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Exploding Sudakov form factors

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Summary. — Contrary to what naively expected, we find that in spontaneously broken gauge theories the resummations of Sudakov double logs (in the presence of an energy scale Q much larger than the mass scale v of the spontaneous gauge breaking) are exponentiated also with a positive coefficient $e^{+\alpha \ \log^2 \frac{Q^2}{v^2}}$. The amplitudes affected by such a term are proportional to v and have a non-zero total gauge charge. As a working example we consider a model with two Abelian gauge groups $U'(1) \otimes U(1)$ with large mass splitting $M_{Z'} \gg M_Z$, and we compute leading radiative corrections to the decay of the heavy extra Z' boson into light fermions.

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1. – Description

The observation that the leading behavior of electroweak corrections at energies much larger than the electroweak scale $\sim 100\,\text{GeV}$ are dominated by the infrared structure of the theory [1] has brought considerable interest in the infrared structure of broken gauge theories [2-6]. This interest is motivated phenomenologically by the possibility of having in a hopefully nearby future colliders operating at such very high energies. Infrared/collinear logarithms account in this regime for a large fraction of the one-loop radiative corrections and provide non-negligible higher-order corrections. Previous analyses mostly convey their attention on charge-conserving amplitudes, which are the leading terms for high-energy phenomenology. The present paper is a review of the results founded in collaboration with Marcello and Paolo Ciafaloni in the more complete paper [7]. Here we investigate, at double-log level, the Sudakov form factor due to the insertion of soft gauge bosons belonging to a broken gauge theory, for amplitudes whose total gauge charge is not zero. These amplitudes are proportional to the vev responsible of the breaking of the gauge group and in the high-energy regime are necessarily higher twist. One may say they are unimportant, but their radiative corrections turn out to have curious properties which make them at least theoretically relevant. The basic model

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we consider contains two chiral spontaneously broken gauge groups $U'(1)\otimes U(1)$. We assume a large mass splitting ($M \gg m_Z$, M being the Z' mass), so that the Z'-boson does not participate to the IR dynamics. Thus, the $U'(1)$ allows to construct simple amplitudes with total gauge charge violation, in our case induced by the operator $\bar\psi\, \mathcal{Z}'_{\mu\nu}\sigma^{\mu\nu}\,\psi$ ($\mathcal{Z}'_{\mu\nu} = \partial_\mu Z'_\nu - \partial_\nu Z'_\mu$) describing the magnetic dipole moment of the Z' gauge boson. More explicitly, since left and right fermion $U(1)$ hypercharges need not be the same, the amplitude connecting the Z' with a left fermion and a right antifermion violates $U(1)$ (hyper)charge conservation. We compute the all order $\left(\alpha \log^2 \frac{M^2}{m_Z^2}\right)$ double leading logs (DLL), taking care of the leading mass suppressed corrections of order $\mathcal{O}\left(\frac{m^2}{M^2}\right)$, m being the fermion mass. We find that, among the form factors describing the effective couplings of the Z' to the two light fermions, only the magnetic one can develop exponentially growing Sudakov-like corrections. We start by writing the most general Lagrangian describing the gauge bosons-fermion interactions (we assume usual kinetic terms for the Abelian gauge bosons):

$$\bar\psi_L(\partial\!\!\!/ + i\,g\,y_L\,Z\!\!\!/ + i\,g'\,f_L\,Z\!\!\!/')\psi_L + \bar\psi_R(\partial\!\!\!/ + i\,g\,y_R\,Z\!\!\!/ + i\,g'\,f_R\,Z\!\!\!/')\psi_R, \tag{1}$$

where $\psi_{L/R} = \frac{1\pm\gamma_5}{2}\psi$, $f_{L/R}$ ($y_{L/R}$) are the $U'(1)$ ($U(1)$) hypercharges for left/right fermions. To implement, in a natural way, the spontaneous breaking of the gauge groups $U'(1)\otimes U(1)$ we need at least two complex Higgs fields, one, let us call $\phi' = \frac{1}{\sqrt{2}}(h' + v' + i\,\varphi')$ with v' the vev breaking $U'(1)$ and another scalar field, $\phi = \frac{1}{\sqrt{2}}(h + v + i\,\varphi)$ with v involved into the breaking of $U(1)$. The hierarchy $M_{Z'} \gg m_Z$ implies necessarily $v' \gg v$. The fermionic mass m being of order m_Z will be induced by the Yukawa interaction $h_f\,\bar\psi_R\,\phi\,\psi_L + \text{h.c.}$ so that $m = \frac{h_f}{\sqrt{2}}\,v$ and for charge conservation we need both $f_\phi = f_R - f_L = 2\,f_A$ and $y_\phi = y_R - y_L = 2\,y_A$. Note that if $\underline{f_A \neq 0}$ also the scalar field ϕ will participate to the breaking of $U'(1)$ and it will induce mixing between the gauge bosons $Z - Z'$ and the Goldstone modes $\varphi' - \varphi$. To be as simple as possible we decided to study a vector like $U(1)$ group where $f_L = f_R \to f_\phi = 0$. In order to clarify the above considerations we write the Lagrangian for the scalar sector([1])

$$|(\partial_\mu + i\,g'\,f_{\phi'}\,Z'_\mu)\phi'|^2 + |(\partial_\mu + i\,g\,y_\phi\,Z_\mu)\phi|^2 + (h_f\,\phi\,\bar\psi_R\,\psi_L + \text{h.c.}) + V(\phi) + \mathcal{V}(\phi') \tag{2}$$

and the Feynman Gauge as gauge fixing. Working in the limit $\frac{m_{f,z}}{M} \ll 1$ we prefer to use the gauge eigenstate basis as propagating free fields with the mass shifts used as perturbations. The Z' field has mass $M^2 = g^{'\,2}\,f^2_{\phi'}\,v^{'\,2}$ while the Z field has mass $m_Z^2 = g^2\,y_\phi^2\,v^2$.

2. – Form factors for the vertex $Z' \to \bar f\,f$

The amplitude for Z' decay $Z'_\mu(p_1+p_2) \to \bar f(p_2)\;f(p_1)$ is given by $\varepsilon_\mu(p_1 + p_2)\bar u(p_1)\Gamma^{(Z')}_\mu v(p_2)$ where $\varepsilon_\mu(p)$ is the physical Z' polarization satisfying $\sum_a \varepsilon^a_\mu \varepsilon^a_\nu = -g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}$. In order to compute the effect induced by the multi loops generated

([1]) The scalar potentials $V(\phi)$ and $\mathcal{V}(\phi')$ are responsible for the generation of the spontaneous symmetry-breaking scales v and v'.

by integrating over soft Z-gauge bosons, we introduce the more general CP invariant vertex:

$$\bar{u}(p_1)\Gamma_\mu^{(Z')}v(p_2) = i\,g'\,\bar{u}(p_1)\left[\gamma_\mu(F_L P_L + F_R P_R) + \frac{m\,(p_1-p_2)_\mu}{(p_1\cdot p_2)}\,F_M + \frac{m\,(p_1+p_2)_\mu}{(p_1\cdot p_2)}\,F_P\gamma_5\right]v(p_2), \tag{3}$$

where $P_{R,L} = \frac{1}{2}(1\pm\gamma_5)$ and F_M is usually named *magnetic form factor* and $(p_1+p_2)^2 = M^2$, $p_{1,2}^2 = m^2$. We also introduce $F_V = \frac{1}{2}(F_R + F_L)$ and $F_A = \frac{1}{2}(F_R - F_L)$, the same relationships hold also for the tree level charges f_i and y_i. Defining $\rho = \frac{m^2}{p_1 p_2}$, the amplitudes squared for the various positive $(+)$ and negative $(-)$ helicity fermions, summed over the Z' polarizations, are given by

$$\frac{|\mathcal{M}_{++}|^2}{4(p_1p_2)} = \left(F_V - F_A\sqrt{1-\rho}\right)^2 \qquad \frac{|\mathcal{M}_{--}|^2}{4(p_1p_2)} = \left(F_V + F_A\sqrt{1-\rho}\right)^2, \tag{4}$$

$$\frac{|\mathcal{M}_{+-}|^2}{4(p_1p_2)} = \frac{|\mathcal{M}_{-+}|^2}{4(p_1p_2)} = \rho\,\left[F_A^2 + (F_V - F_M(1-\rho))^2\right]. \tag{5}$$

The corresponding widths can be obtained multiplying by the appropriate phase space factors. Notice that, since $(p_1+p_2)^\mu\varepsilon_\mu(p_1+p_2) = 0$, the form factor F_P does not contribute to physical amplitudes. In the next section we calculate the on-shell one-loop form factors in the limit $M \gg m_Z$, m, retaining only the DLL contributions. Since we want to calculate the decay rates and the cross-sections up to $\mathcal{O}(\rho)$, we need the values of F_M to order ρ^0 and of $F_{L,R}$ to order ρ^1.

2·1. *Form factors at one loop.* – Let us see the result of the evaluation of the one-loop amplitude at DLL approximation.

$$F_L^{(1)} = f\left(-y_L^2 + \frac{\rho}{2}\,(y_R^2 - y_L^2)\right)L^2; \qquad F_R^{(1)} = f\left(-y_R^2 - \frac{\rho}{2}\,(y_L^2 - y_R^2)\right)L^2; \tag{6}$$

$$F_M^{(1)} = y_A\,f(y_L - y_R)\,L^2; \qquad F_P^{(1)} = y_A\,f(y_L + y_R)\,L^2; \tag{7}$$

where $\frac{\alpha}{4\pi}\log^2\frac{M^2}{m_Z^2} \equiv L^2$; $\alpha = \frac{g^2}{4\pi}$. One can see that the IR double logs affect both $\mathcal{O}(\rho^0)$ and $\mathcal{O}(\rho)$ corrections; the latter are proportional to the y_A charge of the fermions, that is non-zero only for chiral $U(1)$ gauge theories (clearly such double logs are not present in QED [8] and QCD). There is another class of diagrams coming from Z-Z' mixing and from the Higgs/Goldstone sector that gives DLL at order ρ potentialy interesting. We demonstrated that they do not generate corrections to the anomalous Sudakov and in any case in the limit of $f_\phi = 0$ and for $h_f \ll g^2$ they are totally negligible.

3. – All-order resummed form factors

Since we work in the regime $M \gg m_Z$, our first-order calculations cannot be trusted because $L^2 \gg 1$, and we have to proceed to the resummation of all the DLL (L^{2n}). The dressing by soft boson insertions of the eikonal type can be explicitly taken into account at all orders by making use of the eikonal identity (see [9] for instance). We illustrate this calculation by adopting the method of $k_\perp$-ordering: the leading terms in the resummed series are given by "ladder" insertions ordered in the soft variable $k_\perp$, which is the transverse momentum of the soft gauge boson([2]). The resummation of the soft gauge bosons for momenta in the range $k_\perp^{\rm inf} \le k_\perp \le k_\perp^{\rm sup}$ is given by the following Sudakov form factor [10]:

$$S_{i,j}[k_\perp^{\rm sup}, k_\perp^{\rm inf}] = \exp\left[-\frac{\alpha}{2\pi} y_i\, y_j \int_{k_\perp^{2\,\rm inf}}^{k_\perp^{2\,\rm sup}} \frac{{\rm d}k_\perp^2}{k_\perp^2} \log\frac{M^2}{k_\perp^2}\right], \tag{8}$$

where y_i, y_j are the relevant $U(1)$ charges. In general terms we have only three possible Sudakov structures: S_{LL}, S_{RR} and $S_{L,R}$ that after momenta integration will generate three kinds of Sudakov exponents: $e^{-y_L^2\, L^2}$, $e^{-y_R^2\, L^2}$ and $e^{-y_L\, y_R\, L^2}$. While the first two are exponentially suppressing their multiplicative factors, the last one ($e^{-y_L\, y_R\, L^2}$), that we call *Anomalous Sudakov*, depending on the sign of the charges $y_{L,R}$ can generate exponential growing corrections (for $y_L\ y_R < 0$).

We obtain the following results coming from pure Z boson exchanges:

$$F_L^{(Z)} = f\left(e^{-y_L^2 L^2} - \frac{\rho}{2}\left(e^{-y_R^2 L^2} - e^{-y_L^2 L^2}\right)\right), \tag{9}$$

$$F_R^{(Z)} = f\left(e^{-y_R^2 L^2} - \frac{\rho}{2}\left(e^{-y_L^2 L^2} - e^{-y_R^2 L^2}\right)\right), \tag{10}$$

$$F_M^{(Z)} = \frac{f}{2}\left(e^{-y_L^2 L^2} + e^{-y_R^2 L^2}\right) - f\ e^{-y_L\, y_R\, L^2}, \tag{11}$$

$$F_P^{(Z)} = \frac{f}{2}\left(e^{-y_L^2 L^2} - e^{-y_R^2 L^2}\right) \tag{12}$$

from which we can extract the following results:

– the axial and vector form factors related to F_L and F_R receive, after resummation, only "standard" Sudakov form factors ($e^{-y_L^2\, L^2}, e^{-y_R^2\, L^2}$) that exponentially suppress the amplitudes at very large energies.

– The magnetic dipole moment form factor F_M gets dressed also with the *Anomalous Sudakov* ($e^{-y_L\ y_R\ L^2}$) whose exponent can be *positive* if $y_L\ y_R < 0$. If this is the case, F_M asymptotically dominates over F_L, F_R.

4. – Asymptotic dynamics

If $y_L\ y_R < 0$, the terms proportional to the exponentially growing form factor F_M in the squared amplitudes (4), (5) dominate over the terms in $F_{L,R}$ for $M \gg m_z$, m. At what energy scales M does this happen?

The helicity changing decay rate Γ_{+-} becomes

$$\Gamma_{+-} \simeq \Gamma_{+-}^0 \frac{1}{4}\left(4e^{-2y_L y_R L^2} + e^{-2y_R^2 L^2} - 2e^{-(y_L^2+y_R^2)L^2} + e^{-2y_L^2 L^2}\right) + \mathcal{O}(\rho^2), \tag{13}$$

([2]) We have checked that the explicit computation using the eikonal identity produces the same results obtained by $k_\perp$-ordering.

where Γ^0_{+-} is the tree level rate. The resummed expression is a combination of decreasing and one potentially increasing (for $y_L\ y_R < 0$) exponentials. In the limit $L^2 \gg 1$ and for $y_L\ y_R < 0$ quickly the resummed value becomes twice as big as the tree level one, giving a 100% radiative correction that puts in evidence the importance of the resummation. This happens for scales such that

$$e^{-2\ y_L y_R L^2} = 2 \Rightarrow \frac{M}{m_Z} = \exp\left[\sqrt{\frac{\pi \log 2}{-2 y_L y_R \alpha}}\right]. \tag{14}$$

For $y_L = -y_R = 1$, $\alpha \sim 1/30$ and $m_Z \sim 100\,\text{GeV}$ one obtains energies of the order of 30 TeV, which is a relatively low scale value!

For other observables like the full decay rate $\bar{\Gamma}$ the expansion in ρ gives

$$\begin{aligned}\bar{\Gamma} \propto\ & f^2\left(e^{-2y_R^2 L^2} + e^{-2y_L^2 L^2}\right) \\ & +2\rho f^2\left(e^{-2y_L y_R L^2} - e^{-(y_L^2+y_R^2)L^2} + \frac{1}{2}\left(e^{-2y_L^2 L^2} + e^{-2y_R^2 L^2}\right)\right)\end{aligned} \tag{15}$$

In this case the anomalous Sudakov is always multiplied by a power of ρ.

If we compare the $\rho = 0$ terms with the anomalous exponential corrections, we see that they are of the same order when

$$\rho\, e^{-2y_L y_R L^2} \sim e^{-2y_{R,L}^2 L^2} \tag{16}$$

and for $m \sim m_Z$ (just to have the order of magnitude) this happens at mass scales

$$M \sim m\, e^{\overline{\frac{2\,\pi}{\alpha\,(y_L y_R - y_{L,R}^2)}}}, \tag{17}$$

that is of the same order of the Landau-Pole (LP) energy $E_{\text{LP}} \sim m\, e^{\frac{\pi}{\beta\alpha}}$ (where β is the beta-function of the $U(1)$ gauge group). Implementing the above results for the Standard Model, it is straightforward to identify the chiral gauge group $U(1)$ with $U(1)_Y$, with m_Z exactly the gauge boson Z mass of 91 GeV. Then, from the analysis of the quantum number of the SM fields we see that $U(1)$ "anomalous" Sudakov form factors are present only for the down quark sector where $y_L = \frac{1}{6}$ and $y_R = -\frac{1}{3}$ so that $y_L\ y_R = -\frac{1}{18} < 0$.

The phenomenological relevance of the above effects in this case results quite suppressed first of all for the smallness of the gauge coupling $\alpha_Y \sim \frac{1}{60}$ and secondarily also for the smallness of the charges $y_L\ y_R = -\frac{1}{18}$.

The presence of anomalous Sudakov for the non-Abelian $SU(2)$ part is at present under study and results quite interesting because we naively expect phenomenological relevant effects already at TeV scale (see eq. (14)) mainly due to the fact that the gauge coupling is large ($\alpha_W \sim 2\,\alpha_Y$) and the non-Abelian charges are naturally $\mathcal{O}(1)$ [11].

5. – Conclusions

In this work we showed the results for the form factors of a very heavy Z' gauge boson of mass M into a fermion-antifermion pair in a simple $U(1) \otimes U'(1)$ model, performing the calculation up to order m^2 in the fermion mass m and to all orders in the $U(1)$ gauge coupling at the double-log level $(\alpha\ \log^2 \frac{M^2}{m_Z^2})^n$. We conclude that while the axial and vector form factors feature a "standard", energy decreasing Sudakov form factor, the magnetic dipole moment features an "anomalous" exponential $\sim \exp[-\alpha\, y_L\, y_R\, \log^2 \frac{M^2}{m_Z^2}]$ term, which *grows* with energy for fermions having opposite left-right $U(1)$ charges ($y_L\ y_R < 0$). This feature belongs exclusively to broken gauge theories like the electroweak sector of the Standard Model, and is a very unusual one. In fact the magnetic dipole moment corresponds to the insertion of an effective dimension-five operator of the form $\bar\psi_L\, \mathcal{Z}'_{\mu\nu}\sigma^{\mu\nu}\, \psi_R$ ($\mathcal{Z}'_{\mu\nu} = \partial_\mu Z'_\nu - \partial_\nu Z'_\mu$), which explicitly breaks $U(1)$ if $y_L \neq y_R$ and is (must be) proportional to the $U(1)$ vacuum expectation value. The expectation is that at large energy scales, where symmetry is recovered, this symmetry-violating operator gives negligible contribution to observables: this is by no means the case. While the contribution is truly suppressed by fermion masses at tree level, the dressing by IR dynamics around the light Z mass makes this operator the leading one at very high energies. This is a kind of "non-decoupling" in the sense that very high energies observables are sensitive to the very low IR cutoff scale, whatever the ratio of the scales. This is due to the high-energy behavior being dictated by the IR dynamics, and therefore sensitive to symmetry breaking at *any* scale. These results open new questions about the realization of the cancellation theorems [12] involving real and virtual corrections.

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