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Precision determination of the top quark mass

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Summary. — The mass of the top quark is a fundamental parameter of the Standard Model and its measurement allows both to verify the consistency of the model predictions and to set constraints on possible, still unobserved physics. In this paper we present a selected review of the most recent or relevant results obtained by the CDF and D0 Collaborations using up to about 3.6 fb^{-1} of proton-antiproton collisions at $\sqrt{s} \simeq 1.96\,\mathrm{TeV}$ produced at the Tevatron collider.

PACS `14.65.Ha` – Top quarks.
PACS `12.15.Ff` – Quark and lepton masses and mixing.

1. – Introduction

The first observation of the top quark by the CDF and D0 experiments at the Fermilab Tevatron collider in 1995 [1, 2] was somehow expected because, in the framework of the Standard Model (SM), a weak isospin partner of the bottom quark, previously observed in 1977, is necessary. However, since its early measurement, the large value of the top quark mass (M_{top}) represented a really striking property of this particle, giving to the top a special role within the SM and suggesting also possible links to new physics. In fact, apart being itself a fundamental parameter of the SM, M_{top} is by far the largest mass among the ones of the observed fermions, and this makes the top quark contribution to higher-order corrections to many electroweak observables dominant. Therefore M_{top} plays a central role in checking the consistency of theoretical predictions of the SM. The radiative corrections apply also to the W-boson propagator, and therefore to the W mass, M_W, so that, as this also depends logaritmically on the mass of the hypothesized Higgs boson, precise measurements of M_W and M_{top} allow to set indirect constraints on the unpredicted value of the mass of this fundamental, but still unobserved particle of the SM. Moreover, possible contributions due to some unknown physics might also be constrained. Finally, the present value of M_{top} makes the Yukawa coupling to the Higgs field of $\mathcal{O}(1)$ and this could indicate a special role of the top quark in the mechanism of electroweak symmetry breaking.

All the reasons listed above make the accurate knowledge of M_{top} a really important issue and push the CDF and D0 Collaborations to measure the top quark mass in all possible topologies related to $t\bar{t}$ events production. Improvements of the results are obviously due to the increasing statistics, but also to innovative techniques used in the analyses.

2. – Top quark production, decay and signatures

At the Tevatron collider bunches of protons and antiprotons are brought into collision with a center-of-mass energy, $\sqrt{s}$, equal to 1.96 TeV. Data are collected by the multipurpose CDF and D0 detectors [3, 4] which have currently recorded on tape an integrated luminosity of about $6\,\text{fb}^{-1}$ each, even if the most updated analyses reported here use only up to $3.6\,\text{fb}^{-1}$. The goal for the end of Tevatron Run II is to collect up to $8\,\text{fb}^{-1}$ per experiment.

At this energy top quarks are predominantly produced in $t\bar{t}$ pairs by $q\bar{q}$ annihilation ($\sim 85\%$ of the times) or gluon-gluon fusion ($\sim 15\%$). In the SM framework they decay to a W boson and a b-quark with a branching ratio (BR) very close to 100% and, because of their large mass, this happens before any hadronization effect can take place. This implies that informations concerning the top quark can be obtained directly from its decay products. The different final states and signatures of $t\bar{t}$ events are defined by the subsequent decays of the W^+ and W^- bosons and their usual classification is as follows:

- *Di-lepton channel*, where both the W's decay to a charged lepton and a neutrino $t\bar{t} \to W^+ b\, W^- \bar{b} \to (l^+\nu) b\, (l^-\bar{\nu}) \bar{b}$. This represents about 9% of the $t\bar{t}$ events.
- *Lepton + jets channel*, with one of the W's decaying to leptons while the other one to hadrons, *e.g.*, $t\bar{t} \to W^+ b\, W^- \bar{b} \to (l^+\nu) b\, (q_1 \bar{q}_2) \bar{b}$, and a total BR of 45%.
- *All-hadronic channel* (or all-jets channel), where both the W's decay to quarks. This final state has a BR of 46%.

The current theoretical predictions for the $t\bar{t}$ ("signal") production cross-section at $\sqrt{s} = 1.96\,\text{TeV}$ are in the range 6.7–8.0 pb for $M_{\text{top}} = 172\,\text{GeV}/c^2$(¹) [5] so that one pair of top quarks is produced out of about 10^{10} inelastic $p\bar{p}$ collisions. This makes the measure of top quark properties a really challenging task, requiring tools and selection techniques exploiting at the best the peculiar features of the signal. In particular, these include algorithms for the efficient identification of high transverse momentum (p_T) charged leptons coming from W decay and for the reconstruction of hadronic jets by an appropriate clustering of energy depositions in the calorimeters. Identification of jets generated by b-quarks ("b-tagging") is fundamental in reducing the presence of background events and also the combinatoric problem related to possible jet-to-quark assignments, and is provided by vertex detectors allowing reconstruction of secondary vertices related to the decay of b-hadrons. In measuring M_{top}, the reconstruction of the kinematics of the event, and therefore of the energies of quarks and leptons in the final state is crucial. The estimate of the parton energy requires an accurate knowledge of the correction to be applied to the measured jet energy, because of the instrumental effects as well as the definition of jet clustering algorithms. The uncertainty on this factor (called Jet Energy Scale, JES) is currently of order 2–3% and represents the largest source of systematic uncertainty in most of M_{top} measurements.

(¹) This range of values takes into account the uncertainties and is calculated for CTEQ6.6 parton distribution functions.

3. – $M_{\rm top}$ measurement techniques

Apart from the peculiarities of each individual measurement and with a few exceptions, two main techniques are used by the CDF and D0 Collaborations to extract the value of $M_{\rm top}$ from a sample of selected events: the *Matrix Element Method* (ME) and the *Template Method* (TMT).

In the ME, the probability that an event, where a set $\vec{y}$ of variables is measured, come from pair production of top quarks with mass $M_{\rm top}$ or from a background process is defined by considering a possible kinematics $\vec{x}$ at parton level, evaluating its leading order differential cross-section $\mathrm{d}\sigma(\vec{x})$, which includes the calculation of the matrix element for the process, and multiplying it by the "transfer function" $\mathcal{W}(\vec{y}, \vec{x})$, representing the probability that the set of *observed* variables $\vec{y}$ corresponds to the parton level kinematics $\vec{x}$, taking into account the detector effects and the event reconstruction. This function obviously strictly depends on JES too. Integration over possible initial and final states as well as a sum over assignments of observed jets to the partons and solutions for undetected neutrinos momenta are then performed.

In the TMT, a set of n event observables, $\vec{y}$, sensitive to $M_{\rm top}$ is reconstructed and the event probability is simply defined by the distributions *expected* for these variables. These distributions, called "templates" are built from simulated background and $t\bar{t}$ events with various input values of $M_{\rm top}$ for the signal.

In both methods a likelihood for the total sample, written as the product of individual event probabilities, is then usually maximized as a function of $M_{\rm top}$ to extract its value as the one which gives the largest probability to observe the selected set of events.

The power of the ME comes from exploiting a lot of information from the reconstructed event, while its main disadvantage is represented by the intensive usage of computing resources required by the numerical integrations. On the contrary the TMT is computationally much less problematic, but has also a reduced statistical power as, usually, no more than one or two event observables are used to build the templates. Both the methods strictly depend on reliable Monte Carlo event generation and simulation of detector effects. Before the technique is applied to real data, results are usually calibrated by large sets of simulated experiments corresponding to known true values of the variables to be measured.

An important feature of most recent analyses in the lepton + jets and all-hadronic channels is that, in reconstructing the event kinematics, the four-momenta of jets assigned to the W-boson decaying hadronically can be used to constrain the JES, as their invariant mass must equal, within the uncertainties, the well-known mass of the W. This can be exploited through the dependence of the transfer function on JES in the ME and introducing some kind of reconstructed W mass among the templates in the TMT, so that the likelihood can be maximized as a function of $M_{\rm top}$ and JES simultaneously, providing the *in situ* calibration of the latter variable. This technique makes the largest part of the JES uncertainty a component of the statistical uncertainty on $M_{\rm top}$, therefore scaling down with the increasing of collected luminosity.

4. – Measurements in the di-lepton channel

The fully leptonic channel provides the candidate samples with the best signal-to-background ratio (S/B) because of the presence of two energetic, high-p_T leptons and the b-jets. Moreover the combinatoric problem in assigning jets to partons is small. Unfortunately it suffers of a small BR (about 5% if only channels including electron

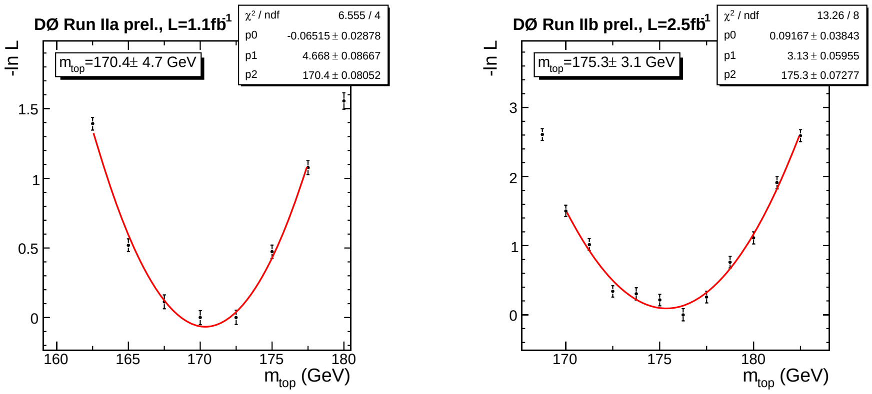

Fig. 1. – Uncalibrated ME likelihood evaluated by D0 analysis [6] in the di-lepton channel on data from two different periods. The minima of the curves denote the measured M_{top} before any calibration.

and/or muons from direct W's decays are considered as usual). The kinematics of the events is underconstrained because the reconstructed transverse missing energy, $\not\!\!E_T$, results from two undetected neutrinos so that assumptions and integrations are needed on unmeasured quantities.

Typical event selections, both in CDF and D0 analyses, require two identified oppositely charged leptons (e or μ) with large E_T ($E_{T,\,l} \geq 15\,\mathrm{GeV}$), at least two energetic hadronic jets ($E_{T,\,\mathrm{jet}} \geq 20\,\mathrm{GeV}$), and a large amount of missing transverse energy ($\not\!\!E_T \geq 25\,\mathrm{GeV}$). Further topological variables may also be used for additional cuts. The S/B can reach a value of about 10 when also b-tagging is required, where the main backgrounds are represented by di-boson events (ZZ, WW, WZ), Drell-Yan process and $W+\mathrm{jets}$ events where one of the jet is misidentified as a lepton.

The most updated result in this channel comes from the D0 experiment [6] and is obtained by a ME analysis performed in the channel including both an electron and a muon. Apart from direct decays of the W's to $e\nu$ or $\mu\nu$, also the possibility that this final state arise from $W \to \tau\nu_\tau \to (l\nu_l)\nu_\tau$, $l = e, \mu$, is considered, so that the total BR is about 3.2% and the dominant background contribution is the Z+jets production with the decay chain $Z \to \tau\tau \to (e\nu_e\nu_\tau)(\mu\nu_\mu\nu_\tau)$. In a data sample corresponding to a total of $3.6\,\mathrm{fb}^{-1}$, 154 candidates are selected with an expectation of about 118 $t\bar{t}$ and 23 background events. Figure 1 shows the likelihood function of M_{top} in two samples from different periods. The calibrated measurement yields $M_{\mathrm{top}} = 174.8 \pm 3.3\,(\mathrm{stat.}) \pm 2.6\,(\mathrm{syst.})\,\mathrm{GeV}/c^2$, with a relative precision $\delta M_{\mathrm{top}}/M_{\mathrm{top}} \approx 2.4\%$.

The ME result has been also combined with two TMT analyses using different algorithms to build the templates (the "Neutrino Weighting Algorithm" (NWA) and the "Matrix Weighting Method") and applied to about $1\,\mathrm{fb}^{-1}$ of data [7]. These analyses include a wider cathegory of di-lepton final states, with specific event selections, and templates are defined starting from event weights evaluated by the agreement between possible solutions of the underconstrained kinematics and the observed event topology. The result obtained in [7] is $M_{\mathrm{top}} = 174.7 \pm 4.4\,(\mathrm{stat.}) \pm 2.0\,(\mathrm{syst.})\,\mathrm{GeV}/c^2$, while the combination with [6] yields $M_{\mathrm{top}} = 174.7 \pm 2.9\,(\mathrm{stat.}) \pm 2.4\,(\mathrm{syst.})\,\mathrm{GeV}/c^2$ with a relative precision $\approx 2.2\%$.

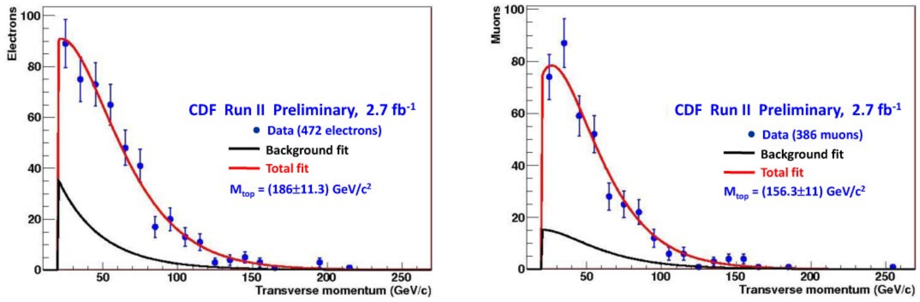

Fig. 2. – Fits of lepton p_T templates to the data in the electron (left) and muon (right) channels in the CDF analysis [10]. The separate results are then combined.

5. – Measurements in the lepton + jets channel

The final state including one charged lepton and jets (among which two are b-quark jets), is considered the "golden channel" as it concerns the measurements of top quark properties, including its mass. In fact it offers the best compromise between the purity of selected samples, reflected in S/B values up to 10 depending on the b-tag requirements, and the available statistics because of its BR of about 45% (30% if only electrons and muon channels are included). This allows both the Tevatron experiments to achieve the best results in this channel.

In particular the best results for M_{top} come from ME analyses [8, 9]. These are based on data samples typically selected by requiring that an event contains an energetic lepton (e or μ), four energetic jets ($E_{T,\text{jet}} \geq 20\,\text{GeV}$) and a good amount of missing energy ($\not{E}_T \geq 20\,\text{GeV}$). Moreover, to further reduce the background, at least one of the four jets must be tagged as a b jet. The main background sources are represented by $W +$jets events and multijet QCD events where one of the jet is misidentified as a lepton.

The CDF analysis [8] considers a data sample corresponding to $3.2\,\text{fb}^{-1}$, where 578 candidates are selected with an expected background of 134.1 ± 32.0 events. Here only the event probability for the $t\bar{t}$ process is explicitly calculated for each event and the average contribution of background events is subtracted to obtain a signal-only likelihood. After the calibration the latter is evaluated on the data sample and maximized to obtain $M_{\text{top}} = 172.1 \pm 0.9\,(\text{stat.}) \pm 1.1\,(\text{syst.})\,\text{GeV}/c^2$. The analysis from D0 [9] is applied to a total of $3.6\,\text{fb}^{-1}$ of data and 835 events are selected. The background probability is explicitly calculated by the matrix element of the dominant $W +$ jets process. The measurement yields $M_{\text{top}} = 173.7 \pm 0.8\,(\text{stat.}) \pm 1.6\,(\text{syst.})\,\text{GeV}/c^2$. Both results include the *in situ* calibration of the JES and represent the best individual measurements from each experiment, achieving a relative precision of ≈ 1% on the top quark mass.

In the same channel, a TMT analysis has been recently performed by the CDF experiment using $2.7\,\text{fb}^{-1}$ of data [10]. The variable used to build the templates is the transverse momentum of the lepton (electron or muon) identified in the event, so that interesting features of this analysis are that no event reconstruction is required and the uncertainty due to the JES is negligible because hadronic jets are not directly considered. Fitting the templates to distributions obtained by 472 and 382 events selected in the electron and muon channels respectively and then combining the results a value of $M_{\text{top}} = 172.1 \pm 7.9\,(\text{stat.}) \pm 3.0\,(\text{syst.})\,\text{GeV}/c^2$ is obtained, with $\delta M_{\text{top}}/M_{\text{top}} \approx 4.9\%$. Figure 2 shows results of fits to the data.

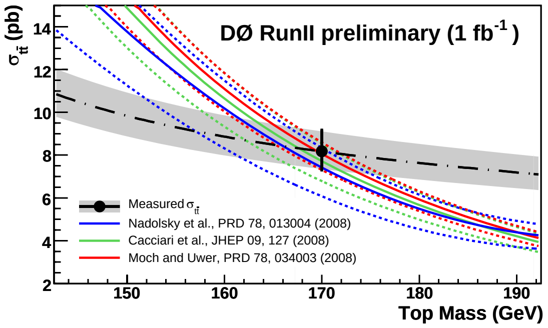

Fig. 3. – Parametrizations of measured and predicted $t\bar{t}$ cross-section, including uncertainties, as a function of the top quark mass, as presented in [12].

6. – Other measurements in the di-lepton and lepton + jets channels

Other interesting results have been recently obtained by the two collaborations exploiting simultaneously information from the di-lepton and lepton+jets channels. A CDF TMT analysis [11] performed with 3.2 fb^{-1} uses more variables to build two-dimensional templates. In particular, in the lepton + jets channel a top mass, m_t^{rec} is reconstructed for each event by a χ^2 fit constraining the event kinematics to the $t\bar{t}$ topology, while a second variable m_{jj}, related for signal events to the mass of the W boson, is used to have a constraint on the JES, allowing the *in situ* calibration. In the di-lepton channel two variables sensitive to M_{top} are used: m_{T2}, defined by reconstructed tranverse masses of the top quarks in the event, and m_t^{NWA}, defined by applying the NWA previously quoted. A simultaneous fit of templates to the observed distributions provides therefore *in situ* calibration of the JES also for events in the di-lepton channel and gives $M_{\mathrm{top}} = 171.7^{+1.4}_{-1.5}\,(\mathrm{stat.}) \pm 1.1\,(\mathrm{syst.})\,\mathrm{GeV}/c^2$.

A different method is applied by the D0 Collaboration [12]. In fact D0 measures M_{top} comparing a measurement of the $t\bar{t}$ production cross-section on 1 fb^{-1} of data to various theoretical predictions, including [5]. The cross-section values and their uncertainty are parametrized as a function of M_{top}, obtaining curves $\sigma^{\mathrm{obs}}(M_{\mathrm{top}}) \pm \delta\sigma^{\mathrm{obs}}(M_{\mathrm{top}})$ and $\sigma^{\mathrm{theo}}(M_{\mathrm{top}}) \pm \delta\sigma^{\mathrm{theo}}(M_{\mathrm{top}})$, as shown in fig. 3.

A likelihood including Gaussian terms both for the observed and theoretical cross-sections is maximized with respect to the unknown "true" values $\sigma_{t\bar{t}}$ and M_{top}. This yields, considering, *e.g.*, the calculation from Moch and Uwer in [5], $M_{\mathrm{top}} = 169.1^{+5.9}_{-5.2}\,\mathrm{GeV}/c^2$, but all the results are in a good agreement with the current World Average top quark mass from direct measurements [13]. This method provides complementary information, with different sensitivity to systematic uncertainties, with respect to the direct measurements and therefore represents also a consistency check.

7. – Measurements in the all-hadronic channel

The all-hadronic channel has the advantages of a large BR of about 46%, and of a fully reconstructed kinematics because ideally no particle from the $t\bar{t}$ system escapes the detector. The major downside is the huge background from QCD multijet production which dominates the signal by three orders of magnitude even after the application of

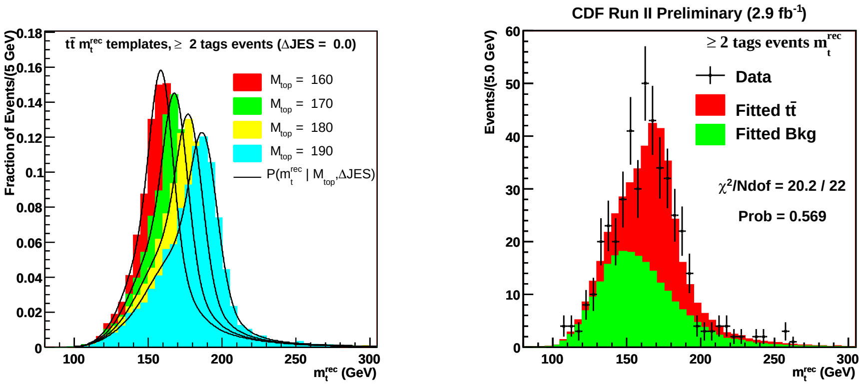

Fig. 4. – Left: examples of signal templates used in the CDF M_{top} measurement in the all-hadronic channel. Sensitivity to input M_{top} is apparent. Right: m_t^{rec} distribution as observed in the data sample with at least 2 b-tagged jets is plotted together with the fitted templates.

specific triggers. Therefore accurate kinematic selections and b-tag requirements are necessary to obtain samples such that $S/B \approx \mathcal{O}(1)$. The former usually require that no energetic lepton is identified in an event, the presence of a large number of jets (≥ 6) and a small amount of missing energy.

The most recent and precise measurement of M_{top} in this channel has been obtained by the CDF experiment by a TMT analysis on $2.9\,\mathrm{fb}^{-1}$ of data [14]. Beyond preselection cuts, a neural net, including both kinematical and jet shape variables, is exploited to select candidate events together with the requirement of at least one b-tagged jet. Kinematic fits are then performed to reconstruct, for each event a "top mass" m_t^{rec} and a "W mass" m_W^{rec}, and distributions of these variables are then used as templates to be fitted to the data in order to obtain the M_{top} measurement with simultaneous JES calibration. Examples of $t\bar{t}$ signal templates are shown on the left of fig. 4. A total of 3452 events with exactly one tagged jet and 441 with at least two tagged jets are selected, with an expected background of 2785 ± 83 and 201 ± 29 events respectively, and the calibrated likelihood fit yields $M_{\mathrm{top}} = 174.8 \pm 1.7\,(\mathrm{stat.}) \pm 1.9\,(\mathrm{syst.})\,\mathrm{GeV}/c^2$ corresponding to $\delta M_{\mathrm{top}}/M_{\mathrm{top}} \approx 1.5\%$. Observed data and fitted templates are shown on the right of fig. 4.

8. – Systematic uncertainties

Given the increasing data collection and the improvements in the selection techniques, the most precise measurements of M_{top} at the Tevatron are now limited by the systematic uncertainties. The *in situ* calibration allows to reduce greatly the uncertainty due to the knowledge of the JES, which partially becomes statistical, but its purely systematic component still represents the dominant contribution for most of the analyses and for the World Average [13]. Other important sources are primarily related to Monte Carlo generation (*e.g.*, initial- and final-state gluon radiation, hadronization model, parametrization of parton density functions). The CDF and D0 Collaborations are performing a joint effort to define a common way to evaluate the systematics, to improve the knowledge of important effects, to avoid possible overlaps and double counting, but also to study possible sources neglected so far. As an example, uncertainty coming from modeling of color reconnection effects has been introduced in the most recent analyses presented here.

9. – Tevatron combination

CDF and D0 combine their best results from each channel both internally (*i.e.* within each experiment separately) [15] and in a joint number representing the World Average for the value of M_{top} [13]. In such combinations correlations among uncertainties for different results are properly taken into account. As it concerns the World Average, the updated value, including many of the results reported here, is $M_{\text{top}} = 173.1 \pm 0.6\,(\text{stat.}) \pm 1.1\,(\text{syst.})\,\text{GeV}/c^2 = 173.1 \pm 1.3\,\text{GeV}/c^2$ with a χ^2/d.o.f. probability of 79%, denoting a good agreement among all measurements. The relative precision is $\approx 0.75\%$. The values from the different channels are also calculated obtaining $M_{\text{top}}^{\text{di}-1} = 171.4 \pm 2.7\,\text{GeV}/c^2$, $M_{\text{top}}^{\text{l+jets}} = 172.7 \pm 1.3\,\text{GeV}/c^2$, $M_{\text{top}}^{\text{all}-\text{had}} = 175.1 \pm 2.6\,\text{GeV}/c^2$. Also these results show a good agreement to each other.

10. – Conclusions

The CDF and D0 Collaborations have both conducted a robust set of analyses performed in order to better and better measure the value of the top quark mass, a fundamental parameter of the Standard Model. Well established techniques are applied to candidates in all channels corresponding to different $t\bar{t}$ final states. Results from individual channels and experiments are combined to obtain the best estimate of M_{top}, whose updated value is $M_{\text{top}} = 173.1 \pm 1.3\,\text{GeV}/c^2$. The precision of this measure (about 0.75%) is already limited by systematic uncertainties, and the two collaborations are working together to reach a common, complete and reliable evaluation of all the effects. Considering that the most updated analyses are now using about half of the final statistic of Tevatron Run II, the precision on M_{top} could reach the 1 GeV level before the collider final shutdown.

REFERENCES

[1] Abe F. *et al.* (CDF Collaboration), *Phys. Rev. Lett.*, **74** (1995) 2626.

[2] Abachi S. *et al.* (D0 Collaboration), *Phys. Rev. Lett.*, **74** (1995) 2632.

[3] Acosta D. *et al.* (CDF Collaboration), *Phys. Rev. D*, **71** (2005) 032001.

[4] Abazov V. M. *et al.* (D0 Collaboration), *Nucl. Instrum. Methods A*, **565** (2006) 463.

[5] Cacciari M. *et al.*, *JHEP*, **127** (2008) 0809; Kidonakis N. and Vogt R., *Phys. Rev. D*, **78** (2008) 074005; Moch S. and Uwer P., *Nucl. Phys. Proc. Suppl.*, **183** (2008) 75.

[6] D0 Collaboration, public notes 5897 (March 2009) and 5743 (July 2008).

[7] Abazov V. M. *et al.* (D0 Collaboration), *Phys. Rev. D*, **80** (2009) 092006.

[8] CDF Collaboration, public note 9692 (February 2009).

[9] D0 Collaboration, public notes 5877 (March 2009) and 5750 (July 2008).

[10] CDF Collaboration, public note 9683 (February 2009).

[11] CDF Collaboration, public note 9679 (March 2009).

[12] Abazov V. M. *et al.* (D0 Collaboration), *Phys. Rev.D*, **80** (2009) 071102.

[13] CDF and D0 Collaborations, FERMILAB-TM-2427-E (2009), CDF public note 9717 (March 2009), DØ public note 5899 (March 2009), arXiv:0903.2503.

[14] CDF Collaboration, public note 9694 (February 2009).

[15] CDF Collaboration, public note 9714 (March 2009), D0 Collaboration, public note 5900 (March 2009).