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Measurements of CKM parameters at the B factories

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Summary. — In the past 10 years our knowledge of the elements of the Cabibbo-Kobayashi-Maskawa matrix has improved substantially. This article reviews some of the many contributions from the B factories to this progress, and discusses their implication in terms of understanding CP violation in the Standard Model and beyond.

PACS 11.30.Er – Charge conjugation, parity, time reversal, and other discrete symmetries.
PACS 12.15.Hh – Determination of Kobayashi-Maskawa matrix elements.
PACS 13.25.Hw – Decays of bottom mesons.

1. – The Unitarity Triangle

According to Kobayashi and Maskawa [1], CP violation in the Standard Model (SM) is due to a complex phase appearing in the quark mixing matrix, the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Following Wolfenstein's notation [2], the CKM matrix can be expressed in terms of the four real parameters λ, A, ρ and η as

$$(1)\quad \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4).$$

While the parameters λ and A have been precisely known for a long time, the parameters ρ and η were poorly measured until recently. The parameter η is of particular interest, because if $\eta = 0$ the Standard Model would not be able to explain CP violation. If the CKM matrix is unitary, then $V^+V = 1$. This implies six unitarity conditions that

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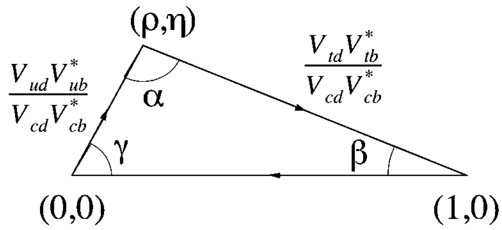

Fig. 1. – The Unitarity Triangle.

relate the nine elements of the matrix. The condition that relates the first and third columns of the matrix can be written as

$$\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} + 1 = 0. \tag{2}$$

This equation represents a triangle in the complex (ρ, η)-plane with the base normalized to 1. This triangle, knows as the Unitarity Triangle (UT), is depicted in fig. 1. The angles (α, β, and γ) and sides of the triangle are defined in fig. 1.

The study of B-meson decays allows us to perform a number of measurements that set constraints in the (ρ, η)-plane. In the Standard Model all measurements must be consistent. The presence of New Physics could cause inconsistencies for some of the measurements of $\approx 10\%$. A redundant and precise set of measurements providing constraints in the (ρ, η)-plane is therefore essential to test the CKM mechanism and probe for New Physics beyond the Standard Model.

The main contributors to this physics program are the two experiments at the asymmetric B factories, BABAR [3] and Belle [4]. Collectively, these experiments recorded to date over one billion $B\overline{B}$ pairs in e^+e^- interactions at the $\Upsilon(4S)$ resonance. The large data set and clean experimental environment allowed the B factories to measure all sides and angles of the UT. The two Tevatron experiments, CDF and DO, add constraints from their measurement of B_s^0 mixing. In addition, several kaon experiments provide complementary information by measuring the CP-violating parameter ϵ_K in K^0 decays.

2. – CP violation in B^0 decays

The angles of the UT can be determined through the measurement of the time dependent CP asymmetry, $A_{CP}(t)$. This quantity is defined as

$$A_{CP}(t) \equiv \frac{N(\overline{B}^0(t) \to f_{CP}) - N(B^0(t) \to f_{CP})}{N(\overline{B}^0(t) \to f_{CP}) + N(B^0(t) \to f_{CP})}, \tag{3}$$

where $N(\overline{B}^0(t) \to f_{CP})$ is the number of $\overline{B}^0$ that decay into the CP-eigenstate f_{CP} after a time t.

In general, this asymmetry can be expressed as the sum of two components:

$$A_{CP}(t) = S_f \sin(\Delta m t) - C_f \cos(\Delta m t), \tag{4}$$

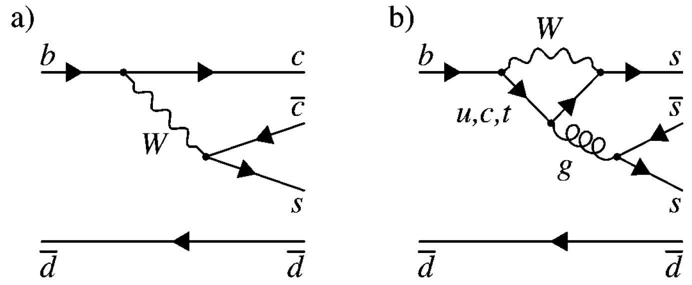

Fig. 2. – Feynman diagrams that mediate the B^0 decays used to measure the angle β: a) $B^0 \to \text{charmonium} + K^0$; b) penguin-dominated B decays.

where Δm is the difference in mass between B^0 mass eigenstates. The sine coefficient S_f is related to an angle of the UT, while the cosine coefficient C_f measures direct CP violation.

When only one diagram contributes to the final state, the cosine term in eq. (4) vanishes. As an example, for decays such as $B \to J/\psi K^0$, $S_f = -\eta_f \times \sin 2\beta$, where η_f is the CP eigenvalue of the final state, negative for charmonium $+ K_S$, and positive for charmonium $+ K_L$. It follows that

$$A_{CP}(t) = -\eta_f \sin 2\beta \sin(\Delta m t), \quad (5)$$

which shows how the angle β is measured by the amplitude of the time dependent CP asymmetry.

The measurement of $A_{CP}(t)$ utilizes decays of the $\Upsilon(4S)$ into two neutral B-mesons, of which one (B_{CP}) can be completely reconstructed into a CP eigenstate, while the decay products of the other (B_{tag}) identify its flavor at decay time. The time t between the two B decays is determined by reconstructing the two B decay vertices. The CP asymmetry amplitudes are determined from an unbinned maximum likelihood fit to the time distributions separately for events tagged as B^0 and $\overline{B}^0$.

3. – The angle β

The most precise measurement of the angle β of the UT is obtained in the study of the decay $B^0 \to \text{charmonium} + K^0$. These decays, known as "golden modes," are dominated by a tree level diagram $b \to c\bar{c}s$ with internal W boson emission (fig. 2a). The leading penguin diagram contribution to the final state has the same weak phase as the tree diagram, and the largest term with different weak phase is a penguin diagram contribution suppressed by $O(\lambda^2)$. This makes $C_f = 0$ in eq. (4) a very good approximation.

Besides the theoretical simplicity, these modes also offer experimental advantages because of their relatively large branching fractions ($\sim 10^{-4}$) and the presence of narrow resonances in the final state, which provide a powerful rejection of combinatorial background. The CP eigenstates considered for this analysis are $J/\psi K_S$, $\psi(2S)K_S$, $\chi_{c1}K_S$, $\eta_c K_S$ and $J/\psi K_L$.

The asymmetry between the two Δt distributions, clearly visible in fig. 3 is a striking manifestation of CP violation in the B system. The same figure also displays the corresponding raw CP asymmetry with the projection of the unbinned maximum-likelihood

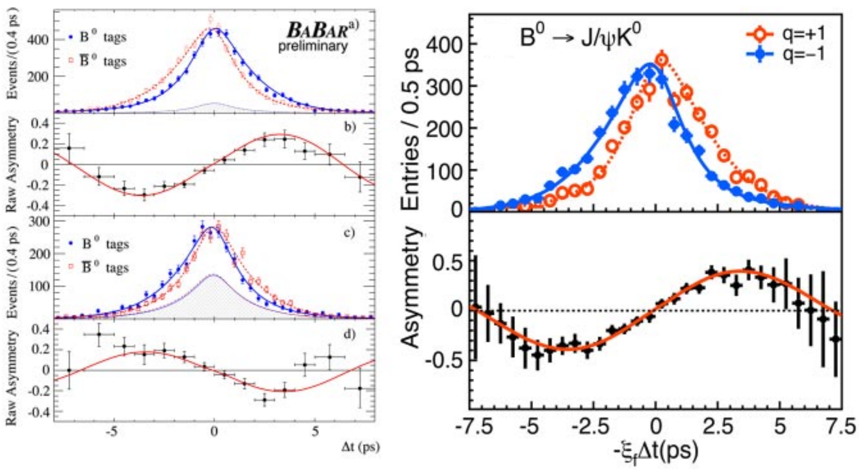

Fig. 3. – Measurements of $\sin 2\beta$ in the "golden modes" by *BABAR* (left) and Belle (right). Left plot (*BABAR*): a) time distributions for events tagged as B^0 (full dots) or $\overline{B}^0$ (open squares) in CP odd (charmonium K_S) final states; b) corresponding raw CP asymmetry with the projection of the unbinned maximum-likelihood fit superimposed; c) and d) corresponding distributions for CP even ($J/\psi K_L$) final states. Right plot (Belle): top) time distributions for events tagged as B^0 (open dots) or $\overline{B}^0$ (full dots) in charmonium K_S final states; bottom) corresponding raw CP asymmetry with the projection of the unbinned maximum-likelihood fit superimposed.

fit superimposed. The measurements from *BABAR* [5] and Belle [6] are averaged to obtain $\sin 2\beta = 0.670 \pm 0.023$ [7]. This measurement provides the strongest constraints in the (ρ, η)-plane.

An independent measurement of the angle β through the study of B decays dominated by penguin diagrams allows us to search for physics beyond the Standard Model. In the SM, final states dominated by $b \to s\bar{s}s$ or $b \to s\bar{d}d$ decays offer a clean and independent way of measuring $\sin 2\beta$ [8]. Examples of these final states are ϕK^0, $\eta' K^0$, $f_0 K^0$, $\pi^0 K^0$, ωK^0, $K^+K^-K_S$ and $K_S K_S K_S$. These decays are mediated by the gluonic penguin diagram illustrated in fig. 2b. In presence of physics beyond the Standard Model, new particles such as squarks and gluinos, could participate in the loop and affect the time dependent asymmetries [9].

A summary of the measurements of $A_{CP}(t)$ in penguin modes by the *BABAR* [10-12] and Belle [13] experiments is reported in fig. 4. Each channel as well as the average of all the penguin modes are in agreement with the value of $\sin 2\beta$ measured in the golden mode within the experimental error.

4. – The angle α

If the decay $B^0 \to \pi^+\pi^-$ were dominated by the $b \to u$ tree level diagram, the amplitude of the time-dependent CP asymmetry in this channel would be a clean measurement of the parameter $\sin 2\alpha$. Unfortunately, the analysis is complicated by sizable contributions from the gluonic $b \to d$ penguin amplitudes to this final state. As a result, the fit to the time-dependent CP asymmetry (eq. (4)) must include both the sine and the cosine terms. The coefficient of the sine term measures the parameter $\alpha_{\rm eff}$, which

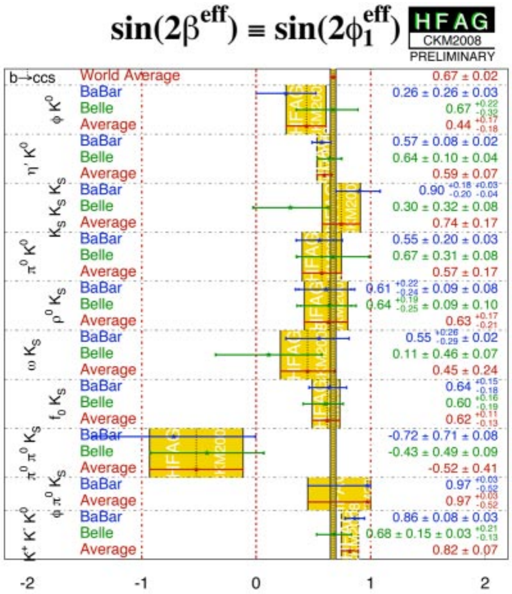

Fig. 4. – (Colour on-line) *BABAR* and Belle measurements of "$\sin 2\beta$" in the penguin-dominated channels. The narrow yellow band indicates the world average of the charmonium + K^0 final states $\pm 1\sigma$.

is related to the angle α of the UT through the correction $\Delta\alpha = \alpha - \alpha_{\rm eff}$. $\Delta\alpha$ can be extracted from an analysis of the branching fractions and CP asymmetries of the full set of isospin-related $b \to u\bar{u}d$ channels [14].

A similar measurement can be performed using the decays $B \to \rho\rho$. This analysis is complicated by the fact that since the ρ is a vector meson, $\rho^+\rho^-$ final states are characterized by three possible angular-momentum states, and therefore they are expected to be an admixture of $CP = +1$ and $CP = -1$ states. However, polarization studies [15-17] indicate that this final state is almost completely longitudinally polarized, and therefore almost a pure CP eigenstate, which simplifies the analysis.

A recent measurement [17] of the branching fraction of $B^+ \to \rho^+\rho^0$ by the *BABAR* Collaboration has substantially improved our knowledge of the UT angle α. The improvement is primarily due to the increase in the measured value of $\mathcal{B}(B^+ \to \rho^+\rho^0)$ compared to previous results. $B^+ \to \rho^+\rho^0$ determines the length of the common base of the isospin triangles for the B and $\overline{B}$ decays. The increase in the base length flattens both triangles, making the four possible solutions [14] nearly degenerate.

Additional constraints are obtained by the study of $B \to \rho\pi$ decays.

Combining all *BABAR* and Belle results, we measure $\alpha = (92^{+6.0}_{-6.5})^\circ$ [18]. This new result represents a substantial improvement over previous measurements of α.

5. – The angle γ

The angle γ is measured exploiting the interference between the decays $B^- \to D^{(*)0}K^{(*)-}$ and $B^- \to \overline{D}^{(*)0}K^{(*)-}$, where both D^0 and $\overline{D}^0$ decay to the same final state. This measurement can be performed in three different ways: utilizing decays

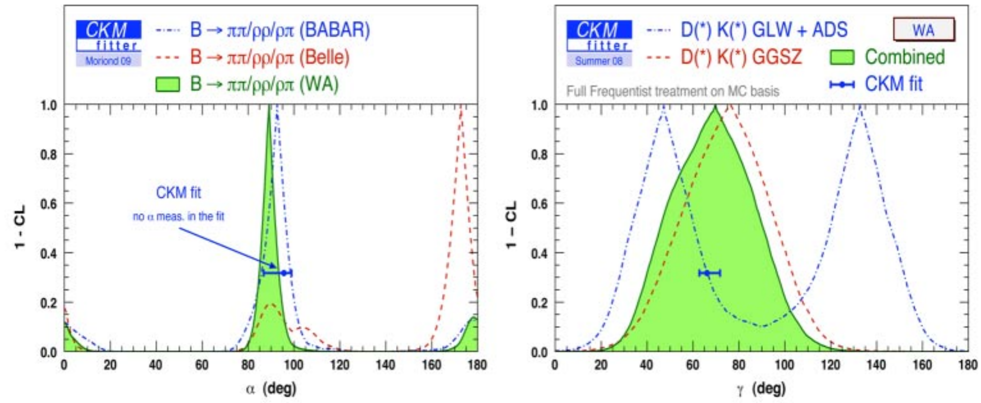

Fig. 5. – Constraints on the UT angles α (left) and γ (right) from various direct measurements compared with indirect constraints [18].

of D-mesons to CP eigenstates [19], utilizing doubly Cabibbo-suppressed decays of the $\overline{D}$-meson [20], and exploiting the interference pattern in the Dalitz plot of $D \to K_S\pi^+\pi^-$ decays [21]. Combining all results from *BABAR* and Belle, we measure $\gamma = (70^{+27}_{-29})^\circ$ [18] (see fig. 5).

6. – The left side of the Unitarity Triangle

The left side of the Unitarity Triangle is determined by the ratio of the CKM matrix elements $|V_{ub}|$ and $|V_{cb}|$. Both are measured in the study of semi-leptonic B decays. The measurement of $|V_{cb}|$ is already very precise, with errors of the order of 1–2% [7]. The determination of $|V_{ub}|$ is more challenging, mainly due to the large background coming from $b \to c\ell\nu$ decays, about 50 times more likely to occur than $b \to u\ell\nu$ transitions.

Two approaches, inclusive and exclusive, can be used to determine $|V_{ub}|$. In inclusive analyses of $B \to X_u\ell\nu$, the $b \to c\ell\nu$ background is suppressed by cutting on a number of kinematical variables. This implies that only partial rates can be directly measured, and theoretical assumptions are used to infer the total rate and extract $|V_{ub}|$. Averaging all inclusive measurements from the *BABAR*, Belle, and CLEO experiments we determine $|V_{ub}| = (3.96 \pm 0.20^{+0.20}_{-0.23}) \times 10^{-3}$ [7, 22], where the first error is experimental and the second theoretical.

In exclusive analyses, $|V_{ub}|$ is extracted from the measurement of the branching fraction $B \to \pi\ell\nu$. These analyses are usually characterized by a good signal/background ratio, but lead to measurements with larger statistical errors due to the small branching fractions of the mode studied. In addition, the theoretical errors are also larger, due to the uncertainties in the form factor calculation. Both experimental and theoretical errors are expected to decrease in the future, making this approach competitive with the inclusive method.

Further discussion on the measurement of $|V_{ub}|$ and $|V_{cb}|$ can be found in ref. [23].

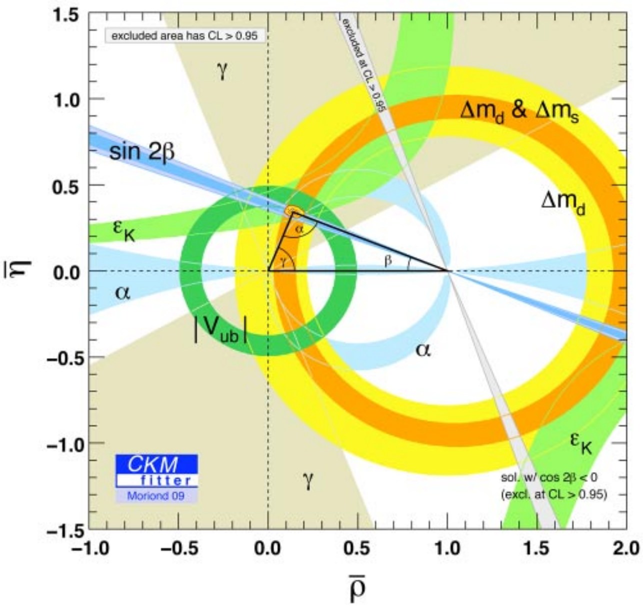

Fig. 6. – Constraints on the apex of the Unitarity Triangle resulting from all measurements.

7. – The right side of the Unitarity Triangle

The right side of the Unitarity Triangle is determined by the ratio of the CKM matrix elements $|V_{td}|$ and $|V_{ts}|$. This ratio can be determined with small ($\approx 4\%$) theoretical uncertainly from the measurement of ratio of the B_d^0 and B_s^0 mixing frequencies. Combining the measurements of Δm_s from the Tevatron [24] and world average Δm_d, we extract $|V_{td}/V_{ts}| = 0.2060 \pm 0.0007(\text{exp})^{+0.0081}_{-0.0060}(\text{theo})$ [25].

An independent determination of $|V_{td}/V_{ts}|$ can be obtained by the measuring the ratio of the branching fractions $BF(B \to \rho\gamma)/BF(B \to K^*\gamma)$. Recent measurements of the branching fractions of $B \to \rho\gamma$ from BABAR [26] and Belle [27] yield $|V_{td}/V_{ts}| = 0.210 \pm 0.015(\text{exp}) \pm 0.018(\text{theo})$. The comparison between the two measurements of $|V_{td}/V_{ts}|$ allows for an independent test of the Standard Model.

8. – Conclusion

Precise and redundant measurements of the sides and angles of the Unitarity Triangle provide a crucial test of CP violation in the Standard Model. The present constraints on the (ρ, η) plane are illustrated in fig. 6. The measurements of the angles β and α from the B factories provide two of the most precise constraints. The comparison shows good agreement between all measurements, as predicted by the CKM mechanism.

The accuracy of several measurements is now of the order of a few percent. This is about the level of precision needed for detecting $O(0.1)$ effects expected from New Physics. Final results from the B factories, results from new-generation flavor experiments, and progress in theory, especially lattice QCD, will be key to observing physics beyond the Standard Model in the flavor sector.

REFERENCES

[1] Kobayashi M. and Maskawa T., *Prog. Theor. Phys.*, **49** (1973) 652.
[2] Wolfenstein L., *Phys. Rev. Lett.*, **51** (1983) 1945.
[3] Aubert B. *et al.* (*BABAR* Collaboration), *Nucl. Instrum. Methods A*, **479** (2002) 1.
[4] Abashian A. *et al.* (Belle Collaboration), *Nucl. Instrum. Methods A*, **479** (2002) 117.
[5] Aubert B. *et al.* (*BABAR* Collaboration), *Phys. Rev. D*, **79** (2009) 072009.
[6] Chen K. F. *et al.* (Belle Collaboration), *Phys. Rev. Lett.*, **98** (2007) 031802.
[7] The Heavy Flavor Averaging Group, `http://www.slac.stanford.edu/xorg/hfag/`.
[8] London D. and Peccei R. D., *Phys. Lett. B*, **223** (1989) 257; Deshpande N. G. and Trampetic J., *Phys. Rev. D*, **41** (1990) 895; Fleischer R., *Z. Phys. C*, **62** (1994) 81; Deshpande N. G. and He X. G., *Phys. Lett. B*, **336** (1994) 471; Grossman Y., Ligeti Z., Nir Y. and Quinn H., *Phys. Rev. D*, **68** (2003) 015004; Gronau M. and Rosner J. L., *Phys. Lett. B*, **564** (2003) 90.
[9] Carter A. B. and Sanda A. I., *Phys. Rev. D*, **23** (1981) 1567; Bigi I. I. and Sanda A. I., *Nucl. Phys. B*, **193** (1981) 85; Fleischer R. and Mannel T., *Phys. Lett. B*, **511** (2003) 240; Grossman Y., Isidori G. and Worah M. P., *Phys. Rev. D*, **58** (1998) 057504; Grossman Y., Ligeti Z., Nir Y. and Quinn H., *Phys. Rev. D*, **68** (2003) 015004; Grossman Y. and Worah M. P., *Phys. Lett. B*, **395** (1997) 241; Fleischer R., *Int. J. Mod. Phys. A*, **12** (1997) 2459; London D. and Soni A., *Phys. Lett. B*, **407** (1997) 61.
[10] Aubert B. *et al.* (BABAR Collaboration), arXiv:0808.0700 [hep-ex]; Aubert B. *et al.* (BABAR Collaboration), *Phys. Rev. D*, **79** (2009) 052003.
[11] Aubert B. (The BABAR Collaboration), arXiv:0905.3615 [hep-ex].
[12] Aubert B. *et al.* (BABAR Collaboration), *Phys. Rev. D*, **76** (2007) 071101.
[13] Chen K. F. *et al.* (Belle Collaboration), *Phys. Rev. Lett.*, **98** (2007) 031802; Adachi I. *et al.* (Belle Collaboration), arXiv:0809.4366 [hep-ex]; Dalseno J. *et al.* (Belle Collaboration), *Phys. Rev. D*, **79** (2009) 072004; Abe K. *et al.* (Belle Collaboration), *Phys. Rev. D*, **76** (2007) 091103.
[14] Gronau M. and London D., *Phys. Rev. Lett.*, **65** (1990) 3381.
[15] Aubert B. *et al.* (BaBar Collaboration), *Phys. Rev. D*, **76** (2007) 052007.
[16] Aubert B. *et al.* (BaBar Collaboration), *Phys. Rev. D*, **78** (2008) 071104.
[17] Aubert B. *et al.* (BaBar Collaboration), arXiv:0901.3522 [hep-ex].
[18] The CKMfitter Collaboration, `http://ckmfitter.in2p3.fr/`.
[19] Gronau M. and Wyler D., *Phys. Lett. B*, **265** (1991) 172; Gronau M. and London D., *Phys. Lett. B*, **253** (1991) 483.
[20] Atwood D., Dunietz I. and Soni A., *Phys. Rev. Lett.*, **78** (1997) 3257.
[21] Giri A., Grossman Y., Soffer A. and Zupan J., *Phys. Rev. D*, **68** (2003) 054018.
[22] Gambino P., Giordano P., Ossola G. and Uraltsev N., *JHEP*, **0710** (2007) 058.
[23] Gambino P., these proceedings.
[24] Abulencia *et al.* (CDF Collaboration), *Phys. Rev. Lett.*, **97** (2006) 242003.
[25] Amsler C. *et al.* (Particle Data Group), *Phys. Lett. B*, **667** (2008) 1.
[26] Aubert B. *et al.* (BABAR Collaboration), *Phys. Rev. Lett.*, **98** (2007) 151802.
[27] Taniguchi N. *et al.* (Belle Collaboration), *Phys. Rev. Lett.*, **101** (111801) 2008; 129904 (Erratum).