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The phase evolution of the Universe during its cooling down in 2HDM

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Summary. — Two Higgs Doublet Model at different values of parameters realizes ground state (vacuum) with different properties. The parameters of the Gibbs potential are varied during cooling down of the Universe after Big Bang. At this variation properties of vacuum state can vary, Universe suffers phase transitions. The evolution of phase states and chains of phase transitions can be much more diverse than in Standard Model with single Higgs doublet. We analyzed phase history of earlier Universe for each set of parameters and find sets of modern parameters, responsible for different chains of thermal phase transitions.

PACS `11.10.Wx` – Finite-temperature field theory.
PACS `11.30.Qc` – Spontaneous and radiative symmetry breaking.
PACS `12.15.-y` – Electroweak interactions.
PACS `98.80.Cq` – Particle-theory and field-theory models of the early Universe (including cosmic pancakes, cosmic strings, chaotic phenomena, inflationary universe, etc.).

More complete version of this paper is given in [1]. We use some equations and notations from [2, 3].

1. – Lagrangian

The spontaneous electroweak symmetry breaking via the Higgs mechanism is described by the Lagrangian

$$\mathcal{L} = \mathcal{L}^{\rm SM}_{gf} + \mathcal{L}_H + \mathcal{L}_Y \qquad \text{with} \qquad \mathcal{L}_H = T - V. \tag{1}$$

Here $\mathcal{L}^{\rm SM}_{gf}$ describes the $SU(2)\times U(1)$ Standard Model interaction of gauge bosons and fermions, $\mathcal{L}_Y$ describes the Yukawa interactions of fermions with Higgs scalars $\varphi_i = \binom{\varphi_i^\dagger}{\varphi_i^0}$ and $\mathcal{L}_H$ is the Higgs scalar Lagrangian; T is the Higgs kinetic term and V is the Higgs potential.

The most general renormalizable Higgs potential of Two Higgs Doublet Model (2HDM) has form

$$V = -V_2(x_i) + V_4(x_i); \qquad V_2(x_i) = \left[m_{11}^2 x_1 + m_{22}^2 x_2 + \left(m_{12}^2 x_3 + \text{h.c.}\right)\right]/2, \tag{2}$$
$$V_4(x_i) = \frac{\lambda_1 x_1^2 + \lambda_2 x_2^2}{2} + \lambda_3 x_1 x_2 + \lambda_4 x_3 x_3^\dagger + \left[\frac{\lambda_5 x_3^2}{2} + \lambda_6 x_1 x_3 + \lambda_7 x_2 x_3 + \text{h.c.}\right],$$
$$x_1 = \varphi_1^\dagger \varphi_1, \qquad x_2 = \varphi_2^\dagger \varphi_2, \qquad x_3 = \varphi_1^\dagger \varphi_2.$$

1˙1. *Z_2 symmetry of potential and its violation. Natural choice.* – At $m_{12}^2 = 0$, $\lambda_6 = \lambda_7 = 0$, the (ϕ_1, ϕ_2) mixing is forbidden at all distances. This potential is invariant under transformations $\phi_1 \times \phi_2 \to -\phi_1 \times \phi_2$. Such invariance is usually called as Z_2 *symmetry.*

If $m_{12}^2 \neq 0$ and $\lambda_6 = \lambda_7 = 0$, the (ϕ_1, ϕ_2) mixing is allowed but only at large distances. That is the case of *softly violated Z_2 symmetry.* In this case renormalization in all orders does not generate nonzero λ_6 and λ_7. The generalized rotation in the (ϕ_1, ϕ_2)-space makes λ_6, $\lambda_7 \neq 0$. That is the case of *hidden softly violated Z_2 symmetry.* In this case mixing angles restoring softly Z_2 violated form of potential do not vary in iterations of perturbation theory and with the scale of distances [4].

If $\lambda_6 \neq 0$ and (or) $\lambda_7 \neq 0$ and these coefficients cannot be eliminated by suitable generalized rotation in the (ϕ_1, ϕ_2)-space, we are dealing with the case of *hardly violated Z_2 symmetry.* In this case the mixed kinetic term also appears in the iterations of perturbation theory with running coefficient, the mixing angles, the restoring diagonal form of the kinetic term become running [4]. We consider this situation as *un-natural.*

1˙2. *Working potential.* – That is why we consider evolution of the vacuum for the potential with softly broken Z_2-symmetry. We limit ourself by *explicitly CP-conserving potential* which is realized at all real λ_i, m_{ij}^2:

$$V_2(x_i) = -\frac{m_{11}^2 x_1 + m_{22}^2 x_2 + m_{12}^2 (x_3 + x_3^\dagger)}{2}, \tag{3a}$$
$$V_4(x_i) = \frac{\lambda_1 x_1^2 + \lambda_2 x_2^2}{2} + \lambda_3 x_1 x_2 + \lambda_4 x_3 x_3^\dagger + \frac{\lambda_5 (x_3^2 + x_3^{\dagger 2})}{2}.$$

All phases and phase transitions that can happen in the most general case can be mapped to phases and transitions in this model [5]. In particular, the classifications of regions in the parameter space and sequences of phase transitions derived below coincide with those obtained in the general case.

To make some equations shorter, we also introduce the following notation for certain combinations of λ_i:

$$\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5, \qquad \tilde\lambda_{345} = \lambda_3 + \lambda_4 - \lambda_5, \qquad \Lambda_{3\pm} = \sqrt{\lambda_1\lambda_2} \pm \lambda_3. \tag{3b}$$
$$\Lambda_{345\pm} = \sqrt{\lambda_1\lambda_2} \pm \lambda_{345}, \qquad \tilde\Lambda_{345\pm} = \sqrt{\lambda_1\lambda_2} \pm \tilde\lambda_{345}.$$

The description of many properties of minimum of potential at given m_{ij}^2 depends not on their absolute values, but on their ratios. Therefore, it is useful to introduce a special parametrization for the V_2 term:

$$m_{11}^2 = m^2(1-\delta), \qquad m_{22}^2 = k^2 m^2 (1+\delta), \qquad m_{12}^2 = \mu k m^2; \quad k \stackrel{\text{def}}{=} \sqrt[4]{\lambda_2/\lambda_1}. \tag{4}$$

1·3. *Positivity constraints.* – To have a stable vacuum, the potential must be positive at large quasi-classical values of fields $|\phi_k|$ (*positivity constraints*) for an arbitrary direction in the (ϕ_1, ϕ_2)-space. This translates into $V_4 > 0$ for all non-zero values of the fields, which places restrictions on possible values of λ_i. For the potential (3) such restrictions have a simple form (see, *e.g.*, [6, 7])

$$\lambda_1 > 0, \qquad \lambda_2 > 0, \qquad \Lambda_{3+} > 0, \qquad \Lambda_{345+} > 0, \qquad \tilde{\Lambda}_{345+} > 0. \tag{5}$$

1·4. *Yukawa sector.* – In order for the Z_2 symmetry to survive through the perturbation series, the Yukawa interactions $\mathcal{L}_Y$ must connect each right-handed fermion to only one scalar field ϕ_1 or ϕ_2 (in particular, Models I or II for Yukawa sector, see [8] for details of the definitions).

2. – Temperature dependence

At finite temperature, the ground state of a system is given by the minimum of the Gibbs potential

$$V_G = \mathrm{Tr}\left(Ve^{-\hat{H}/T}\right) / \,\mathrm{Tr}\left(e^{-\hat{H}/T}\right) \equiv V + \Delta V \equiv -V_2(T) + V_4(T). \tag{6}$$

Corrections ΔV, to the first nontrivial approximation, are given by simple loop diagrams. It is calculated with the Matsubara diagram technique at $T^2 \gg m_i^2$ [9]. In this approximation the quartic term $V_4(T)$ coincides with that of basic potential (3), $V_4(T) = V_4$, while the mass term $V_2(T)$ evolves with temperature:

$$\begin{gathered} m_{11}^2(T) = m_{11}^2(0) - 2c_1 m^2 w, \quad m_{22}^2(T) = m_{22}^2(0) - 2k^2 c_2 m^2 w, \quad m_{12}^2(T) = m_{12}^2(0), \\ c_i = c_i^s + c_i^g + c_i^f; \quad w = T^2/(12m^2). \end{gathered} \tag{7a}$$

The scalar loop contributions c_i^s and the gauge boson loop contributions c_i^g are

$$c_1^s = (3\lambda_1 + 2\lambda_3 + \lambda_4)/2, \quad c_2^s = (3\lambda_2 + 2\lambda_3 + \lambda_4)/(2k^2), \quad c_1^g = k^2 c_2^g = 3(3g^2 + g'^2)/8, \tag{7b}$$

with g and g' being the standard electroweak coupling constants. The fermion loop contributions c_i^f depend on the form of the Yukawa sector. For the Model II and Model I, the main contributions to these coefficients can be written in natural notation as

$$c_1^f(\mathrm{II}) = 3g_t^2/2, \quad c_2^f(\mathrm{II}) = 3g_b^2/(2k^2); \qquad c_1^f(\mathrm{I}) = 3(g_t^2 + g_b^2)/2, \quad c_2^f(\mathrm{I}) = 0. \tag{7c}$$

Simple algebra allows us to express temperature-dependent parameters of the potential (4) $m(T)$, $\delta(T)$, $\mu(T)$ via their "zero-point" (*i.e.* modern zero-temperature) values m, δ, μ:

$$m^2(T) = m^2[1 - (c_2 + c_1)w], \quad \mu(T) = \mu\frac{m^2}{m^2(T)}, \quad \delta(T) = \frac{m^2}{m^2(T)}[\delta - (c_2 - c_1)w]. \tag{8}$$

The thermal evolution of system is described by a straight rays in the (μ, δ)-plane.

3. – Extrema and phase transitions

The Higgs potential can have several extrema. The extremum with the lowest value of the energy, the global minimum of potential, realizes the vacuum state. The other extrema can be either saddle points or maxima or local minima of the potential.

The extrema of the potential define the values $\langle\phi_{1,2}\rangle$ of the fields $\phi_{1,2}$ via equations

$$\partial V/\partial\phi_i|_{\phi_i=\langle\phi_i\rangle}=0,\qquad \partial V/\partial\phi_i^\dagger|_{\phi_i=\langle\phi_i\rangle}=0. \tag{9}$$

For each extremum one can choose the z-axis in the weak isospin space along $\langle\phi_1\rangle$ direction, so that

$$\langle\phi_1\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v_1\end{pmatrix},\qquad \langle\phi_2\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}u\\ v_2e^{-i\xi}\end{pmatrix};\quad u,\,v_1,\,v_2>0. \tag{10}$$

In the considered case, the classification of different extrema can be established:

- *Electroweak symmetry extremum (EWs):* $u=v_1=v_2=0$. In this extremum electroweak symmetry is not broken, gauge bosons and fermions are massless.
- *Neutral CP conserving (CPc) extremum:* $u=0$, $\xi=0$.
- *Neutral spontaneously CP violating (sCPv) extremum:* $u=0$, $\xi\neq 0$.
- *Charged extremum (ch):* $u\neq 0$.

4. – CPc vacua and the first-order phase transition

The CPc extrema are realized in the entire space of parameters λ_i. One can transform two cubic equations representing conditions (9) into relations for quantities $v^2=v_1^2+v_2^2$ and $\tau=kv_1/v_2$. For potential (3) these equations, together with expression for extremum energy can be written as

$$\sqrt{\lambda_1\lambda_2}\mu\tau^4+(\Lambda_{345-}-\delta\cdot\Lambda_{345+})\tau^3-(\Lambda_{345-}+\delta\cdot\Lambda_{345+})\tau-\sqrt{\lambda_1\lambda_2}\mu=0, \tag{11}$$

$$v^2=m^2(k^2+\tau^2)\frac{1-\delta+\mu\tau}{\lambda_{345}\tau^2+\sqrt{\lambda_1\lambda_2}}\,, \tag{12}$$

$$\mathcal{E}_{CPc}=-\frac{m^4k^2}{8}\cdot\frac{(1-\delta+\mu\tau)[1-\delta+2\mu\tau+\tau^2(1+\delta)]}{\lambda_{345}\tau^2+\sqrt{\lambda_1\lambda_2}}\,.$$

At $\delta=0$ eq. (11) is factorized and easily solved:

$$(\tau^2-1)\left(\sqrt{\lambda_1\lambda_2}\mu(\tau^2+1)+\Lambda_{345-}\tau\right)=0\Rightarrow\tau_{A\pm}=\pm 1, \tag{13}$$

$$\tau_{B\pm}=\frac{-\Lambda_{345-}}{2\mu\sqrt{\lambda_1\lambda_2}}\left(1\pm\sqrt{1-\frac{4\mu^2\lambda_1\lambda_2}{\Lambda_{345-}^2}}\right).$$

The energies of extrema, that correspond to the solutions $\tau_{A\pm}$, $\tau_{B\pm}$ are

$$\mathcal{E}_{CPcA}=-\frac{m^4k^2}{4}\cdot\frac{(1\pm\mu)^2}{\Lambda_{345+}}\,,\qquad \mathcal{E}_{CPcB\pm}=-\frac{m^4k^2}{4}\left(\frac{1}{2\sqrt{\lambda_1\lambda_2}}-\frac{\mu^2}{\Lambda_{345-}}\right). \tag{14}$$

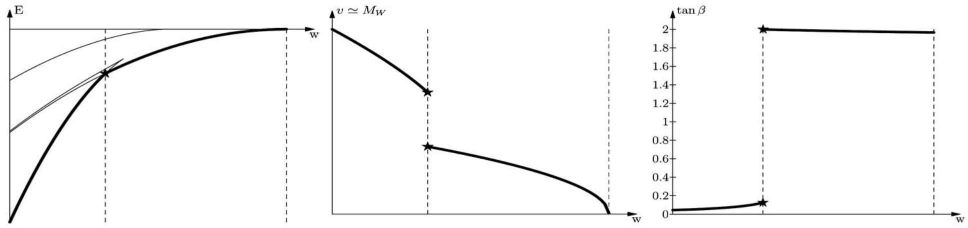

Fig. 1. – Evolution of physical parameters at first-order phase transition.

One can see that the solutions B_+ and B_- are degenerate in energy. This degeneracy is a result of additional discrete symmetry of potential (not Lagrangian!) $\phi_1 \leftrightarrow k\phi_2$, that appears at $\delta = 0$.

Consider the case when $\delta(T)$ crosses $\delta(T) = 0$ at certain temperature, and at that temperature $B\pm$ is a vacuum. Then one can show that the phase with the lowest extremum energy switches from B_+ to B_- (or vice versa). That is a phase transition from the phase $B_\pm$ to the phase $B_\mp$.

Figure 1 shows the dependence of extremum energy (left plot), and v (middle plot), $\tan\beta = v_1/v_2$ (right plot) on temperature parameter w (7a) in the considered case. Thick line represents the values in vacuum state. Thin lines on the left plot show the energies of other extrema.

Points of phase transitions are marked by a small star. The first-order nature of the phase transition is evident on these plots, as the quantities v and $\tan\beta$ jump at the transition point.

Realization of the degenerate situation can only happen at certain parameters of potential. The constraints for $B\pm$ to be minimum together with the constraint on the existence of solution $\tau_{B\pm}$ in (13) can be written as

$$(15) \qquad \Lambda_{345-} < 0, \quad \Lambda_{3-} < 0, \quad \tilde{\Lambda}_{345-} < 0; \qquad 2|\mu| \le |\Lambda_{345-}|/\sqrt{\lambda_1\lambda_2}, \quad \delta = 0.$$

Note that constraints on quartic parameters λ_i are independent of quadratic parameters μ and δ. The same pattern will follow for the other types of phase transitions.

5. – Transition through sCPv vacuum

In the case of sCPv extremum one can find the value of $\cos\xi$ first. After that the system (9) transforms into a system of two linear equations (see [10]). The solution of this system can be written explicitly:

$$(16) \quad v_1^2 = k^2m^2\left[\frac{1}{\tilde{\Lambda}_{345+}} - \frac{\delta}{\tilde{\Lambda}_{345-}}\right], \quad v_2^2 = m^2\left[\frac{1}{\tilde{\Lambda}_{345+}} + \frac{\delta}{\tilde{\Lambda}_{345-}}\right], \quad \cos\xi = \frac{\mu k m^2}{2\lambda_5 v_1 v_2}.$$

To make sure that sCPv extremum exists, one must require that $|\cos\xi| < 1$. This constraint and the conditions for this extremum to be vacuum are

$$(17) \quad \lambda_5 > 0, \quad \lambda_5 > \lambda_4, \quad \tilde{\Lambda}_{345-} > 0; \qquad \frac{\mu^2}{b_1^2} + \frac{\delta^2}{b_2^2} < 1, \quad b_1 = \frac{2\lambda_5}{\tilde{\Lambda}_{345+}}, \quad b_2 = \frac{\tilde{\Lambda}_{345-}}{\tilde{\Lambda}_{345+}}.$$

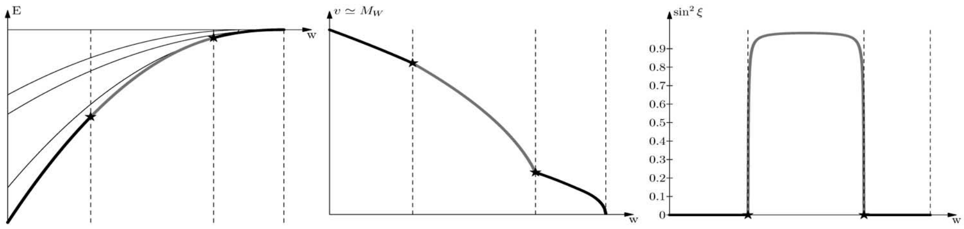

Fig. 2. – Evolution of physical parameters in case of transition through sCPv vacuum.

One can say that if quartic parameters satisfy the presented relations, then the quadratic parameters μ and δ should lie inside the ellipse with semiaxes b_1 and b_2 on a (μ, δ)-plane.

Evolution of physical parameters during transition through sCPv vacuum is presented in fig. 2. Notation and description of these plots are the same as for the previous case fig. 1. The rightmost figure now represents the behavior of the order parameter for sCPv case, $\sin^2 \xi$, which is non-zero only for the sCPv phase, and shows typical behavior of the order parameter during 2nd-order phase transitions.

6. – Transition through charged vacuum

The charged vacuum is not a modern state of our Universe, but it is possible that the Universe passes this state during cooling after Big Bang. The conditions for this extremum can be rewritten as a linear system on quadratic combinations of vev's. The solution of this system can be written explicitly:

$$(18)\quad v_1^2 = k^2 m^2 \left[\frac{1}{\Lambda_{3+}} - \frac{\delta}{\Lambda_{3-}} \right], \qquad u^2 + v_2^2 = m^2 \left[\frac{1}{\Lambda_{3+}} + \frac{\delta}{\Lambda_{3-}} \right], \qquad v_1 v_2 = \frac{m^2 k \mu}{\lambda_4 + \lambda_5}.$$

It can be proven (see [10]) that if charged extremum exists, then it is a vacuum state of the model. The conditions for the existence of the extremum can be reduced to the form similar to (17):

$$(19)\qquad \lambda_4 \pm \lambda_5 > 0, \quad \Lambda_{3-} > 0; \qquad \frac{\mu^2}{a_1^2} + \frac{\delta^2}{a_2^2} < 1, \quad a_1 = \frac{\lambda_4 + \lambda_5}{\tilde{\Lambda}_{3+}}, \quad a_2 = \frac{\tilde{\Lambda}_{3-}}{\tilde{\Lambda}_{3+}}.$$

Figure 3 presents the evolution of physical parameters in case of transition through charged vacuum. On the right figure the evolution of the order parameter $\zeta = u/v_2$ is presented.

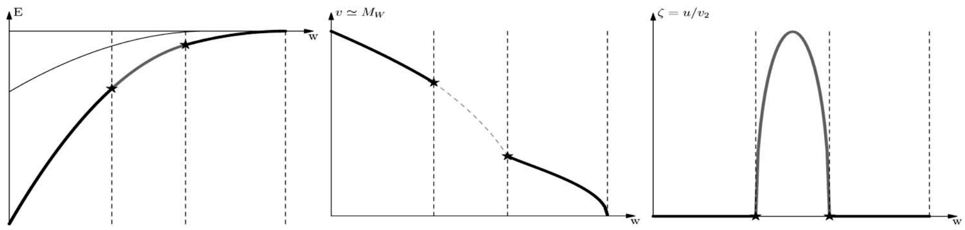

Fig. 3. – Evolution of physical parameters in case of transition through charged vacuum.

7. – Discussion

We presented the phases and thermal phase transitions in 2HDM with softly broken Z_2 symmetry. This case is representative for the most general form of 2HDM Lagrangian [5]. We obtained a rich picture of possible phase transitions. Higher-order effects will modify our results, but we believe that with these modifications the picture can only become richer. And even the picture obtained looks very interesting.

Let us now discuss some general features of the picture obtained.

7·1. *Possible sequences of phase states.* – Thermal evolution can be split into two stages whose properties are rather decoupled from each other. First—is the evolution at very high temperatures at $m^2(T) < 0$. At these temperatures restoration of EW symmetry can occur. Or it is possible to have non-restoration of EW symmetry or even transition through EW symmetric vacuum (see analysis in [1]). At the second stage it is possible to have different sequences of transitions between EW-violating extrema, which was discussed above.

$$\left.\begin{array}{ll} (a) & EWs \\[2ex] (b) & CPc \\[2ex] (c) & CPc \xrightarrow{\text{II}} EWs \end{array}\right\} \xrightarrow{\text{II}} \begin{array}{ll} CPc & \text{(I)} \\ CPc \xrightarrow{\text{II}} \text{charged} \xrightarrow{\text{II}} CPc & \text{(II)} \\ CPc1 \xrightarrow{\text{I}} CPc2 & \text{(III)} \\ CPc \xrightarrow{\text{II}} sCPv & \text{(IV)} \\ CPc \xrightarrow{\text{II}} sCPv \xrightarrow{\text{II}} CPc & \text{(V)} \end{array} \tag{20}$$

The sequence (EW $\xrightarrow{\text{II}}$ CPc $\xrightarrow{\text{II}}$ charged) was omitted from this list. Each possible sequence from the left column can be combined with any possible sequence from the right column. In the case (b) + (I), the history of the Universe contains no phase transition at all.

7·2. *Regions of parameters allowing for different paths of phase evolution.* – In ref. [1] we presented a method how to describe regions in the space of the zero-temperature parameters of the model that lead to each specific type of thermal evolution of the Universe. To cast them into the corresponding regions of observables, such as masses and couplings constants, is a natural task for continuation of this work.

7·3. *Rearrangement of particle mass spectrum.* – In most examples considered here the value of $\tan\beta \equiv v_1/v_2$ changes strongly. If the Model II for Yukawa sector is considered, then the fermion mass spectrum within one generation can be rearranged. Under these circumstances, it is possible that in the past the decay $t \to Wb$ was suppressed, and $W \to tb$ decay was allowed or even that the b-quark was heavier than t-quark.

Another interesting opportunity, which can be realized in many cases, is a non-monotonic dependence of masses of particle on temperature, when they start from zero at EWSB phase transition, grow and overshoot their today's values and drop down after subsequent phase transitions.

This rearrangement of fermion masses can be viewed as yet another phase transition in fermion subsystem, with own fluctuations, etc. The study of this possible phase transition goes beyond the approach developed in this paper.

There exists a possibility that the last phase transition took place relatively lately in the history of the Universe. In this case the possible rearrangement of the quark mass spectrum could have even more spectacular effects. For example, if the Universe lived long enough in an intermediate phase with $m_d < m_u$, then the proton could be lighter than the neutron and could even decay into it during this intermediate stage, which has profound cosmological consequences.

7˙4. *Possible relations to cosmology*

1) Different phenomena discussed here can give rise to new effects in the structure of Cosmic Microwave Background radiation and other cosmological observables. Feasibility of their observation is a subject for future studies.

   The cases with new phase transitions in addition to the standard EWSB lead to additional stages in the early history of the Universe with strong fluctuations near the phase transition points. For example, in many cases there exist either a metastable local minimum state or other extrema just above the vacuum state. Possible virtual transitions to these states can enhance fluctuations and their observable effects.

2) If the charge-breaking vacuum state was indeed an intermediate stage of the evolution of the Universe, then a number of unexpected effects appear and they can strongly influence the modern situation. First of all, in the charge-breaking phase all the gauge bosons are massive, and electric charge is not conserved, the local electric neutrality of medium can be strongly violated. After a phase transition to the modern charge-conserving vacuum, strong deviations from the local electroneutrality, originating from the charge-breaking phase, can occur. They will result in a strong relative motion of separate parts of the Universe, which can result either in strong mixing and averaging of matter or in production of structures like caustics (proto-galaxies). The restoration of the electric neutrality can go on during a very long time after the phase transition to neutral vacuum.

3) In the standard approach the temperature of the phase transition is unavoidably set by the electroweak scale. In our model the same is valid for the first EWSB transition. However, the temperature of the last phase transition can be sufficiently low. Certainly, for a detailed description of such situation our approximation must be improved.

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