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Application of the Post-Widder Laplace inversion algorithm to postseismic rebound models

V. Cannelli(1), D. Melini(1), A. Piersanti(1) and G. Spada(2)

(1) *Istituto Nazionale di Geofisica e Vulcanologia, Sezione di Sismologia e Tettonofisica Rome, Italy*
(2) *Istituto di Fisica, Università di Urbino "Carlo Bo" - Urbino, Italy*

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Summary. — The postseismic response of a viscoelastic Earth can be computed analytically with a normal-mode approach, based on the application of propagator methods. This framework suffers from many limitations, mostly connected with the solution of the secular equation, whose degree scales with the number of viscoelastic layers so that only low-resolution models can be practically solved. Recently, a viable alternative to the normal-mode approach has been proposed, based on the Post-Widder inversion formula. This method allows to overcome some of the intrinsic limitations of the normal-mode approach, so that Earth models with arbitrary radial resolution can be employed and general linear non-Maxwell rheologies can be implemented. In this work, we test the robustness of the method against a standard normal-mode approach in order to optimize computation performance while ensuring the solution stability. As an application, we address the issue of finding the minimum number of layers with distinct elastic properties needed to accurately describe the postseismic relaxation of a realistic Earth model.

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1. – Introduction

Besides transient phenomena related to seismic wave propagation, earthquakes induce a series of permanent effects, which can be detected with modern instrumental techniques. Among these, one of the most studied is the permanent coseismic deformation field, which for exceptionally large events can be still of the order of millimeter at thousands of kilometers from the epicenter [1]. This permanent deformation field evolves with time due to the relaxation of ductile astenospheric layers, giving a continous postseismic deformation whose features are directly related to the physical properties of the involved layers. A precise modelistic estimate of deformation patterns induced by a seismic event on global scale is therefore a valuable tool to investigate coseismic and postseismic deformations, allowing to obtain information on the characteristics of the seismic event and on the physics of the Earth's interior.

Estimating deformation effects induced by giant earthquakes clearly requires the use of spherical Earth models, taking into account self-gravitation. This class of models has been developed theoretically in the last decade [2-4] within the normal-mode framework (hereafter NM), which was originally introduced by Peltier [5]. The main shortcoming of this approach is the solution of the "secular equation", whose polynomial degree scales with rheological model complexity; as a consequence, only coarse models can be employed in order to avoid numerical instabilities. Moreover, the introduction of realistic rheological laws results in algebraic complexities that can be (eventually) tackled only with the aid of symbolic manipulators, so that in practice only a simple Maxwell law can be modeled.

To overcome these limitations, several workarounds have been proposed in the literature [6-8] either by invoking purely numerical stages in the solution scheme or by introducing *a priori* assumptions on some characteristics of the solution. Recently, a new solution scheme based on the application of the "Post-Widder formula" [9, 10] has been proposed [11]; with this method, the structure of the NM formalism is preserved but the explicit solution of the secular equation is not needed, so that stratification models with arbitrary layering resolution can be employed. At the same time, the Post-Widder algorithm (hereafter PW) permits a straightforward implementantion of general (possibly transient) linear rheological laws in addition to the Maxwell law.

2. – Theoretical background and numerical issues

The analytic solution of a postseismic rebound model with Maxwell viscoelastic rheology is usually carried out in the Laplace domain because, owing to the Correspondence Principle of linear viscoelasticity [12], the governing equations of the model become formally identical to the elastic case and a solution can be obtained with standard propagation techniques. However, once the solution is obtained, it has to be transformed back to the time domain. This inversion can be performed explicitly by expressing it in terms of a Bromwich path integral and invoking the Residue Theorem; this requires the knowledge of the poles of the Laplace-transformed solution, which are the (isolated) roots of the equation

$$\left| \mathbf{P} \left[\prod_{i=1}^{N} \mathbf{Y}_i(r_i, s) \mathbf{Y}_i^{-1}(r_{i+1}, s) \right] \mathbf{I}_c \right| = 0, \tag{1}$$

where N is the number of layers, r_i and $\mathbf{Y}_i$ are, respectively, the top radius and the fundamental matrix of each layer, $\mathbf{P}$ is a projector matrix and $\mathbf{I}_c$ represents boundary conditions at the core-mantle boundary [13]. It can be shown that, if a linear rheological law is assumed, the polynomial degree of eq. (1) is $6N$. This represents a serious limitation to the range of practically solvable Earth models, since for high polynomial degrees all root-finding algorithms become numerically unstable.

The PW inversion algorithm, in its discretized form, provides an approximate expression to evaluate the Laplace inverse of a function by sampling the values of the transform on the positive real axis. Since for a stably stratified incompressible Earth the roots of the secular equation are placed along the real negative axis [14], the sampling region is singularity free, which makes the PW formula a viable alternative to the NM approach. The main shortcoming of the PW inversion method is its slow (less than logarithmic) convergence. Moreover, in the discretized form of the PW method, the antitransform is expressed in terms of a sum of oscillating terms which may lead to catastrophic cancellation. To avoid this possibility, the computation must be performed with high-precision floating point arithmetic, through the use of a high-level multiprecision library, which leads to a considerable performance degradation with respect to native hardware formats.

In order to minimize the computational requests of a PW code, it is necessary to carefully find the minimum floating point precision needed to carry out the whole computation without numerical degenerations. To this aim, a comprehensive set of numerical benchmarks has been performed by comparing the results of the PW algorithm with an independent NM code [11]. As a result, it has been obtained that NM results are correctly reproduced if a system precision ranging from 30 and 40 digits is employed. With these parameters, the evaluation of a single degree of the harmonic expansion of a postseismic deformation field requires about 1.7 s on a 1.6 GHz Intel Itanium2 CPU. Since the harmonic expansion of postseismic fields usually requires several thousands of terms to reach convergence [15], the computational requirements of the PW codes can be handled only by high-performance parallel systems.

3. – Impact of layering resolution on coseismic and postseismic rebound modeling

One of the key issues in postseismic deformation modeling is the effect of lithospheric and mantle layering. The application of the PW method to postseismic deformation models allows to investigate models with arbitrary layering resolution; it is therefore possible to quantitatively assess the minimum resolution needed to reproduce a realistic layering within a predefined precision. To this aim, we computed the postseismic deformation field with models of increasing radial resolution and studied their convergence to results obtained with a realistic reference Earth model, obtained as a discretization of the Preliminary Reference Earth Model [16], hereafter PREM. We defined three different approaches to the layered model definition: i) uniform layering from the core-mantle boundary to Earth surface; ii) homogeneous lithosphere and uniformly layered mantle; iii) high-resolution uniformly layered lithosphere and low-resolution uniformly layered mantle. For each layering strategy, we seek the minimum number of layers needed to reproduce reference results within a 5% threshold. As a result, we find that the fastest and most regular convergence to PREM results is obtained with layering approach (iii) with 25 layers. This is in agreement with the well-known importance of lithospheric structure in modeling surface coseismic displacements [17], and allows to estimate quantitatively the minimum radial resolution needed to model experimental data within their typical uncertainties [18].

4. – Conclusions

We discussed the application of the Post-Widder inversion formula to normal-mode analytical modeling of postseismic rebound. The PW approach allows to overcome the most striking limitations of this class of models, allowing to investigate layering structures of arbitrary resolution and generalized linear rheologies. These modelistic enhancements come at the cost of a considerable increase of computational requests, that can be fulfilled owing to the growing availability of high-performance computing facilities.

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