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Neutralino Dark Matter in non-universal and non-minimal SUSY

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Summary. — We discuss neutralino dark matter in non-universal SUSY including the NUHM, $SU(5)$ with non-universal gauginos. In the MSSM we argue from naturalness that non-universal soft mass parameters are preferred, with non-universal gaugino masses enabling supernatural dark matter beyond the MSSM, we also discuss neutralino dark matter in the USSM and E_6SSM. In the E_6SSM a light neutralino LSP coming from the inert Higgsino and singlino sector is unavoidable and makes an attractive dark matter candidate.

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1. – Introduction

Supersymmetry (SUSY) has been considered to be the best candidate beyond the standard model (SM) from a viewpoint of both the hierarchy problem and the gauge coupling unification. Recent astrophysical observations showing the existence of a substantial amount of non-relativistic and non-baryonic dark matter seem to make SUSY even more promising. The lightest SUSY particle (LSP) of R-parity conserving models, in most cases the lightest neutralino, can serve as a good candidate for Dark Matter (DM). Indeed a key motivation for TeV scale supersymmetry (SUSY) is that it provides a natural dark matter candidate if the lightest neutralino is the LSP. However the regions of parameter space that yield neutralino dark matter in agreement with WMAP look very restricted. We have studied the naturalness of dark matter in the MSSM and CMSSM in [1].

Questions of fine-tuning have long been considered in the case of electroweak symmetry breaking. In many of these studies the degree of fine-tuning required was quantified through a measure of the sensitivity of m_Z^2 to the input parameters of the MSSM a_{MSSM}. We use a similar measure to quantify the degree of fine-tuning required of the MSSM parameters to produce an LSP that reproduces the observed dark matter relic density:

$$\Delta^{\Omega}_{a_{\mathrm{MSSM}}} = \left| \frac{\partial \ln\left(\Omega_{\mathrm{CDM}} h^2\right)}{\partial \ln\left(a_{\mathrm{MSSM}}\right)} \right| . \qquad (1)$$

We take the total tuning of a point to be $\max(\Delta^{\Omega}_{a})$.

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The calculation of $\Omega_{\rm CDM}h^2$ primarily depends on $a_{\rm MSSM}$ through their effect on the annihilation cross-section of the lightest neutralino $\tilde{\chi}_1^0$. This is primarily determined by the mass and composition of $\tilde{\chi}_1^0$. This in turn is determined by diagonalising the neutralino mass matrix at low energy. The matrix depends upon M_1, M_2, and μ. If one of these is much lighter than the others, $\tilde{\chi}_1^0$ will be primarily of that form. This allows us to divide up the MSSM parameter space depending on the composition of $\tilde{\chi}_1^0$. A wino LSP ($M_2 \ll M_1,\, \mu$) or higgsino LSP ($\mu \ll M_1,\, M_2$) annihilates very efficiently, resulting in too little dark matter, $\Omega_{\rm CDM}h^2 \ll \Omega_{\rm CDM}^{\rm WMAP}h^2$. A bino LSP ($M_2 \ll M_1,\, \mu$) annihilates primarily via t-channel slepton exchange. This process is only efficient for light sleptons and so bino LSPs generally result in too large a relic $\Omega_{\rm CDM}h^2 \gg \Omega_{\rm CDM}^{\rm WMAP}h^2$.

Therefore to fit the observed dark matter density we are required to move to unusual regions of the parameter space. One possibility is to consider a mixed LSP. If we have a bino LSP with just enough of either wino or higgsino mixed in, we can fit $\Omega_{\rm CDM}^{\rm WMAP}h^2$. Alternatively we can consider a bino LSP in which the annihilation cross-section is enhanced via some means. This can occur in a few different ways. Firstly, if there are light sfermions, t-channel sfermion is enhanced. Secondly if $2m_{\tilde{\chi}_1^0} = m_{Z,A,h}$ the neutralinos can annihilate to an on-shell boson. Finally if the NLSP is quasi-degenerate in mass with the LSP, there will be a significant NLSP number density at freeze out and we must factor in annihilations of the NLSP into our calculations of the SUSY relic density. All of these effects can enhance annihilation of a bino LSP to the extent that we fit the observed dark matter relic density. We would expect each region to exhibit a different sensitivity to the MSSM input parameters.

To study these regions we take 4 different sets of boundary conditions on the MSSM input parameters $a_{\rm MSSM}$ at $m_{\rm GUT}$, beginning with the familiar case of the constrained minimal supersymmetric standard model (CMSSM).

2. – Neutralino dark matter in the CMSSM

The CMSSM has 4 free inputs:

$$a_{\rm CMSSM} \in \{m_0,\ m_{1/2},\ A_0,\ \tan\beta \text{ and } \mathrm{sign}(\mu)\}. \tag{2}$$

m_0 is a common scalar mass that sets the soft masses of the sfermion and Higgs sectors. $m_{1/2}$ is a common gaugino mass. A_0 sets the soft SUSY breaking trilinear coupling. $\tan\beta$ is the ratio of the Higgs VEVs. Finally the requirement that the model provide radiative electroweak symmetry breaking determines the magnitude of the SUSY conserving Higgs mass μ but leaves the sign as a free parameter.

The mass and composition of the lightest neutralino is determined by diagonalising a mass matrix that depends upon the parameters M_1, M_2 and μ at the electroweak scale. In the CMSSM $M_1 = M_2 = m_{1/2}$ at the GUT scale. As running effects mean that $M_1(m_Z) \approx 0.4M_1(m_{\rm GUT})$ and $M_2(m_Z) \approx 0.8M_2(m_{\rm GUT})$, gaugino mass unification sets $M_1(m_Z) \approx 0.5M_2(m_Z)$. Therefore, unless $\mu < M_1$, the lightest neutralino will be dominantly bino.

In fig. 1 we consider the $(m_0, m_{1/2})$-plane of the CMSSM parameter space with $\tan\beta = 10$, $A_0 = 0$. Across this parameter space $\tilde{\chi}_1^0$ is bino. This generally results in $\Omega_{\rm CDM}h^2 \gg \Omega_{\rm CDM}^{\rm WMAP}h^2$. However at low m_0 the $\tilde{\tau}$ becomes light. In the light (green) region $m_{\tilde{\tau}} < m_{\tilde{\chi}_1^0}$ and the region is ruled out as this would result in a charged LSP. Along the edge of this region $m_{\tilde{\tau}} \approx m_{\tilde{\chi}_1^0}$. This means that at the time of freeze out there would have been a large number density of $\tilde{\tau}$ alongside the $\tilde{\chi}_1^0$, allowing many more annihilation

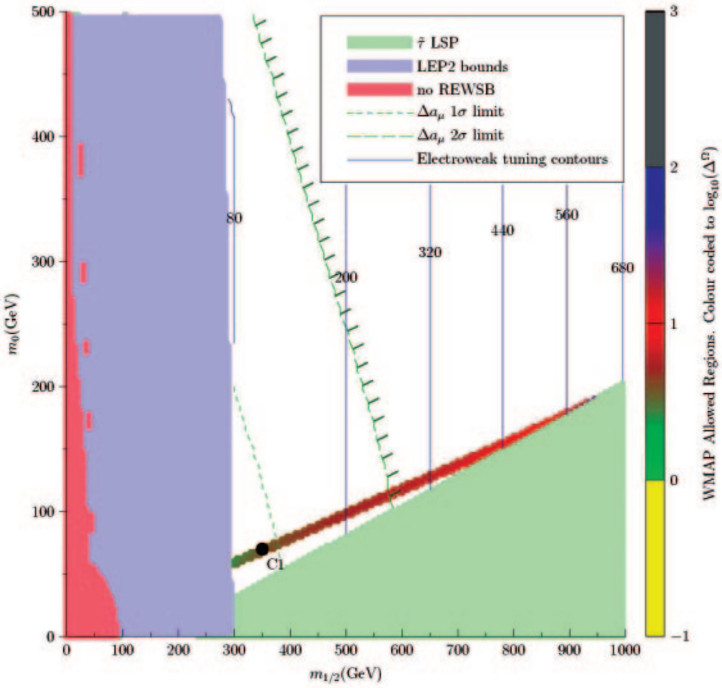

Fig. 1. – (Colour on-line) The $(m_{1/2}, m_0)$-plane for the CMSSM with $A_0 = 0$, $\tan\beta = 10$.

channels than are open for neutralinos alone. This results in a significant decrease of the neutralino relic density. In the multicoloured strip that lies alongside the $\tilde{\tau}$ LSP region, this coannihilation process results in $\Omega_{\rm CDM}h^2 = \Omega_{\rm CDM}^{\rm WMAP}h^2$. The varying colours of this strip represent the value of Δ^Ω, defined by the colour legend on the right.

Coannihilation occurs when the LSP and NLSP are close in mass. As a result, the efficiency of coannihilation processes is highly sensitive to the mass difference $\delta m = m_{\rm NLSP} - m_{\rm LSP}$. If these masses are determined by separate parameters we would expect that a large degree of tuning would be required to fit the observed dark matter density. In the coannihilation strip of fig. 1 the NLSP is the stau. The stau mass is set at the GUT scale by m_0 and the neutralino mass is set by $m_{1/2}$. As these are independent parameters, we would expect the $\tilde{\tau} - \tilde{\chi}_1^0$ coannihilation region to exhibit considerable fine-tuning.

The colour coding of the coannihilation strip shows a tuning of 3–15, considerably lower than would be expected if $m_{\tilde{\tau}}$ and $m_{\tilde{\chi}_1^0}$ were unrelated. The smallness of the tuning comes from the fact that along the coannihilation strip $m_0 < m_{1/2}$. The running of the right-handed slepton masses is strongly dependent on M_1. When m_0 is small, the dominant contribution to the low energy $\tilde{\tau}$ mass is via this running contribution from M_1. Thus in this region of the CMSSM $m_{\tilde{\tau}}$ depends strongly on $m_{1/2}$, resulting in a correlation of the masses of the neutralino and the stau at low energy. It is this correlation of the masses that results in the low tuning observed.

Though we do not show it here, we have also investigated the other regions of the CMSSM that fit $\Omega_{\rm CDM}^{\rm WMAP}h^2$. For large m_0 μ becomes small and we have a bino/higgsino LSP. We find that such regions exhibit a tuning $\Delta^\Omega = 30$–60, less natural than the

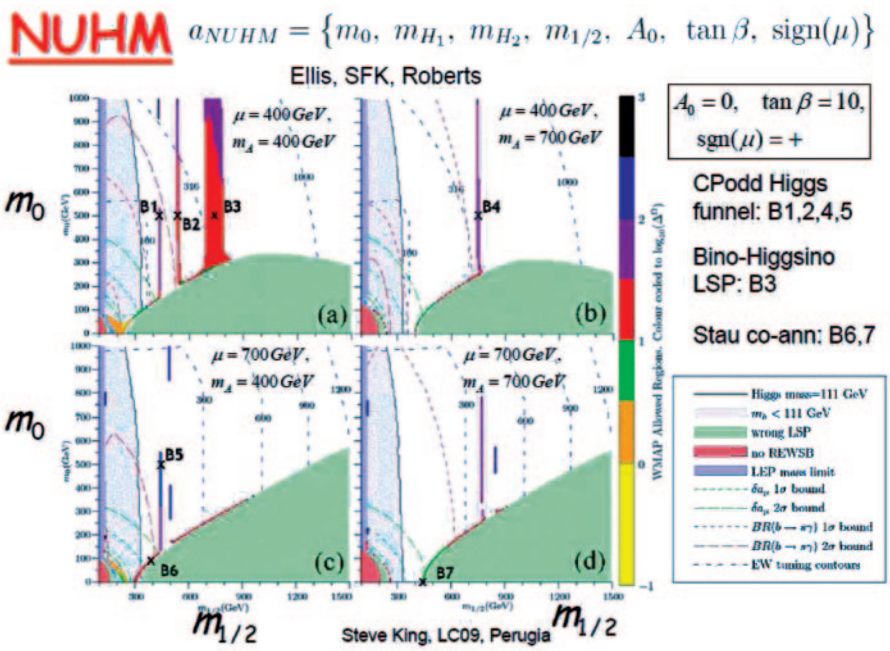

Fig. 2. – Neutralino dark matter in the NUHM.

coannihilation strip. For large $\tan\beta$ we can also access a region in which $m_A \approx 2m_{\tilde{\chi}_1^0}$. This allows for neutralino annihilation via the production of an on-shell pseudoscalar Higgs boson. We find such an annihilation channel to require a tuning $\Delta^{\Omega} = 80$–300. Finally at large $\tan\beta$ the running of the $\tilde{\tau}$ mass is no longer dominated by M_1. This breaks the correlation between $m_{\tilde{\chi}_1^0}$ and $m_{\tilde{\tau}}$ at low energies and results in the coannihilation strip that requires a tuning $\Delta^{\Omega} \approx 50$. This bears out our expectations that coannihilation should require significant tuning to achieve in normal circumstances.

3. – Neutralino dark matter in the NUHM

We have extended the analysis of fine-tuning to the non-universal Higgs model (NUHM) where the two Higgs doublet soft masses are taken to be free parameters [2]. The NUHM has two additional free inputs:

$$a_{\text{CMSSM}} \in \{m_0,\ m_{H_1},\ m_{H_2},\ m_{1/2},\ A_0,\ \tan\beta \text{ and } \text{sign}(\mu)\}. \quad (3)$$

m_0 is a common scalar mass that sets the soft masses of the sfermion sector while the Higgs sectors are described by two Higgs doublet soft masses m_{H_1}, m_{H_2} taken to be free parameters. As in the MSSM $m_{1/2}$ is a common gaugino mass and A_0 sets the soft SUSY breaking trilinear coupling. $\tan\beta$ is the ratio of the Higgs VEVs. Finally the requirement that the model provide radiative electroweak symmetry breaking determines m_{H_1}, m_{H_2} in terms of the magnitude of the SUSY conserving Higgs mass μ and the CP odd Higgs mass m_A, leaving the sign of μ as a free parameter.

In fig. 2 we consider the $(m_0, m_{1/2})$-plane of the NUHM parameter space with $\tan\beta = 10$, $A_0 = 0$ and various (positive) μ and CP odd Higgs mass m_A. Successful regions are shown corresponding to the CP odd Higgs funnel, the bino-Higgsino LSP as well as the stau co-annihilation region. However significant dark matter fine-tuning sensitivity remains for all these regions.

4. – Neutralino dark matter with non-universal gauginos

The best way to reduce fine-tuning sensitivity in SUSY models is to access the so-called "bulk region" coresponding to a bino LSP in which the annihilation cross-section is enhanced by having light sfermions which can be exchanged in the t-channel. In the CMSSM the bulk region is not accessible since the Higgs mass requires typically $M_{1/2} > 300\,\mathrm{GeV}$ leading to the t-channel sleptons being too heavy. However if gaugino mass universality is relaxed, with typically $M_{1,2} < M_3$, then lighter sleptons may be obtained while maintaining a sufficiently heavy Higgs boson and the bulk region may be accessed, leading to "natural" or even "supernatural" neutralino dark matter [1]. For example, non-universal gaugino masses may arise from type I string theory [3].

One obvious question is whether non-universal gaugino masses are consistent with grand unified theories (GUTs). Indeed it is perfectly possible to achieve this providing there is at least one SUSY breaking field in a non-singlet representation of the GUT group whose F-term acquires a VEV. The Lagrangian term leading the non-universal gaugino masses is of the form

$$\frac{\langle F_\Phi \rangle_{ij}}{M_{\mathrm{Planck}}} \lambda_i \lambda_j. \tag{4}$$

For example we have considered $SU(5)$ where SUSY is broken by a combination of a singlet and one of the 24, 75 or 200 representations of $SU(5)$ [2]. The results in fig. 3 clearly show that the supernatural regions of neutralino dark matter where bulk annihilation becomes possible (corresponding to the yellow regions) can easily be accessed in these models.

5. – Neutralino dark matter in the USSM

Nobody tells us that SUSY is realized in the guise of the MSSM, with or without universal soft parameters. Indeed the MSSM also suffers from the so-called μ problem: why the dimensionful parameter μ of the supersymmetric Higgs mass term $\mu \hat{H}_1 \hat{H}_2$ has to be of EW scale. This problem can be solved in the next-to-MSSM (NMSSM) by promoting the μ parameter to a new singlet superfield S coupled to Higgs doublets, $\lambda \hat{S} \hat{H}_1 \hat{H}_2$. This triple-Higgs coupling term also helps to push up the mass of the lightest CP-even Higgs boson, relaxing the fine-tuning necessary to comply with the LEP bounds. Postulating an additional $U_X(1)$ gauge symmetry avoids a massless axion, or domain wall problems of the NMSSM. Such a $U(1)$-extended MSSM, the so-called USSM [4], can be considered as an effective low-energy approximation of a more complete E_6SSM model [5], with other E_6SSM fields assumed heavy.

In addition to the MSSM superfields, the USSM contains a chiral superfield $\hat{S}$ and an Abelian gauge superfield B'. Thus the MSSM particle spectrum is extended by a new CP-even Higgs boson S, a gauge bozon Z' and two neutral –inos: a singlino $\tilde{S}$ and a bino′ $\tilde{B}'$; other sectors are not enlarged. The extended neutralino mass matrix then resembles the MSSM but with an extra $\tilde{S}$, $\tilde{B}'$ sector appended which mix together as

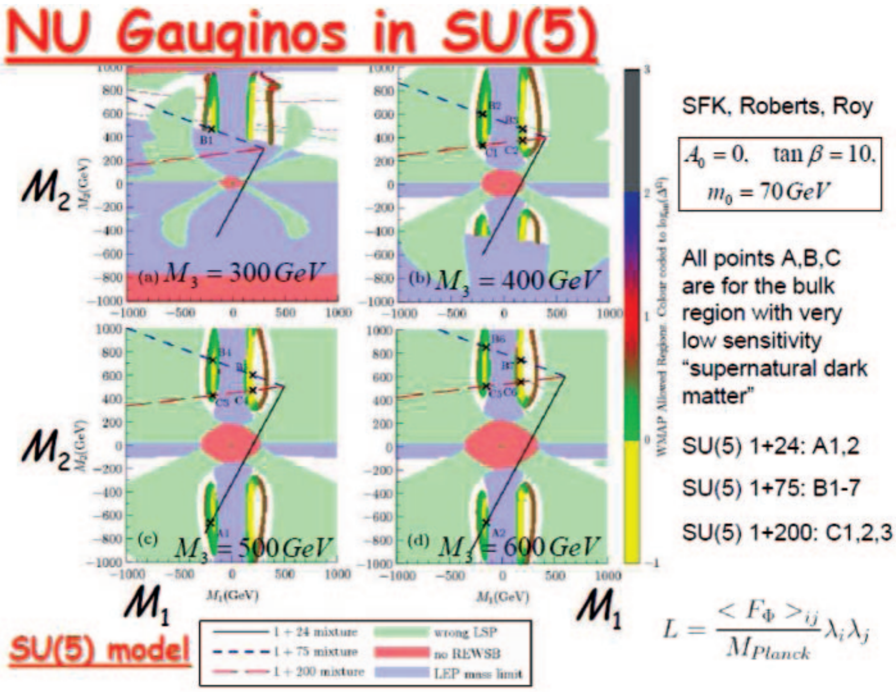

Fig. 3. – Neutralino dark matter in the NU $SU(5)$.

a 2 by 2 matrix in the $\tilde{S}$, $\tilde{B}'$ basis:

$$\begin{pmatrix} 0 & M_{Z'} \\ M_{Z'} & M_1' \end{pmatrix}, \tag{5}$$

where $M_{Z'}$ is the mass of the Z' which sets the scale for $\tilde{S} - \tilde{B}'$ mixing. For very large $\tilde{B}'$ mass M_1' a sort of see-saw mechanism sets occurs resulting in a light singlino $\tilde{S}$, while for small M_1' all the states in this sector are heavy.

The results in fig. 4 are for non-universal gaugino masses with the $U(1)'$ soft bino′ mass M_1' allowed to vary independently along the x-axis of these plots, while M_1, M_2 are held fixed, along with the effective $\mu = \lambda\langle S\rangle$ parameter and $\tan\beta$ fixed as shown. For low M_1' the LSP starts out as MSSM Higgsino with efficient annihilation leading to too little dark matter. Then as M_1' increases the LSP becomes increasingly singlino-like (due to the see-saw mechanism described above) and this inhibits the annihilation thereby increasing the relic abundance until it crosses the WMAP line. Thus successful dark matter can be achieved in this model. Figure 4 also shows the corresponding spin-independent and spin-dependent cross-sections whose behaviour as a function of M_1' can be readily understood from its Higgsino-singlino component nature.

6. – Neutralino dark matter in the E_6SSM

The USSM is not a complete model since it is not anomaly free. As mentioned it must be regarded as an effective low-energy approximation of a more complete E_6SSM

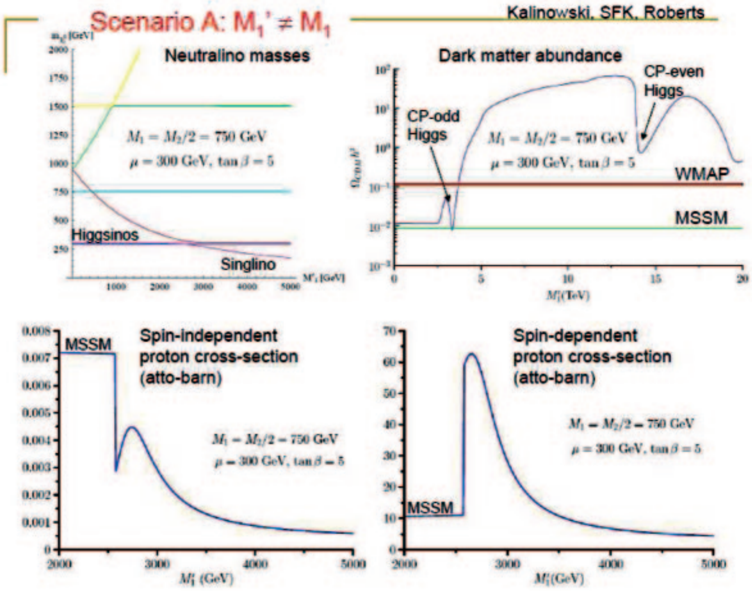

Fig. 4. – Neutralino dark matter in the USSM.

model [5]. The E$_6$SSM [5] is an E_6 inspired model with an extra gauged $U(1)_N$ symmetry at low energies, defined by $U(1)_N = 1/4U(1)_\chi + \sqrt{15}/4U(1)_\psi$, with $U(1)_\chi$ and $U(1)_\psi$ in turn, defined by the breaking, $E_6 \to SO(10) \times U(1)_\psi$ and $SO(10) \to SU(5) \times U(1)_\chi$. The low energy gauge group of the E$_6$SSM is then $SU(3) \times SU(2) \times U(1)_Y \times U(1)_N$. The matter content fills three generations of 27plet representations of E$_6$, leading to an automatic cancellation of anomalies. Each 27plet contains one generation of ordinary matter; singlet fields, S_i; up and down type Higgs like field, $H_{u,i}$ and $H_{d,i}$ and exotic squarks, D_i, $\bar{D}_i$. The model also contains two extra $SU(2)$ doublets, H' and $\bar{H}'$, which are required for gauge coupling unification.

From the point of view of neutralino dark matter, the important point about this model is that there are two inert families of Higgsinos and singlinos whose scalar partners do not couple to fermions and do not get VEVs (that is why we call them inert) and which are naturally light [6]. Consider one such inert family consisting of neutral components $\tilde{H}^0_{u,1}$, $\tilde{H}^0_{d,1}$ and a singlino $\tilde{S}_1$. They are almost decoupled from the USSM neutralinos and form a mass matrix in the basis $\tilde{H}^0_{d,1}$, $\tilde{H}^0_{u,1}$, $\tilde{S}_1$ [6]:

$$\begin{pmatrix} 0 & \lambda' s & f v \sin\beta \\ \lambda' s & 0 & f v \cos\beta \\ f v \sin\beta & f v \cos\beta & 0 \end{pmatrix}. \tag{6}$$

Diagonalizing the matrix leads to an LSP mass of order

$$m_{\mathrm{LSP}} \sim \frac{f^2 v^2}{\lambda' s} \sin 2\beta \tag{7}$$

which clearly shows that a light LSP is unavoidable since $s = \langle S \rangle \sim M_{Z'}$ is large

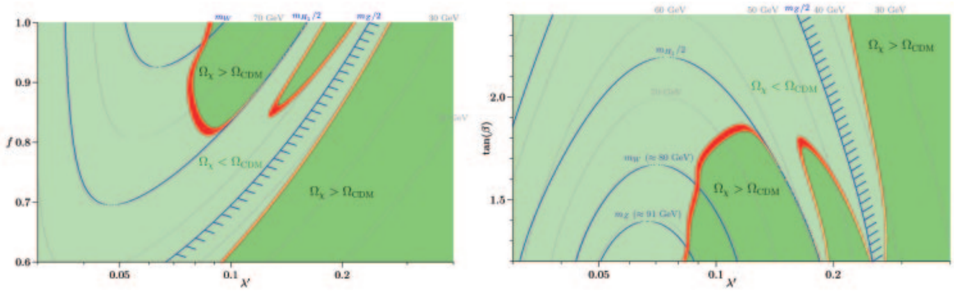

Fig. 5. – (Colour on-line) Neutralino dark matter in the USSM. Left panel is for $\tan\beta = 1.5$. Right panel is for $f = 1$.

compared to the electroweak Higgs VEV v. Successful dark matter is indicated by the red regions in fig. 5 which is taken from [6] where the other parameters are defined. The contours of LSP masses are also shown where an LSP mass which exceeds $M_Z/2$ is required to avoid conflict with the Z invisible width as measured at LEP.

7. – Conclusion

We have discussed neutralino dark matter in non-universal SUSY including the NUHM, $SU(5)$ with non-universal gauginos. In the MSSM we argued from naturalness that non-universal soft mass parameters are preferred, with non-universal gaugino masses enabling supernatural dark matter beyond the MSSM, we also discussed neutralino dark matter in the USSM and E_6SSM. In the E_6SSM a light neutralino LSP coming from the inert Higgsino and singlino sector is unavoidable and makes an attractive dark matter candidate.

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