IL NUOVO CIMENTO Vol. 32 C, N. 3-4 Maggio-Agosto 2009
DOI 10.1393/ncc/i2009-10441-5

Colloquia: IFAE 2009

On the critical end point of the QCD and the NJL model phase diagrams

M. Ruggieri

Università di Bari - Bari, Italy
INFN, Sezione di Bari - Bari, Italy

(ricevuto il 19 Settembre 2009; pubblicato online il 28 Ottobre 2009)

Summary. — In this paper I compare the knowledge on the critical end point of the QCD phase diagram grasped from lattice calculations, with that obtained from Nambu–Jona-Lasinio (NJL) model computations.

PACS 21.65.Qr – Quark matter.
PACS 12.38.Mh – Quark-gluon plasma.
PACS 12.38.Lg – Other nonperturbative calculations.

The major knowledge on the QCD phase transitions at zero baryon density comes from first principle calculations made on supercomputers, namely from the lattice. When simulations are run with physical quark masses, it is well known that lattice predicts the restoration of chiral symmetry, which is spontaneously broken by the quark condensate in the vacuum, at a finite value of the temperature $170\,\mathrm{MeV} \leq T \leq 200\,\mathrm{MeV}$. The chiral restoration in the vacuum is actually a smooth crossover, the reason being that finite values of the quark masses break explicitly chiral symmetry, hence there is not a true phase transition. For simplicity, from now on I will call the chiral crossover, as well as the true phase transition, the chiral restoration. In correspondence of the chiral restoration, lattice shows that a deconfinement transition occurs. This has suggested that chiral restoration and deconfinement of color are two intimately connected transitions of QCD, (see ref. [1] and references therein).

Lattice investigations at finite baryon chemical potential, μ, suffer the (in)famous sign problem in three color QCD. To this end, several approximated methods have been used to circumvent it. By means of one of these methods, namely the two-parameter reweighting, it has been predicted, some time ago [2], that the chiral crossover becomes a first-order transition at a certain value of μ. The couple (μ_E, T_E) in the (μ, T)-plane at which this occurs is called the Critical End Point (CEP) of the QCD phase diagram. The numerical simulations of ref. [2] predict $\mu_E \approx 350\,\mathrm{MeV}$ and $T_E \approx 160\,\mathrm{MeV}$.

An interesting alternative to the reweighting analysis of the QCD phase diagram, with particular reference to the existence of a CEP, has been performed [3] (see also references therein). The reasoning on which the investigations of [3] lies is very simple

to understand: at $\mu = 0$, it is known, from lattice studies, that the chiral transition is a true first-order transition, if quarks are taken in the chiral limit. Moreover, as the quark masses are increased above a critical value, the transition becomes a crossover. It happens that at the physical point, defined as the couple of values for the up- and strange-quark mass, (m_u, m_s), which gives the physical spectrum of mesons, the transition is a crossover. Hence, there exists a critical line in the (m_u, m_s)-plane which is the border between an inner region, in which the chiral transition is of first order, and an outer region, in which the transition is a crossover. As μ is increased, one can study the evolution of the critical line in this plane. In order to circumvent the sign problem, the authors of ref. [3] performed a Taylor expansion in powers of μ/T, computing all the coefficients at $\mu = 0$ (where the sign problems is absent). Within the Taylor expansion, the critical line is expressed as

$$m_c(\mu) = m_c(0) \left[1 + \sum_{k=1}^{N} c_k \left(\frac{\mu}{T_c} \right)^{2k} \right]. \tag{1}$$

The coefficient c_1 governs the behavior of the critical line at small values of μ. Nowadays, the coefficients c_k have been determined up to the 8th order. Surprisingly enough, the results of ref. [3] are that the critical line moves towards lower values (hence to less realistic) of the quark masses, as μ is increased. This means that at finite (but small, see below) values of μ the crossover remains crossover, if quarks are taken in the chiral limit. The analysis performed in ref. [3] should be reliable, by author's admission, up to $\mu \approx 500\,\mathrm{MeV}$. As a consequence, their results are consistent with the scenery in which a CEP, if it there exists, is located at values of μ larger than that predicted in [2]. The discrepancy is probably due to the fact that the reweighting method suffers of large systematic errors at large μ.

It is of a certain interest to compare this scenario with that of some model calculation. Among the various models, the NJL model (or its improved version, the Polyakov–Nambu–Jona-Lasinio (PNJL) one) is a very popular one (for review see [4]). The NJL model Lagrangian shares the same global symmetries of the QCD Lagrangian. Since we can describe the numerous (expected) phases of the QCD phase diagram in terms of broken/restored global symmetries, the hope is that the NJL calculations grasps, for the property specified above, at least the main characters of the QCD phase diagram in the μ-T plane. Moreover, determining the ground state of the model at any temperature and/or chemical potential is a very easy task, which requires only some numerics. On the other hand, first principle calculations are not feasible at finite μ both numerically (for the infamous sign problem of three color QCD) and analytically (for weak coupling approximation might break down in the range of temperature/chemical potential relevant for heavy-ion collisions as well as for compact star phenomenology). Therefore, the NJL model might be helpful in depicting the main aspects of the QCD phase diagram.

The NJL (or PNJL) phase diagram has been discussed in several papers. Here I refer to [5]. First of all, I need to specify the model Lagrangian density,

$$\mathcal{L} = \sum_f \bar{\psi}_f \left(i D_\mu \gamma^\mu - m_f + \mu \gamma_0 \right) \psi_f + \mathcal{L}_4 + \mathcal{L}_6, \tag{2}$$

where the sum is over the three flavors f ($= 1, 2, 3$ for u, d, s). In the above equation the background gauge field $A_\mu = g \delta_{\mu 0} A_{a\mu} T_a$ is coupled to quarks via the covari-

ant derivative $D_\mu = \partial_\mu - iA_\mu$ and A_μ will be specified later; m_f is the current mass (we assume $m_u = m_d$). The quark chemical potential is denoted by μ. The NJL four-fermion and six-fermion interaction Lagrangians are as follows [4]:

$$\mathcal{L}_4 = G\sum_{a=0}^{8}\left[(\bar\psi\lambda_a\psi)^2 + (i\bar\psi\gamma_5\lambda_a\psi)^2\right], \tag{3}$$

$$\mathcal{L}_6 = -K\big[\det\bar\psi_f(1+\gamma_5)\psi_{f'} + \det\bar\psi_f(1-\gamma_5)\psi_{f'}\big], \tag{4}$$

where λ_a are the Gell-Mann matrices in flavor space ($\lambda_0 = \sqrt{2/3}\,\mathbf{1}_f$) and the determinant is in flavor space as well. The parameters are

$$m_{u,d} = 5.5\,\text{MeV}, \quad m_s = 140.7\,\text{MeV}, \quad G\Lambda^2 = 1.835, \quad K\Lambda^5 = 12.36, \quad \Lambda = 602.3\,\text{MeV}.$$

From these parameters one gets $m_\pi \simeq 135\,\text{MeV}$, $m_K \simeq 498\,\text{MeV}$, $m_{\eta'} \simeq 958\,\text{MeV}$, $m_\eta \simeq 515\,\text{MeV}$ and $f_\pi \simeq 92\,\text{MeV}$.

Once the Lagrangian is specified, the thermodynamic potential at temperature T is obtained after integration over the fermion fields in the partition function:

$$\Omega = \mathcal{U}\big[T,\Phi,\bar\Phi\big] + \Omega_q\big[M_f,\Phi,\bar\Phi\big], \tag{5}$$

where Ω_q denotes the free quark contribution, as well as the interaction term of quarks with the Polyakov loop (see [5] for more details). In the thermodynamical potential, the term $\mathcal{U}(T,\Phi,\bar\Phi)$ is the novelty that improves the NJL model and promotes it to the PNJL model [6]. It describes the dynamics of the traced Polyakov loop in the absence of dynamical quarks. The potential $\mathcal{U}$ cannot be determined by first principles: one has to choose a convenient form for it, by trying to reproduce lattice data on thermodynamical quantities of the pure glue theory. Different analytical forms of $\mathcal{U}$ lead to different quantitative predictions, even if the qualitative picture is quite not sensible of the form chosen. In this paper I focus on a model calculation based on the following potential:

$$\frac{\mathcal{U}(T,\Phi,\bar\Phi)}{T^4} = -\frac{\tilde b_2(T)}{2}\bar\Phi\Phi + b(T)\ln\Big[1 - 6\bar\Phi\Phi + 4\big(\Phi^3 + \bar\Phi^3\big) - (\bar\Phi\Phi)^2\Big], \tag{6}$$

where the analytical form of the coefficients has been determined in ref. [7].

In the mean-field approximation, which is formally equivalent to determine only the classical contribution to the partition function, one can get quark condensates σ_f and Polyakov loop for any value of the parameters μ and T simply by looking at the global minima of Ω. Depending on the values of σ_f and Φ, one can characterize the symmetry breaking pattern of the theory in any point of the plane μ-T, hence one can build a phase diagram. The phase diagram of the model is sketched in fig. 1. For simplicity, I have drawn only the chiral crossover line. The dashed line denotes the chiral crossover, the solid line corresponds to a first-order transition. The region denoted symbolically by χ_{SB} denotes the zone of the phase diagram with quark condensate different from zero. In the region $\chi \approx 0$, on the other hand, one has $\langle\bar uu\rangle \approx 0$ but $\langle\bar ss\rangle \neq 0$. It is interesting to notice that, with the parameters at hand that reproduce the vacuum spectra of the pseudoscalar mesons, the CEP is located at quite large values of the quark chemical potential, which is one third of the baryon chemical potential, thus at values of μ larger than the 350 MeV quoted above. The introduction of a vector interaction can shift μ_{CEP}

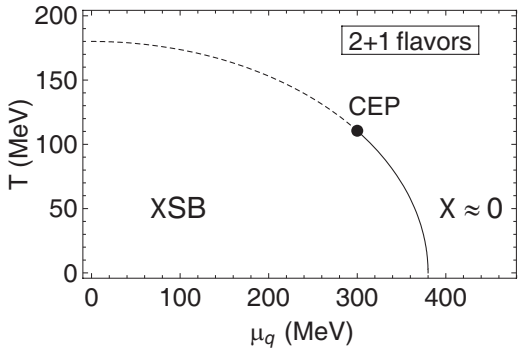

Fig. 1. – Sketch of the phase diagram of the PNJL model with $2+1$ massive flavors. Here μ_q denotes the quark chemical potential, $\mu_q = 3\mu$, where μ corresponds to the baryon chemical potential. For simplicity, I have drawn only the chiral crossover line. The dashed line denotes the chiral crossover, the solid line corresponds to a first-order transition. The region denoted symbolically by χ_{SB} denotes the zone of the phase diagram with quark condensate different from zero. In the region $\chi \approx 0$, on the other hand, one has $\langle \bar{u}u \rangle \approx 0$ but $\langle \bar{s}s \neq 0 \rangle$. Based on [5].

to higher values, depending on its magnitude at finite density [8]. It can even disappear at all, if the vector interaction is repulsive enough. Hence, we can conclude that the PNJL model scenario is in agreement with the newest lattice findings on the absence of a CEP at small values of the baryon chemical potential.

* * *

I acknowledge H. Abuki, M. Ciminale, P. Colangelo, R. Gatto, M. Mannarelli and S. Nicotri for valuable collaboration and interesting discussions on the topics discussed here. This talk is dedicated to the memory of my mentor, Prof. Giuseppe Nardulli.

REFERENCES

[1] Bazavov A. *et al.*, arXiv:0903.4379 [hep-lat].
[2] Fodor Z. and Katz S. D., *JHEP*, **0404** (2004) 050 [arXiv:hep-lat/0402006].
[3] de Forcrand P. and Philipsen O., *PoS LATTICE2008* 208 (2008) [arXiv:0811.3858 [hep-lat]].
[4] Klevansky S. P., *Rev. Mod. Phys.*, **64** (1992) 649; Hatsuda T. and Kunihiro T., *Phys. Rep.*, **247** (1994) 221 [arXiv:hep-ph/9401310]; Buballa M., *Phys. Rep.*, **407** (2005) 205 [arXiv:hep-ph/0402234].
[5] Ciminale M., Nardulli G., Ruggieri M. and Gatto R., *Phys. Lett. B*, **657** (2007) 64 [arXiv:0706.4215 [hep-ph]].
[6] Fukushima K., *Phys. Lett. B*, **591** (2004) 277 [arXiv:hep-ph/0310121].
[7] Roessner S., Ratti C. and Weise W., *Phys. Rev. D*, **75** (2007) 034007 [arXiv:hep-ph/0609281].
[8] Fukushima K., *Phys. Rev. D*, **77** (2008) 114028; **78** (2008) 039902 (Erratum) [arXiv: 0803.3318 [hep-ph]].