IL NUOVO CIMENTO Vol. 32 C, N. 3-4 Maggio-Agosto 2009
DOI 10.1393/ncc/i2009-10514-5

Colloquia: IFAE 2009

Equilibrium sequences of hybrid stars with LOFF matter core

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(ricevuto il 19 Settembre 2009; pubblicato online il 21 Dicembre 2009)

Summary. — We study equilibrium configurations of hybrid stars with inhomogeneous Color SuperConducting (CSC) phases in the inner core and a mantle of nuclear matter.

PACS 12.38.Mh – Quark-gluon plasma.
PACS 24.85.+p – Quarks, gluons, and QCD in nuclear reactions.

Since the publication of the first articles about Color SuperConductivity, it was clear that the primary place where these phases could be searched is in the core of very compact stars, generically named Neutron Stars. Matter in the interiors of neutron stars is compressed by gravity to densities much larger than the density of an ordinary nucleus (by factors up to 10). At such densities baryons are likely to lose their identity and dissolve into deconfined quarks. If compact (hybrid) stars featuring quark cores surrounded by a nuclear mantle exist in nature, they could provide a unique window on the properties of QCD at high baryon densities.

Following the analysis given in [1], we study the possibility that the equation of state of matter at high densities admits stable configurations of self-gravitating objects in General Relativity featuring deconfined quark matter, and if so, we check if the gross parameters of these objects, like their mass and radius, are compatible with the known astronomical bounds.

Because of β-equilibrium in the light quark sector the chemical potentials of u and d quarks obey the constraint $\mu_d = \mu_u + \mu_e$. Furthermore, the Fermi surface of strange quarks is mismatched with the Fermi surfaces of light u and d quarks because of the large strange quark mass. We have already incorporated these features in our previously described studies about the three flavor LOFF phases [2,3].

The nuclear equation of state can be constructed starting from a number of different principles [4]. We tested a large number of equations of state to construct hybrid star configurations. These belong to the classes of i) non-relativistic variational and Bruckner-Hartree-Fock theories which use as an input a non-relativistic potential fitted to the elastic nucleon-nucleon scattering data; ii) relativistic mean-field models which are fitted to the bulk properties of nuclear matter, and iii) relativistic models which include correlations at the level of the covariant scattering amplitude (Dirac-Bruckner-Hartree-Fock theories). In the analysis presented here, it was sufficient to characterize these equations of states by their stiffness; as we shall see, only the stiffest equations of state are admissible for phase equilibrium between nuclear and quark matter.

A straightforward normalization of the quark pressure in the NJL model requires that the pressure vanishes at zero density and temperature. In the terminology of the MIT bag model, this is equivalent to subtraction of a bag constant from the thermodynamic potential. Since the value of the bag constant is related to confinement which is absent in the NJL model, it appears reasonable to consider changes in its value, and hence in the normalization of the pressure. We shall consider the simple case of a constant shift in the asymptotic value of the pressure. In ref. [3] the calculation of masses and condensates has been carried out for $\eta = G_D/G_S = 0.75$. This regime is usually referred to as "intermediate coupling". We shall adopt a "strong coupling" regime with $\eta = 1$, since only in the latter case the matching to the nuclear equations of state can be performed without variations in the bag constant (we will discuss this point in more detail in the following subsection).

Physically, the true nuclear equation of state must go over to some sort of quark equation of state at some density if deconfinement takes place in nature. Since we have only models of deconfined matter and nuclear matter, this transition is modeled by requiring that there exists a baryo-chemical potential at which the pressures of these phases are equal. This is equivalent to the condition that the pressure *vs.* the chemical potential curves $P(\mu)$ for these phases cross (matching). If the $P(\mu)$ curves for the chosen equations of state of nuclear and quark matter do not cross, the models are incompatible in the sense that they cannot describe the desired transition between nuclear and quark matter. The low-density equation of state of nuclear matter and the high-density equation of state of CCS matter are matched at an interface via the Maxwell construction. The phase with largest pressure is the one that is realized at a given chemical potential.

The high-density regime contains two equations of state for crystalline color superconductivity which differ for the normalization of pressure at zero density (or, equivalently, the value of the bag constant). The model A1 is normalized such that the pressure vanishes at zero density. For the models A and B the zero-density pressure is shifted by an amount $\delta p = 10\,\mathrm{MeV/fm}^3$. We are aware of the arbitrariness of the latter procedure, the sole practical purpose of which is to produce an equation of state which can be matched to a particular nuclear equation of state. Yet another possibility is to set $\delta p = 0$, but vary the value of the constituent masses of the light quarks in the fit of the parameters of the NJL model.

A set of nuclear equations of state were tested for matching with the models above; it included about dozen equations of state, listed in refs. [4]. Only two equations of state based on the Dirac-Bruckner-Hartree-Fock approach are suitable to match with the quark equations of state presented above. These are shown in both panels of fig. 1.

We consider equilibrium and stability of cold hybrid stars with LOFF cores. Each equation of state defines a sequence of equilibrium, non-rotating stellar configurations in General Relativity, which can be parameterized in terms of the central density ρ_c of the configuration. It is assumed that the configurations are cold ($T \simeq 0$). The spherically symmetric solutions of Einstein's equations for self-gravitating fluids are given by the well-known Tolman-Opennheimer-Volkoff equations. A generic feature of these solutions is the existence of a maximum mass for any equation of state; as the central density is increased beyond the value corresponding to the maximum mass, the stars become unstable towards collapse to a black hole. One criterion for the stability of a sequence

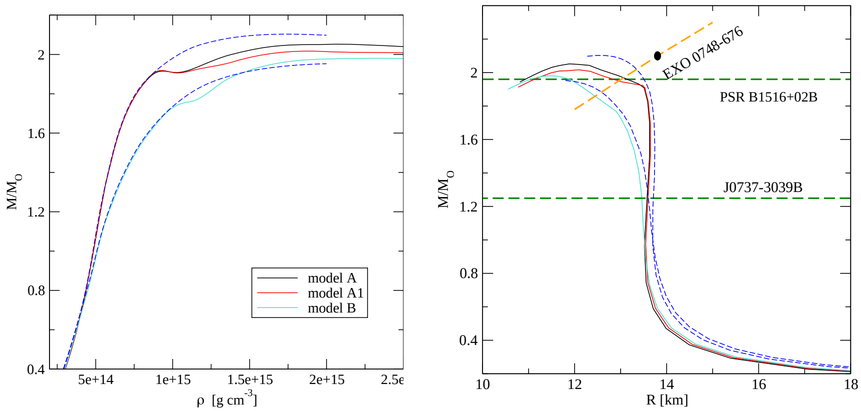

Fig. 1. – Left panel: dependence of masses of hybrid, non-rotating compact stars on their central density for the models A, A1, and B. The dashed lines show the same for the associated nuclear equations of state. M_0 stays for the mass of the Sun. We use indifferently the notations M_0 and $M_\odot$. Right panel: mass-radius diagram for non-rotating configurations including the bounds from EXO 0748-676 and the bounds on the upper and lower pulsar masses. The sequences of hybrid configurations for model A (*heavy, black online*), A1 (*medium-light, red online*), and B (*light, blue online*) are shown by solid lines. Models A and A1 share the same nuclear (low-density) equation of state, while the models A and B share the same quark (high-density) equation of state. The dashed lines are the sequences of purely nuclear stars with underlying nuclear equations of state of model A (equivalently A1) and model B.

of configurations is the requirement that the derivative $\mathrm{d}M/\mathrm{d}\rho_c$ should be positive (the mass of the star should be an increasing function of the central density). At the point of instability the fundamental (pulsation) modes become unstable. If stability is regained at higher central densities, the modes by which the stars become unstable towards the eventual collapse belong to higher-order harmonics.

For configurations constructed from a purely nuclear equation of state the stable sequence extends up to a maximum mass of the order 2 $M_\odot$ (right panel of fig. 1, dashed lines); the value of the maximum mass is large, since our chosen equations of state are rather hard. The hybrid configurations branch off from the nuclear configurations when the central density reaches that of the deconfinement phase transition. The jump in the density at constant pressure causes a plateau of marginal stability beyond the point where the hybrid stars bifurcate. This is followed by an unstable branch ($\mathrm{d}M/\mathrm{d}\rho_c < 0$). Most importantly, the stability is regained at larger central densities: a stable branch of hybrid stars emerges in the range of central densities $1.3 \leq \rho_c \leq 2.5 \times 10^{15}\,\mathrm{g\,cm^{-3}}$. The models A and B feature the same high-density quark matter, whereas the models A and A1 the same nuclear equation of state. It is seen that the effect of having different nuclear equations of state (the models A and B) at intermediate densities is substantial (at densities below $10^{13}\,\mathrm{g\,cm^3}$ all models are matched to the same equation of state). At the same time, the small shift δp by which the models A and A1 differ does not influence the masses of stable hybrid stars, although it is necessary for matching of nuclear and quark EOS in the models A and B. It is evident that there will exist purely nuclear and hybrid configurations with different central densities but the same masses. This is reminiscent of the situation encountered in non-superconducting hybrid stars [5]; the second branch of hybrid stars was called twin, since for each hybrid star there always exists a counterpart with the same mass composed entirely of nuclear matter.

The right panel of fig. 1 displays the astronomical bounds on the masses and radii of compact stars along with the "tracks" for our models on the mass-radius diagram. All bounds are quoted at the 1σ level. The bound inferred from the star EXO0748-676, which combines information from redshifted O and F lines, the emitting area of X-ray radiation, and the Eddington luminosity, constrain the mass and the radius of a compact star to lie on a straight line shown in the right panel of fig. 1 [6]. Both the hybrid stars and their nuclear counterparts have masses and radii within these bounds. For sequences constructed from models A and A1 there is a range of masses and radii that correspond to the new family of twin stars discussed above. The stars belonging to the new family lie to the left from the sharp kink at the point $R \simeq 13.5\,\text{km}$ and $M \simeq 1.9 M_\odot$. The stable branch of this new family of stars is separated from the stable nuclear sequence by an instability region. The sequences corresponding to model B do not show such an instability region.

Some evidence for massive neutron stars with $M \sim 2M_\odot$ has been inferred from recent measurements on the pulsar PSR B1516+02B in the Globular Cluster M5 gave $M = 1.96 \pm 0.1 M_\odot$ [7]. The models A and A1 are consistent with these bounds. For the model B these bounds correspond to the stable configuration with the largest mass. For completeness, the lower bound on the neutron star mass 1.249 ± 0.001 $M_\odot$ is shown in the right panel of fig. 1, which is inferred from the millisecond binary J0737-3039 [8].

We stress that our models of color superconducting hybrid stars with LOFF matter core can be used to obtain quantitative limits on the strain of gravitational wave emission that can be emitted. This topic will be studied in a future work.

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