

Holographic scalar mesons

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Summary. — A holographic description of scalar mesons is presented, in which two- and three-point functions are holographically reconstructed. Mass spectrum, decay constants, eigenfunctions and the coupling of the scalar states with two pseudoscalars are found. A comparison of the results with current phenomenology is discussed.

PACS 11.25.Tq – Gauge/string duality.

PACS 12.38.Lg – Other nonperturbative calculations.

PACS 12.40.Yx – Hadron mass models and calculations.

PACS 14.40.Cs – Other mesons with $S = C = 0$, mass < 2.5 GeV.

1. – Introduction

Many approaches and techniques have been developed to understand QCD in its non-perturbative regime, but, up to now, no one has been completely satisfactory, leaving strongly coupled theories still a mystery. Recently, the possibility to apply AdS/CFT correspondence [1, 2] methods to (the large N limit of) strongly coupled gauge theories has been pointed out. This direction, known as AdS/QCD [3], has been followed along two main approaches. The first is a top-down approach, consisting in the attempt to obtain QCD-like theories as gravity duals of certain limits of well-defined superstring frameworks [4]. The second is a phenomenological approach, consisting in building a higher-dimensional model able to describe certain relevant degrees of freedom of strong interaction, assuming its validity as QCD dual. This is the approach followed in the present discussion. Many aspects of chromodynamics have been studied in this framework, like chiral symmetry breaking [5, 6], deep inelastic scattering [7], deconfinement transition [8, 9] and QQ potential [10, 11], form factors [12] and spectra [13-18]. In this paper, the model for chiral symmetry breaking introduced in [6] and, with a different aim, in [19] is used to investigate the scalar meson sector [15], which is still debated nowadays, due to its features at large N_c [20].

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2. – Model

The model is defined by the five-dimensional action

$$(1) \quad S = -\frac{1}{k} \int d^5x \sqrt{-g} e^{-\Phi(z)} \text{Tr} \left\{ |DX|^2 + m_5^2 X^2 + \frac{1}{2g_5^2} (F_V^2 + F_A^2) \right\}$$

in the five-dimensional Anti-de Sitter spacetime (the bulk), defined by the metric $g_{MN} = (R^2/z^2)\eta_{MN}$, where η_{MN} is the Minkowski metric tensor with signature $-++++$, R is the AdS radius and z is the fifth *holographic* coordinate $0 \leq z < \infty$. Every field is dual to a QCD operator defined on the boundary $z = 0$. $X = (X_0 + S)e^{2i\pi}$ is a scalar field, whose (negative) mass is fixed by the formula $m_5^2 R^2 = (\Delta - p)(\Delta + p - 4)$, where Δ is the dimension of the corresponding operator and p is the order of the p -form (*i.e.* $p = 0$). π is the chiral field and $X_0 = v(z)/2$ is dual to $\langle \bar{q}q \rangle$ and is responsible for chiral symmetry breaking. $S = S^A T^A = S_1 T^0 + S_8^a T^a$ with $T^0 = (1/\sqrt{6})\mathbf{1}$ and T^a the generators of $SU(3)_F$, ($A = \{0, a\}$, with $a = 1, \dots, 8$). S^A is dual to the QCD operator $\mathcal{O}_S^A = \bar{q} T^A q$ representing the scalar mesons. $F_V^{MN} = \partial^M V^N - \partial^N V^M - i[V^M, V^N] - i[A^M, A^N]$ and $F_A^{MN} = \partial^M A^N - \partial^N A^M - i[V^M, A^N] - i[A^M, V^N]$ are the strength tensors of the fields V_M^a and A_M^a , obtained rotating $A_{L,R}^a$, which are inserted to gauge, in the bulk, the global chiral symmetry $SU(3)_L \otimes SU(3)_R$, broken to $SU(3)_V$ by $\langle \bar{q}q \rangle$. $A_{L,R}^a$ are dual to the $\bar{q}_{L,R} \gamma^\mu T^a q_{L,R}$ currents. The field $\Phi(z) = c^2 z^2$, which in fact defines the model, is a non-dynamical field, inserted to break the conformal symmetry in the UV (c being a mass parameter).

Assuming for QCD the validity of the AdS/CFT relation

$$(2) \quad \left\langle \exp \left[i \int d^4x (\mathcal{L} + \varphi_0(x) \mathcal{O}(x)) \right] \right\rangle_{\text{QCD}} = e^{iS[\varphi(x,z)]},$$

where the l.h.s. is the QCD generating functional and $\varphi_0(x)$ is the boundary ($z \rightarrow 0$) value of the $5d$ field $\varphi(x, z)$, the effective action (1) is the only ingredient needed to evaluate correlation functions.

3. – Spectrum

To evaluate the spectrum of the scalar mesons, consider the quadratic action for the field S^A :

$$(3) \quad S_{\text{eff}} = -\frac{1}{2k} \int d^5x \sqrt{-g} e^{-\Phi} (g^{MN} \partial_M S^A \partial_N S^A + m_5^2 S^A S^A).$$

Looking for a solution of the equation of motion which is a plane wave in the $4d$ coordinates, $S^A(x, z) = e^{iq \cdot x} \tilde{S}(z)$, the masses are found solving a second-order linear differential equation, whose normalizable solutions represent the wave functions of the scalar mesons. The spectrum is discrete and the eigensystem is [21]

$$(4) \quad m_n^2 = c^2(4n + 6) \quad \tilde{S}_n(z) = \sqrt{\frac{2}{n+1}} c^3 z^3 L_n^1(c^2 z^2)$$

with L_n^1 the generalized Laguerre polynomials. The scalar mesons are then organized in a Regge trajectory and, fixing the parameter c with the ρ -meson mass, $c = m_\rho/2$ [6], they

turn out to be heavier than vector mesons, with $m_0 = 943 \text{ MeV}$, and in good agreement with the experimental masses, considering $a_0(980)$ and $f_0(980)$ as the lightest scalar states.

4. – Two-point correlation function

The next step is to evaluate the two-point correlation function, defined in QCD as

$$(5) \quad \Pi_{\text{QCD}}^{AB}(q^2) = i \int d^4x e^{iq \cdot x} \langle 0 | T [\mathcal{O}_S^A(x) \mathcal{O}_S^B(0)] | 0 \rangle.$$

Writing $\tilde{S}(q^2, z^2) = S(q^2, z^2) \tilde{S}_0(q^2)$, with $\tilde{S}_0(q^2)$ the Fourier transform of the source of the operator \mathcal{O}_S in the QCD generating functional and $S(q^2, z^2)$ a function called bulk-to-boundary propagator [2], and using (2), one can evaluate (5) deriving twice the effective action (1) with respect to \tilde{S}_0 , obtaining

$$(6) \quad \begin{aligned} \Pi_{\text{AdS}}^{AB}(q^2) &= \delta^{AB} \frac{R^3}{k} S(q^2, z^2) \frac{e^{-c^2 z^2}}{z^3} \partial_z S(q^2, z^2) \Big|_{z=1/\nu \rightarrow 0} \\ &= \delta^{AB} \frac{4c^2 R}{k} \left[\left(\frac{q^2}{4c^2} + \frac{1}{2} \right) \ln(c^2 z^2) + \left(\gamma - \frac{1}{2} \right) + \frac{q^2}{4c^2} \left(2\gamma - \frac{1}{2} \right) \right. \\ &\quad \left. + \left(\frac{q^2}{4c^2} + \frac{1}{2} \right) \psi(q^2/4c^2 + 3/2) \right] \Big|_{z=1/\nu}. \end{aligned}$$

This function has poles at $-q_n^2 = m_n^2 = c^2(4n + 6)$, in agreement with (4), with residues $F_n^2 = 16Rc^4(n + 1)/k$ corresponding to the decay constants of the scalar mesons. The factor R/k can be fixed with a comparison of (6) with the known QCD result, obtaining $R/k = N_c/(16\pi^2)$. The AdS prediction $F_0 = 0.08 \text{ GeV}^2$ can be compared to QCD determinations $F_{a_0} = (0.21 \pm 0.05) \text{ GeV}^2$ and $F_{f_0} = 0.18 \pm 0.015 \text{ GeV}^2$ [22, 23], showing a difference of about a factor of two.

5. – Three-point correlation function and interaction with two pseudoscalars

The three-point correlation function describing the interaction between a scalar meson and two pseudoscalars is defined in QCD by

$$(7) \quad \Pi_{\text{QCD}}^{abc}{}_{\alpha\beta}(p_1, p_2) = d^{abc} \frac{p_{1\alpha} p_{2\beta}}{p_1^2 p_2^2} f_\pi^2 \sum_{n=0}^{\infty} \frac{F_n g_{S_n PP}}{q^2 + m_n^2}$$

with $q = -(p_1 + p_2)$, f_π the pion decay constant and $g_{S_n PP}$ the coupling.

To evaluate it on the AdS side, consider the corresponding interaction term in (1):

$$(8) \quad S_{\text{eff}}^{SPP} = -\frac{R^3}{k} \int d^5x \frac{e^{-\Phi(z)}}{z^3} v(z) \left[\frac{2}{\sqrt{6}} S_1 (\partial\psi)^2 + d^{abc} S_8^a \eta^{MN} (\partial_M \psi^b) (\partial_N \psi^c) \right]$$

where $A_M = A_{\perp M} + \partial_M \phi$ and $\psi^a = \phi^a - \pi^a$ is the pseudoscalar dual field. Differentiating with respect to the sources, the coupling for the $n = 0$ state is given by

$$(9) \quad g_{S_0 PP} = \frac{m_{S_0}^2 R c \sqrt{N_c}}{4\pi f_\pi^2} \int_0^\infty du e^{-u^2} v(u)$$

with $u = cz$. The numerical result is of $\mathcal{O}(10)$ MeV, at odds with experimental values $g_{a_0 \eta \pi} = 12 \pm 6$ GeV and phenomenological determinations $g_{f_0 K^+ K^-} \simeq 7$ GeV [24]. This is an issue of the model and it is due to the fact that the integral in (9) is dominated by the small quark mass parameter. This is related to the difficulty of the model in correctly reproducing both spontaneous and explicit chiral symmetry breaking, difficulty already analyzed in [6].

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