IL NUOVO CIMENTO Vol. 33 C, N. 2 Marzo-Aprile 2010
DOI 10.1393/ncc/i2010-10606-1

Colloquia: LC09

Determination of the strong coupling constant based on event shapes

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(ricevuto il 15 Febbraio 2010; approvato il 30 Marzo 2010; pubblicato online il 5 Luglio 2010)

Summary. — We report on a determination of the strong coupling constant from a fit of QCD predictions for six event-shape variables, calculated at next-to-next-to-leading order (NNLO) and matched to resummation in the next-to-leading-logarithmic approximation (NLLA). We use data collected by ALEPH at centre-of-mass energies between 91 and 206 GeV. We also investigate the role of hadronisation corrections, using both Monte Carlo generator predictions and analytic models to parametrise non-perturbative power corrections.

PACS 12.38.Aw – General properties of QCD (dynamics, confinement, etc.).
PACS 12.38.Bx – Perturbative calculations.
PACS 13.66.Bc – Hadron production in e^-e^+ interactions.
PACS 13.66.Jn – Precision measurements in e^-e^+ interactions.

1. – Introduction

The reaction of e^+e^- annihilation into 3 jets allows a precise determination of the strong coupling constant α_s, since the deviation from two-jet configurations is proportional to it. Not only jet rates, but also the topology of the single events can be studied in a systematic fashion. The so-called event-shape observables became very popular mainly because they are well suited both for experimental measurement and for theoretical description, since many of them are infrared and collinear safe. They describe topological

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properties of hadronic final states by parameterizing the energy-momentum flow of an event. This class of observables is also interesting because it shows a rather strong sensitivity to hadronisation effects, at least in phase-space regions characterised by soft and collinear gluon radiation. We have studied the six event-shape observables: thrust T (respectively, $\tau = 1 - T$), heavy jet mass ρ, wide and total jet broadening B_W and B_T, C-parameter and the two-to-three-jet transition parameter in the Durham algorithm, Y_3. For the definitions of these variables and their historical origin we refer to [1] and references therein. We will denote the variables collectively as y in the following, such that their two jet limit is $y \to 0$.

Event-shape distributions in e^+e^- annihilation have been measured with high accuracy by a number of experiments, most of them at LEP at centre-of-mass energies between 91 and 206 GeV [2-6]. For a long time, the theoretical state-of-the-art description of event-shape distributions over the full kinematic range was based on the matching of the next-to-leading-logarithmic approximation (NLLA) [7] onto the fixed next-to-leading order (NLO) calculation. Recently, NNLO results for event-shape distributions became available [8, 9]. Soon after, the matching of the resummed result in the next-to-leading-logarithmic approximation onto the NNLO calculation has been performed [10] in the so-called ln R-matching scheme [7]. Based on these results several determinations of the strong coupling constant using both NNLO and matched NNLO+NLLA predictions for hadronic event shapes have been carried out [11-14], together with a detailed investigation of Monte Carlo (MC) hadronisation corrections. Next-to-leading order electroweak corrections to event-shape distributions in e^+e^- annihilation were also computed very recently [15].

Apart from distributions of event-shape observables, one can also study mean values and higher moments. They have been measured by several experiments, most extensively by JADE [16, 17] and OPAL [4]. Theoretical predictions are now accurate at NNLO [18,19]. Moments are particularly attractive in view of studying non-perturbative hadronisation corrections to event shapes [20]. In ref. [21], NNLO perturbative QCD predictions have been combined with non-perturbative power corrections in a dispersive model [22-25]. The resulting theoretical expressions have been compared to experimental data from JADE and OPAL, which carried out extensive studies on moments, and new values for both $\alpha_s(M_Z)$ and α_0, the effective coupling in the non-perturbative regime, have been determined.

The two approaches—estimating the hadronisation corrections by general purpose MC programs or modelling power corrections analytically—lead to some interesting insights about hadronisation corrections, which will be summarised in the following.

2. – Theoretical framework

Event-shape distributions. – The fixed-order QCD description of event-shape distributions starts from the perturbative expansion

$$(1)\qquad \frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}y}(y,Q,\mu) = \bar{\alpha}_s(\mu)\frac{\mathrm{d}A}{\mathrm{d}y}(y) + \bar{\alpha}_s^2(\mu)\frac{\mathrm{d}B}{\mathrm{d}y}(y,x_\mu) + \bar{\alpha}_s^3(\mu)\frac{\mathrm{d}C}{\mathrm{d}y}(y,x_\mu) + \mathcal{O}(\bar{\alpha}_s^4),$$

where $\bar{\alpha}_s = \frac{\alpha_s}{2\pi}$ and $x_\mu = \frac{\mu}{Q}$, and where A, B and C are the perturbatively calculated coefficients [8] at LO, NLO and NNLO.

All coefficients are normalised to the tree-level cross-section σ_0 for $e^+e^- \to q\bar{q}$. For massless quarks, this normalisation cancels all electroweak coupling factors, and the dependence of (1) on the collision energy is only through α_s and x_μ. Predictions for the experimentally measured event-shape distributions are then obtained by normalising to $\sigma_{\rm had}$. In all expressions, the scale dependence of α_s is determined according to the three-loop running of $\alpha_s(\mu)$.

We take into account bottom mass effects by retaining the massless $N_F = 5$ expressions and adding the difference between the massless and massive LO and NLO coefficients A and B, where a pole b-quark mass of $m_{\rm b} = 4.5\,{\rm GeV}$ was used.

In the limit $y \to 0$ one observes that the perturbative contribution of order α_s^n to the cross-section diverges like $\alpha_s^n L^{2n}$, with $L = -\ln y$ ($L = -\ln(y/6)$ for $y = C$). This leading logarithmic (LL) behaviour is due to multiple soft gluon emission at higher orders, and the LL coefficients exponentiate, such that they can be resummed to all orders. Assuming massless quarks the event-shape observables considered here could be resummed to next-to-leading logarithmic (NLL) accuracy.

In order to obtain a reliable description of the event-shape distributions over a wide range in y, fixed-order and resummed predictions are matched and double-counted terms are subtracted. A number of different matching procedures have been proposed in the literature, see, *e.g.*, ref. [1] for a review. The most commonly used procedure is the so-called $\ln R$-matching [7], which we used in two different variants for our study on α_s [12]. For more details about the NLLA+NNLO matching we refer to ref. [10].

Moments of event-shape observables. – The n-th moment of an event-shape observable y is defined by

$$\langle y^n \rangle = \frac{1}{\sigma_{\rm had}} \int_0^{y_{\rm max}} y^n \frac{{\rm d}\sigma}{{\rm d}y} {\rm d}y, \tag{2}$$

where $y_{\rm max}$ is the kinematically allowed upper limit of the observable. For moments of event shapes, one expects the hadronisation corrections to be additive, such that they can be divided into a perturbative and a non-perturbative contribution, where the non-perturbative contribution accounts for hadronisation effects.

In ref. [21], the dispersive model derived in refs. [22-25] has been used and extended to NNLO to estimate hadronisation corrections to event-shape moments by calculating analytical predictions for power corrections. It introduces only a single new parameter α_0, which can be interpreted as the average strong coupling in the non-perturbative region:

$$\frac{1}{\mu_I} \int_0^{\mu_I} {\rm d}Q\, \alpha_{\rm eff}(Q^2) = \alpha_0(\mu_I), \tag{3}$$

where below the IR cutoff μ_I the strong coupling is replaced by an effective coupling. This dispersive model for the strong coupling leads to a shift in the distributions

$$\frac{{\rm d}\sigma}{{\rm d}y}(y) = \frac{{\rm d}\sigma_{\rm pt}}{{\rm d}y}\,(y - a_y\, P), \tag{4}$$

where the numerical factor a_y depends on the event shape, while P is believed to be universal and scales with the centre-of-mass energy like μ_I/Q. Insertion of eq. (4) into the definition of the moments leads to

$$(5)\quad \langle y^n \rangle = \int_{-a_y P}^{y_{\max}-a_y P} \mathrm{d}y\,(y+a_y P)^n \frac{1}{\sigma_{\mathrm{tot}}}\frac{\mathrm{d}\sigma_{\mathrm{pt}}}{\mathrm{d}y}(y) \approx \int_0^{y_{\max}} \mathrm{d}y\,(y+a_y P)^n \frac{1}{\sigma_{\mathrm{tot}}}\frac{\mathrm{d}\sigma_{\mathrm{pt}}}{\mathrm{d}y}(y).$$

From this expression one can extract the non-perturbative predictions for the moments of y.

3. – Determination of α_s and α_0

α_s from distributions of hadronic event shapes. – We have used the six event-shape observables listed in sect. **1** for our fits. The measurements we use have been carried out by the ALEPH Collaboration [2] at eight different centre-of-mass energies between 91.2 and 206 GeV. The measurements have been corrected for detector effects and for backgrounds coming from 4-fermion processes above 133 GeV. The experimental uncertainties were estimated by varying event and particle selection cuts. They are below 1% at LEP1 and slightly larger at LEP2.

The perturbative QCD prediction is corrected for hadronisation and resonance decays by means of a transition matrix, which is computed with the MC generators PYTHIA [26], HERWIG [27] and ARIADNE [28], all tuned to global hadronic observables at M_Z [29]. The parton level is defined by the quarks and gluons present at the end of the parton shower in PYTHIA and HERWIG and the partons resulting from the colour dipole radiation in ARIADNE. Corrected measurements of event-shape distributions are compared to the theoretical calculation at particle level. For a detailed description of the determination and treatment of experimental systematic uncertainties we refer to refs. [2, 11].

The value of α_s is determined at each energy using a binned least-squares fit. Combining the results for six event-shape variables and eight LEP1/LEP2 centre-of-mass energies, we obtain

$$\alpha_s(M_Z) = 0.1224 \pm 0.0009\,(\mathrm{stat}) \pm 0.0009\,(\mathrm{exp}) \pm 0.0012\,(\mathrm{had}) \pm 0.0035\,(\mathrm{theo}).$$

For the fitted values of the coupling constant as found from event-shape variables calculated at various orders we refer to the figures and tables of [12]. The central value of the result is slightly lower than the central value of 0.1228 obtained from a fit using purely fixed-order NNLO predictions [11], and slightly larger than the NLO+NLLA results [2]. Furthermore the dominant theoretical uncertainty on $\alpha_s(M_Z)$, as estimated from scale variations, is reduced by 20% compared to NLO+NLLA. However, compared to the fit based on purely fixed-order NNLO predictions, the perturbative uncertainty is *increased* in the NNLO+NLLA fit. The reason is that in the two-jet region the NLLA+NLO and NLLA+NNLO predictions agree by construction and therefore, the renormalisation scale uncertainty is dominated by the resummation in this region, which results in a larger overall scale uncertainty in the α_s fit. As already observed for the fixed-order NNLO results, the scatter among the values of $\alpha_s(M_Z)$ extracted from the six different event-shape variables is substantially reduced compared to the NLO+NLLA case.

Hadronisation corrections from LL+NLO event generators. – In recent years large efforts went into the development of modern MC event generators which include in part NLO corrections matched to parton showers at leading logarithmic accuracy (LL) for various processes. Here we use HERWIG++ [30, 31] version 2.3 for our investigations.

Several schemes for the implementation of NLO corrections are available [32-34]. We studied the MCNLO [32] and POWHEG [33] schemes([1]). Discussing the full details of our study is beyond the scope of this paper; here we only mention some of our observations. For further details we refer to ref. [12].

From the study of hadronisation corrections we make the following important observation. It appears that there are two "classes" of variables. The first class contains T, C and B_T, while the second class consists of the ρ, B_W and Y_3. For the first class, using the standard hadronisation corrections from PYTHIA, we obtain $\alpha_s(M_Z)$ values some 5% higher than those found from the second class of variables. In a study of higher moments of event shapes [18], indications were found that variables from the first class still suffer from sizable missing higher-order corrections, whereas the second class of observables have a better perturbative stability. The PYTHIA result is obtained with tuned parameters, where the tuning to data had been performed at the hadron level. This tuning results in a rather large effective coupling in the parton shower, which might partly explain the larger parton level prediction of PYTHIA compared to pure NNLO+NLLA prediction. As the tuning has been performed at hadron level, this implies that the hadronisation corrections come out to be smaller than what would have been found by tuning a hypothetical MC prediction with a parton level corresponding to the NNLO+NLLA prediction. This means that the PYTHIA hadronisation corrections, applied in the α_s fit, might be too small, resulting in a larger $\alpha_s(M_Z)$ value. Such problems do not appear to exist for the second class of variables.

A determination of α_s based on 3-jet rates calculated at NNLO accuracy has also been performed recently [35], with the result $\alpha_s(M_Z) = 0.1175 \pm 0.0020\,(\text{exp}) \pm 0.0015\,(\text{theo})$, which is lower than the one obtained from fits to distributions of event shapes.

α_s and α_0 from moments of hadronic event shapes. – Now we turn to analytical models to estimate hadronisation corrections. The expressions derived in [21] match the dispersive model with the perturbative prediction at NNLO QCD. Comparing these expressions with experimental data on event-shape moments, a combined determination of the perturbative strong coupling constant α_s and the non-perturbative parameter α_0 has been performed [21], based on data from the JADE and OPAL experiments [17]. The data consist of 18 points at centre-of-mass energies between 14.0 and 206.6 GeV for the first five moments of T, C, Y_3, ρ, B_W and B_T, and have been taken from [36]. For each moment the NLO as well as the NNLO prediction was fitted with $\alpha_s(M_Z)$ and α_0 as fit parameters, except for the moments of Y_3, which have no leading $\frac{1}{Q}$ power correction and thus are independent of α_0.

Compared to previous results at NLO, inclusion of NNLO effects results in a considerably improved consistency in the parameters determined from different shape variables, and in a substantial reduction of the error on α_s. We further observe that the theoretical error on the extraction of $\alpha_S(M_Z)$ from ρ, Y_3 and B_W is considerably smaller than from τ, C and B_T. As mentioned above and discussed in detail in [18], the moments of the former three shape variables receive moderate NNLO corrections for all n, while the NNLO corrections for the latter three are large already for $n = 1$ and increase with n. Consequently, the theoretical description of the moments of ρ, Y_3 and B_W displays a higher perturbative stability, which is reflected in the smaller theoretical uncertainty on $\alpha_s(M_Z)$ derived from those variables.

([1]) We use the notation MCNLO for the *method*, while MC@NLO denotes the *program*.

In a second step, we combine the $\alpha_s(M_Z)$ and α_0 measurements obtained from different event-shape variables. Taking the weighted mean over all values except B_W and B_T, we obtain at NNLO:

$$\begin{aligned} \alpha_s(M_Z) &= 0.1153 \pm 0.0017(\text{exp}) \pm 0.0023(\text{th}), \\ \alpha_0 &= 0.5132 \pm 0.0115(\text{exp}) \pm 0.0381(\text{th}), \end{aligned} \tag{6}$$

The moments of B_W and B_T have been excluded here since their theoretical description requires an additional contribution to the non-perturbative coefficient P [21] which is not available consistently to NNLO.

Comparing the NLO and NNLO combinations [21], it can be seen very clearly that the measurements obtained from the different variables are consistent with each other within errors. The average of $\alpha_s(M_Z)$ is dominated by the measurements based on ρ and Y_3, which have the smallest theoretical uncertainties. From NLO to NNLO, the error on $\alpha_s(M_Z)$ is reduced by a factor of two. Analysing the different sources of the systematical errors, we observe that the error on $\alpha_s(M_Z)$ is clearly dominated by the x_μ variation, while the largest contribution to the error on α_0 comes from the uncertainty on the Milan factor $\mathcal{M}$ [24]. Since this uncertainty has not been improved in the current study, it is understandable that the systematic error on α_0 remains unchanged.

To quantify the difference of the dispersive model to hadronisation corrections from the legacy generators, we analysed the moments of $(1-T)$ with hadronisation corrections from PYTHIA. As a result, we obtained fit results for $\alpha_s(M_\mathrm{Z})$ which are typically 4% higher than by using the dispersive model, with a slightly worse quality of the fit. Comparing perturbative and non-perturbative contributions at $\sqrt{s} = M_\mathrm{Z}$, we observed that PYTHIA hadronisation corrections amount to less than half the power corrections obtained in the dispersive model, thereby explaining the tendency towards a larger value of $\alpha_s(M_\mathrm{Z})$, since the missing numerical magnitude of the power corrections must be compensated by a larger perturbative contribution.

Conclusions

We have compared determinations of the strong coupling constant based on hadronic event shapes measured at LEP using two different approaches:

1) a fit of perturbative QCD results at NNLO, matched to resummation in the NLLA, to ALEPH data where the hadronisation corrections have been estimated using MC event generators;

2) a fit of perturbative QCD results at NNLO matched to non-perturbative power corrections in the dispersive model, providing analytical parametrisations of hadronisation corrections, to JADE and OPAL data.

We find that the second approach results in a lower value of $\alpha_s(M_Z)$ than the first one.

We conclude that apparently there are two "classes" of event-shape variables, the first class containing T, C and B_T, the second class containing ρ, B_W and Y_3. Comparing parton and hadron level predictions from PYTHIA, the first class of variables gives a parton level prediction which is about 10% higher than the NNLO+NLLA prediction, where the PYTHIA curve has been obtained with parameters tuned to data at the hadron level. This may imply that the hadronisation corrections come out to be too small for these variables, resulting in a larger $\alpha_s(M_Z)$ value. This hypothesis is corroborated by the fact that the theoretical description of the moments of the first class of variables displays a lower perturbative stability. Further evidence for the underestimation of hadronisation corrections by the legacy generators is also provided by the fact that they predict power corrections which are less than half of what is obtained in the dispersive model.

Precision QCD studies at colliders may have to address the issue of hadronisation corrections in more detail in the future.

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This research was supported in part by the Swiss National Science Foundation (SNF) under contracts PP0022-118864 and 200020-126691, by the UK Science and Technology Facilities Council, by the European Commission's Marie-Curie Research Training Network MRTN-CT-2006-035505 and by the German Helmholtz Alliance "Physics at the Terascale".

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