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micrOMEGAs: A tool for dark matter studies

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Summary. — `micrOMEGAs` is a tool for cold dark matter (DM) studies in generic extensions of the standard model with a R-parity like discrete symmetry that guarantees the stability of the lightest odd particle. The code computes the DM relic density, the elastic scattering cross-sections of DM on nuclei relevant for direct detection, and the spectra of e^+, $\bar{p}$, γ originating from DM annihilation including propagation of charged cosmic rays. The cross-sections and decay properties of new particles relevant for collider studies are included as well as constraints from the flavour sector on the parameter space of supersymmetric models.

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1. – Introduction

The existence of a dominant dark matter component in the universe has been firmly established by cosmological observations in the last few years notably by SDSS [1] and WMAP [2]. Furthermore, the amount of DM today, the relic density, has been measured with very good precision, $\Omega h^2 = 0.1099 \pm 0.0062$ [3]. A leading candidate for cold DM is a new weakly interacting stable massive particle which naturally provides a reasonable value for the relic density. It is particulary interesting that extensions of the standard model (SM) whose prime goal is to solve the hierarchy problem can in many cases also provide a viable DM candidate. Such particles arise naturally in many extensions of the standard model [4] from the minimal supersymmetric standard model [5, 6] to models of extra dimensions [7, 8], little Higgs models [9] or models with extended gauge or Higgs sectors [10, 11]. In these models which possess a symmetry like R-parity that guarantees the stability of the lightest odd particle (LOP), the DM candidate can be either a Majorana fermion, a Dirac fermion, a vector boson or a scalar. The DM mass ranges anywhere from a few GeV's to a few TeV's.

Astroparticle and collider experiments are being actively pursued to search for DM. Combined studies of DM signals in these different types of experiments should provide the necessary ingredients to unravel the nature of DM. These involve searches for DM either directly through detection of elastic scattering of the DM with the nuclei in a large detector or indirectly trough detection of products of DM annihilation (photons, positrons, neutrinos or antiprotons) in the Galaxy or in the Sun. Furthermore colliders are searching for DM as well as for other new particles predicted in extensions of the standard model.

Sophisticated tools have been developed to perform computations of DM observables within R-parity conserving supersymmetric models, four are publicly available: `micrOMEGAs` [12], `DarkSUSY` [13], `IsaTools` [14] and `superISO` [15]. All include precise computations of the relic density as well as various constraints on the model from precision observables or the flavour sector. In addition the first three compute the direct and indirect detection rates while `micrOMEGAs` and `Isatools` provide collider observables. `micrOMEGAs` has the added important feature that it can be easily generalised to other extensions of the standard model [16]. This is because `micrOMEGAs` is structured around `CalcHEP` [17] a generic program which once given a model file containing the list of particles, their masses and the associated Feynman rules describing their interactions, computes any cross-section in the model. The standard `micrOMEGAs` routines can then be used to compute the relic density as well as other DM observables.

2. – Relic density

A relic density calculation entails solving the evolution equation for the abundance of DM, $Y(T)$, defined as the number density divided by the entropy density [18],

$$(1) \qquad \frac{\mathrm{d}Y}{\mathrm{d}T} = \sqrt{\frac{\pi g_*(T)}{45}} M_p \langle \sigma v \rangle (Y(T)^2 - Y_{\mathrm{eq}}(T)^2),$$

where g_* is an effective number of degree of freedom, M_p is the Planck mass and $Y_{\mathrm{eq}}(T)$ the thermal equilibrium abundance. $\langle \sigma v \rangle$ is the relativistic thermally averaged annihilation cross-section. The dependence on the specific particle physics model enters only in this cross-section which includes all annihilation and coannihilation channels,

$$(2) \qquad \langle \sigma v \rangle = \frac{\sum_{i,j} g_i g_j \int_{(m_i+m_j)^2} \mathrm{d}s \sqrt{s} K_1(\sqrt{s}/T) p_{ij}^2 \sum_{k,l} \sigma_{ij;kl}(s)}{2T \left(\sum_i g_i m_i^2 K_2(m_i/T) \right)^2},$$

where g_i is the number of degree of freedom, $\sigma_{ij;kl}$ the total cross-section for annihilation of a pair of odd particles with masses m_i, m_j into some Standard Model particles (k,l), and $p_{ij}(\sqrt{s})$ is the momentum (total energy) of the incoming particles in their center-of-mass frame.

Integrating eq. (1) from $T = \infty$ to $T = T_0$ leads to the present day abundance $Y(T_0)$ needed in the estimation of the relic density,

$$(3) \qquad \Omega_{\mathrm{LOP}} h^2 = \frac{8\pi}{3} \frac{s(T_0)}{M_p^2 (100\,(\mathrm{km/s/Mpc}))^2} M_{\mathrm{LOP}} Y(T_0) = 2.742 \times 10^8 \frac{M_{\mathrm{LOP}}}{\mathrm{GeV}} Y(T_0),$$

where $s(T_0)$ is the entropy density at present time and h the normalized Hubble constant.

To compute the relic density, `micrOMEGAs` solves the equation for the abundance eq. (1), numerically without any approximation. The computation of all annihilation and coannihilation cross-sections are done exactly at tree-level. For this we rely on `CalcHEP` [17]. In the thermally averaged cross-section, eq. (2), coannihilation channels are compiled and added only when necessary, that is when the mass difference with the LOP does not exceed a value determined by the user.

3. – New models

`micrOMEGAs` was first developed for the minimal supersymmetric standard model (MSSM). In this model the large number of annihilation channels made automation desirable. For this reason `micrOMEGAs` was based on `CalcHEP` [17]. The generalisation to other particle physics models was then straigthforward and only requires specifying the new model file into `CalcHEP` [17].

In order that the program finds the list of processes that need to be computed for the thermally averaged annihilation cross-section, relevant for the relic density calculation, one needs to specify the analogous of R-parity and assign a parity odd or even to every particle in the model. The standard model particles have an even parity. The lightest odd particle will then be identified with the DM candidate. `micrOMEGAs` automatically generates all processes of the type $\chi_i \chi_j \to X, Y$ where χ_i designates all R-parity odd particle and X, Y all R-parity even particles. `micrOMEGAs` then looks for s-channel poles as well as for thresholds to adapt the integration routines for higher accuracies in these specific regions, and performs the relic density calculation. Note that the code can also compute the relic density of a charged particle, this quantity can for example be used to extract the relic density of the gravitino DM.

Our approach is very general and, as long as one sticks to tree-level masses and cross-sections, necessitates minimal work from the user beyond the definition of the model file. However it has been demonstrated that for an accurate relic density calculation it is necessary in many cases to take into account higher-order corrections. In particular corrections to the mass of either the LOP or of any particle that can appear in s-channel are important, for example the corrections to Higgs masses in the MSSM or its extensions. The large QCD corrections to the Higgs width must also be taken into account when annihilation occurs near a Higgs resonance. In general one expects that these loop corrections can be implemented via an effective Lagrangian. In practice then it might be necessary for the user to implement additional routines or interface other programs to take these effects into account. This was done explicitly for the NMSSM [19], the CPVMSSM [20] and the MSSM with Dirac gaugino masses [21].

Another advantage of our approach based on a generic program like `CalcHEP` is that one can compute in addition any cross-section or decay width in the new model considered. In particular, tree-level cross-sections for $2 \to 2$ processes and decay widths of particles are available. Furthermore the cross-sections times relative velocity, σv, for DM annihilation at $v \to 0$ and the yields for the continuum γ, e^+, $\bar{p}$, ν spectra, relevant for indirect detection of DM, are also automatically computed. The procedure to extract the elastic scattering cross-section is described next.

4. – Direct detection

In direct detection, one measures the recoil energy deposited by the scattering of DM (χ) with the nuclei. Generically DM-nuclei interactions can be split into spin-independent

(scalar) and spin-dependent interations. The scalar interactions add coherently in the nucleus so heavy nuclei offer the best sensitivity. On the other hand, spin-dependent interactions rely mainly on one unpaired nucleon and therefore dominate over scalar interactions only for light nuclei unless scalar interactions are themselves suppressed. In both cases, the cross-section for the DM nuclei interaction is typically low, so large detectors are required. Many experiments involving a variety of nuclei have been set up. The best limit for spin-independent interactions was reported recently by CDMS with $\sigma_{\chi p}^{\mathrm{SI}} \approx 3.8 \times 10^{-8}$ pb for a DM mass around $70\,\mathrm{GeV}$ [22]. This limit already probes a fraction of the parameter space of DM models.

Many ingredients enter the calculation of the direct detection rate and cover both astroparticle, particle and nuclear physics aspects. The detection rate depends on the χ-nucleus cross-section which is derived from the interaction at the quark level. The different matrix elements for χq interactions have to be converted into effective couplings of DMs to protons and nucleons, this is done through coefficients that describe the quark content in the nucleons. Note that the DM have small velocities, therefore the momentum transfer is very small as compared to the masses of the DM and/or nuclei and the χ-nucleon elastic cross-sections can be calculated in the limit of zero momentum transfer.

The distribution of the number of events over the recoil energy for spin-independent interactions reads

$$(4) \qquad \frac{\mathrm{d}N^{\mathrm{SI}}}{\mathrm{d}E} = \frac{2M_{\mathrm{det}}t}{\pi}\frac{\rho_0}{M_\chi}F_A^2(q)(\lambda_p Z + \lambda_n(A-Z))^2 I(E),$$

where ρ_0 is the DM density near the Earth, M_{det} the mass of the detector, t the exposure time, $I(E)$ is an integral over the velocity distribution and $F_A(q)$ is the nucleus form factor which depends on the momentum transfer. The coefficients λ_p, λ_n contain all particle physics dependence and are extracted from the matrix elements for χq scattering coupled with coefficients that describe the quark content in the nucleons.

Traditionally the coefficients λ_q of the low-energy effective χq Lagrangian are evaluated symbolically using Fierz identities. Instead in `micrOMEGAs` we use an original approach that allows to handle a generic model. First we expand the χq interactions over a set of basic point-like operators, only a few operators are necessary in the $q^2 \to 0$ limit. The same operators also describe the DM-nucleon interactions. For example for SI interactions of a Majorana fermion with quarks or nucleons the effective Lagrangian reads

$$(5) \qquad \mathcal{L}^{\mathrm{SI}} = \lambda_q \overline{\psi}_\chi \psi_\chi \overline{\psi}_q \psi_q;$$

`micrOMEGAs` creates automatically a new model file that contains these operators as new auxiliary vertices in the model. CalcHEP then generates and calculates symbolically all diagrams for DM-quark/anti-quark elastic scattering at zero momentum transfer. The interference terms between one normal vertex and one auxiliary vertex allow to evaluate numerically the λ_q coefficients [23]. Note that in the file that defines the model all quarks should be defined as massive particles. Vertices that depend on light quark masses, for example the couplings of Higgs to light quarks cannot be neglected. The dominant term for DM quark scalar interactions is proportional to quark masses, when converting to DM nucleon interactions this quark mass will get replaced by a nucleon mass.

5. – Indirect detection

DM annihilation in the Galactic halo produces pairs of standard model particles. These particle then hadronize and decay into stable particles that evolve freely in the interstellar medium. The final states with γ, e^+, $\bar{p}$ and $\bar{D}$ are particularly interesting as they are the subject of indirect searches. Recently many new results from indirect DM searches have been released notably by PAMELA and Fermi [24, 25]. The rate for the production of these particles can be cast into

$$(6) \qquad Q(\mathbf{x}, E) = \frac{1}{2}\langle \sigma v \rangle \left(\frac{\rho(\mathbf{x})}{m_\chi} \right)^2 \frac{\mathrm{d}N}{\mathrm{d}E} \,,$$

where $\langle \sigma v \rangle$ is the annihilation cross-section times the relative velocity of incoming DM particles in the zero velocity limit. m_χ is the mass of the DM candidate, $\rho(\mathbf{x})$ is the DM density at the location $\mathbf{x}$ and $\mathrm{d}N/\mathrm{d}E$ is the energy distribution of the particle produced in one collision. The predictions for the energy spectra depend on non-perturbative QCD and imply the use of Monte Carlo simulations such as `PYTHIA`. The annihilation rates are extracted from `CalcHEP` in any model. The rate for photons observed near the center of the galaxy shows a strong dependence on the halo profile in particular on the DM density near the center, a quantity that still has large uncertainties. Different halo profiles are implemented in `micrOMEGAs`. In addition `micrOMEGAs` provides, in the MSSM, the rate for gamma-rays that come from direct annihilation of DM particles. This model-dependent process is loop-induced and suppressed but is nevertheless interesting because of the dramatic signature, a monochromatic gamma-ray line [26].

The charged particles generated from DM annihilation propagate through the Galactic halo and their energy spectrum at the Earth differs from the one produced at the source. Charged particles are deflected by the irregularities of the galactic magnetic field, suffer energy losses from synchroton radiation and inverse Compton scattering as well as diffusive reacceleration in the disk. Finally galactic convection wipes away charged particles from the disk. Solar modulation can also affect the low energy part of the spectrum. The equation that describes the evolution of the energy distribution for all particles (protons, anti-protons, positrons) reads

$$(7) \qquad \frac{\partial}{\partial z}(V_C \psi) - \boldsymbol{\nabla} \cdot (K(E)\boldsymbol{\nabla}\psi) + \frac{\partial}{\partial E}(b(E)\psi) = Q(\mathbf{x}, E),$$

where $\psi = \mathrm{d}n/\mathrm{d}E$ is the number density of particles per unit volume and energy. Q is the production rate of primary particles per unit volume and energy, eq. (6). $b(E)$ is the energy loss rate and K is the space diffusion coefficient, assumed homogeneous, $K(E) = K_0 \beta \mathcal{R}^\delta$ where β is the particle velocity and R its rigidity. The propagation equation is solved within a semi-analytical two-zone model. Within this approach the region of diffusion of cosmic rays is represented by a thick disk of thickness $2L$ and radius $R = 20\,\mathrm{kpc}$ [27] . The thin galactic disk where primary cosmic rays are accelerated lies in the middle and has a thickness $2h \approx 200\,\mathrm{pc}$ and radius R. The boundary conditions are such that the number density vanishes at $z = \pm L$ and at $r = \pm R$. The galactic wind is directed outward along the z direction so the convective velocity is also vertical and of constant magnitude $V_C(z) = V_C \mathrm{sign}(z)$. The propagation parameters δ, K_0, L, V_C are constrained by the analysis of the boron to carbon ratio, a quantity sensitive to cosmic ray transport [27]. Propagation induces large uncertainties in the prediction of the spectrum from

DM annihilation, furthermore extracting a signal from DM annihilation requires a careful understanding of the background as well as of astrophysical sources. Only the DM signal will be included in the first version of the indirect detection module to be released soon.

6. – Conclusion

`micrOMEGAs` is a comprehensive tool for DM studies in extensions of the SM. The implementation of new models and new features are being pursued. One objective is a more precise computation of the relic density. Within the MSSM [28] it was shown that the one-loop corrections to the annihilation cross-sections could be sizeable as compared with the precision expected from PLANCK measurements. An interface between the loop-improved computations and `micrOMEGAs` is being worked out.

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