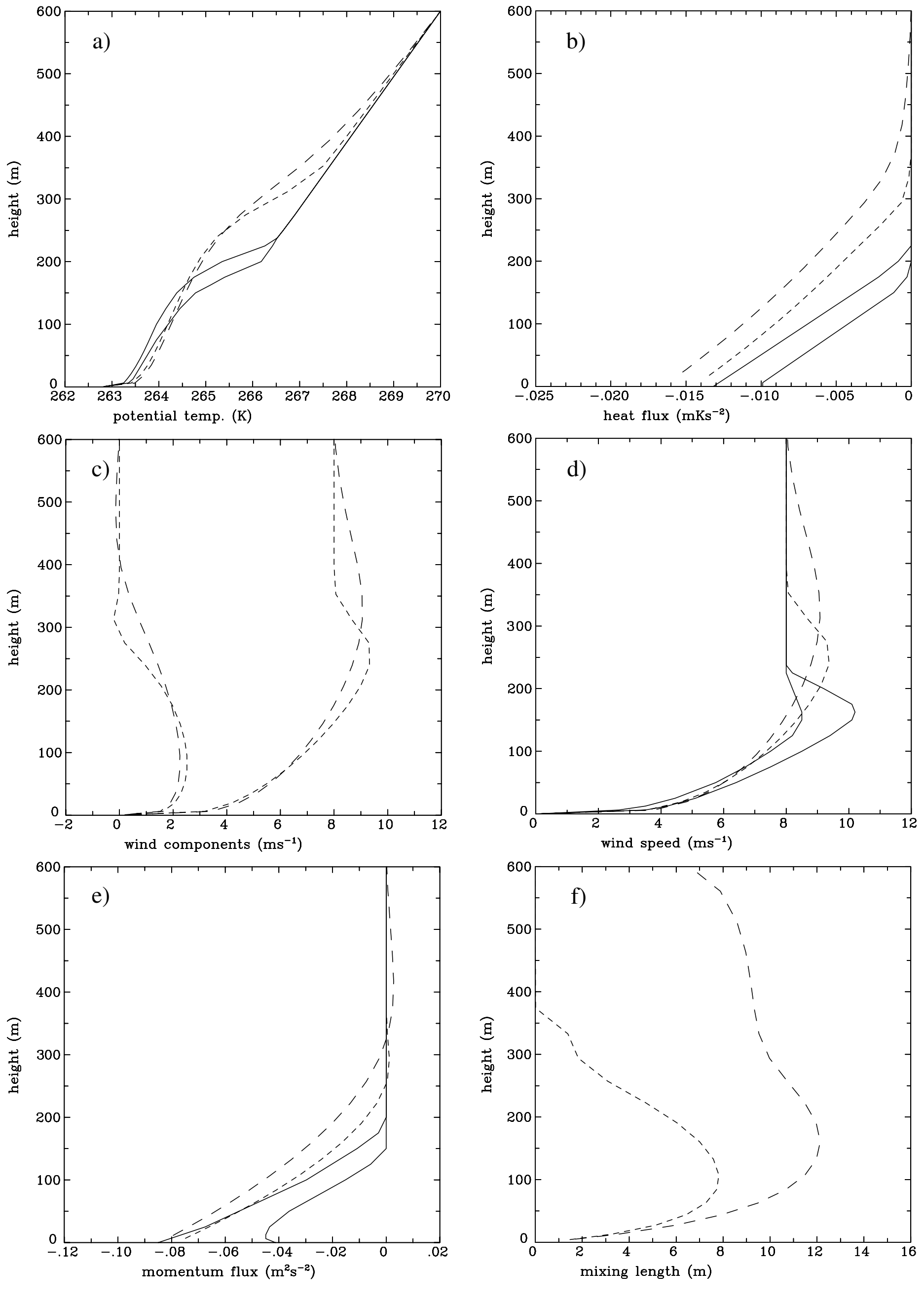

Fig. 2. – Potential temperature (a), heat flux (b), wind components (c), wind speed (d), momentum flux (e), and mixing length (multiplied by stability functions for momentum) (f) vertical profiles in the stable case after 9 hours. Solid line: range of LES results; dashed lines with longer segments: Prandtl model with the Louis stability functions; dashed lines with smaller segments: Prandtl model with the MOST-Consistent stability functions.

6. – Discussion

It may sound rather strange that the new stability functions for the PBL are derived in a very thin layer close to the surface. However, it is a consequence of the constraint of continuity between surface and PBL fluxes, that is absolutely legitimate.

This constraint is imposed, mathematically, by comparing the MOST and the mixing length model. Thus, since the MOST is formulated for a discrete layer, the mixing length model is formulated in terms of standard finite-difference method. This is potentially a great advantage because this is exactly the way in which the model is operationally applied.

The two models are compared at the height for which $\ln(z/z_0) = 1$. The necessity of the identification of this particular layer is motivated by the fact that the mixing length model, after discretization by means of finite differences, takes into account turbulence produced in a linear wind and temperature profile. Since the surface layer is characterized by the logarithmic, rapidly varying, wind and temperature profiles, the mixing length model is not of general validity and its results depend on the height at which it is applied. The constraint of continuity leads to the definition of the particular height at which the mixing length model produces quantitatively the same turbulent fluxes than the MOST. As a consequence, the Richardson number becomes a well-defined quantity, allowing the derivation of the stability functions from the similarity functions.

The new stability functions are based on the Richardson number. Thus, they can be used in the PBL and in the upper troposphere, where the only environmental parameter of interest for turbulence is the Richardson number. Moreover, the new stability functions have no memory of the particular resolution at which they are derived. The only requirement for the resolution is that the Richardson number should be a well-defined variable, *i.e.* the wind and temperature profiles should be well resolved, that is implicit in the finite-difference method.

The performance of the new stability functions is proved to be satisfactory. In fact, from visual comparison of the results from the Cuxart *et al.* intercomparison paper [22], it is possible to see that the model derived here is among the best mixing length models. A better performance is obtained only with the higher-order models, because they provide a more accurate representation of the second-order moments that are less closely dependent on the diagnostic mixing length.

7. – Conclusions

New stability functions for the mixing length model are derived from the observations, in the stably stratified case, using the Monin-Obukhov similarity theory and invoking the continuity between the surface layer and the PBL turbulent fluxes.

The new stability functions are tested in a single column simulation of the stably stratified boundary layer and compared to the Louis stability functions and to an ensemble of large eddy simulation models.

With respect to the Louis model, the forecast of the boundary layer height and the representation of the capping inversion produced by the new model are closer to the LES results.

Moreover, the new model is consistent with the free flow above the boundary layer, thus it is more general than the Louis model and it can be applied, without *ad hoc* restrictions, to the whole troposphere.

The new PBL model is consistent with the Monin-Obukhov similarity theory, thus it is suggested for application in the atmospheric models that implement the similarity theory in the surface layer.

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REFERENCES

[1] Monin A. S. and Obukhov A. M., *Tr. Geofiz. Inst., Akad. Nauk SSSR*, **24** (1955) 163.
[2] Detering H. W. and Etling D., *Boundary-Layer Meteorol.*, **33** (1985) 113.
[3] Troen I. and Mahrt L., *Boundary-Layer Meteorol.*, **37** (1986) 129.
[4] Canuto V. M., Minotti F., Ronchi C., Ypma R. M. and Zeman O., *J. Atmos. Sci.*, **51** (1994) 1605.
[5] Abdella K. and McFarlane N., *J. Atmos. Sci.*, **54** (1997) 1850.
[6] Cheng Y., Canuto V. M. and Howard A., *J. Atmos. Sci.*, **62** (2005) 2189.
[7] Gryanik V. M. and Hartmann J., *J. Atmos. Sci.*, **62** (2005) 2632.
[8] Nieuwstadt F. T. M., *J. Atmos. Sci.*, **41** (1984) 2202.
[9] Stull R. B., *An Introduction to Boundary Layer Meteorology* (Kluwer Academic Publishers, Dordrecht, The Netherlands) 1988.
[10] Prandtl L., *Z. Angew. Math. Mech.*, **5** (1925) 136.
[11] Blackadar A. K., *J. Geophys. Res.*, **67** (1962) 3095.
[12] Mellor G. and Yamada T., *Rev. Geophys. Space Phys.*, **20** (1982) 851.
[13] Canuto V. M., Cheng Y. and Howard A., *J. Atmos. Sci.*, **59** (2002) 1550.
[14] Louis J. F., *Boundary-Layer Meteorol.*, **17** (1979) 18.
[15] Beljaars A. C. M. and Viterbo P., *The Role of the Boundary Layer in a Numerical Weather Prediction Model, in Clear and Cloudy Boundary Layers Mechanics*, edited by Holtslag A. A. M. and Duynkerke P. G. (North Holland Publishers) 1999.
[16] Viterbo P., Beljaars A. C. M., Mahfouf J.-F. and Teixeira J., *Q. J. R. Meteorol. Soc.*, **125** (1999) 2401.
[17] Zampieri M., Buzzi A. and Malguzzi P., *Nat. Hazards Earth Syst. Sci.*, **5** (2005) 603.
[18] Beljaars A. C. M. and Holtslag A. S. M., *J. Appl. Meteorol.*, **30** (1991) 327.
[19] Rodi W., *Turbulence Models and their Applications in Hydraulics - A State of Art Review (Report)* (Institut fur Hydromechanik, University of Karlsruhe) 1980.
[20] Garrat J. R., *The Atmospheric Boundary Layer* (Cambridge University Press, New York) 1994.
[21] Kosovic B. and Curry J. A., *J. Atmos. Sci.*, **57** (2000) 1052.
[22] Cuxart J., Holtslag A. A. M., Beare R. J., Bazile E., Beljaars A., Cheng A., Conangla L., Ek M., Freedman F., Hamdi R., Kerstein H., Kitagawa H., Lenderink G., Lewellen D., Mailhot J., Mauritsen T., Perov V., Schayes G., Steeneveld G-J., Svensson G., Taylor P., Weng W., Wunsch S. and Xu K-M., *Boundary-Layer Meteorol.*, **118** (2006) 273.
[23] Beare R. J., MacVean M. K., Holtslag A. A. M., Cuxart J., Esau I., Golaz J-C., Jimenez M. A., Khairoutdinov M., Kosovic M., Lewellen D., Lund T. S., Lundquist J. K., McCabe A., Moene A. F., Noh Y., Raasch S. and Sullivan P. P., *Boundary-Layer Meteorol.*, **118** (2006) 247.