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Bound states of UED level-1 KK quarks at the Linear Collider

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Summary. — We study the formation and detection at the next linear e^+e^- collider of bound states of level-1 quark Kaluza-Klein excitations $\mathcal{B}_{KK}$ within a scenario of universal extra-dimensions (UED).

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1. – Introduction

It is well known that as early as 1921 Theodore Kaluza proposed a theory that was intended to unify gravity and electromagnetism by considering a space-time with one extra *space-like* dimension [1]. A few years later Oscar Klein proposed that the extra space dimension (the fifth dimension) is in reality compactified around a circle of very small radius [2]. These revolutionary ideas have thereafter been ignored for quite some time. However recent developments in the field of string theory have suggested again the possibility that the number of space-time dimensions is actually different from $D = 4$ (indeed string theory models require $D = 11$, *i.e.* seven additional dimensions). In 1990 it was realized [3] that string theory motivates scenarios in which the size of the extra dimensions could be as large as $R \approx 10^{-17}$ cm (corresponding roughly to electroweak energy scale ($\approx$ TeV)) contrary to naive expectations which relate them to a scale of the order of the Planck length $L_P \approx 10^{-33}$ cm (corresponding to the Planck mass $M_P = \sqrt{\hbar c/G} \approx 10^{19}$ GeV). See also [4].

Subsequently two approaches have been developed to discuss the observable effects of these, as yet, hypothetical extra dimensions. One possibility is to assume that the extra space-like dimensions are flat and compactified to a "small" radius. This is the so-called ADD model [5] where only the gravitational interaction is assumed to propagate in the extra-dimension. A second possibility is contemplated in the Randall-Sundrum type of models where the extra dimensions do have curvature and are embedded in a warped geometry [6, 7].

Universal extra-dimensional models were introduced in ref. [8] and are characterized, as opposed to the ADD model, by the fact that all particles of the Standard Model (SM) are allowed to propagate in the (flat) extra space dimensions, the so-called *bulk*. Here to each SM particle $X^{(0)}$ corresponds in this model a tower of Kaluza-Klein states $X^{(n)}$ (KK-excitations), whose masses are related to the size of the compact extra dimension introduced and the mass of the SM particle via the relation $m^2_{X^{(n)}} \approx m^2_{X^{(0)}} + n^2/R^2$. An important aspect of the UED model is that it provides a viable candidate to the Cold Dark Matter. This would be the lightest KK particle (LKP) which typically is the level 1 photon. Many aspects of the phenomenology of these KK excitations have been discussed in the literature. For reviews see refs. [9-12]. In particular KK production has been considered both at the Cern large hadron collider (LHC) and at the next linear collider (ILC). Direct searches of KK level excitations at collider experiments give a current bound on the scale of the extra-dimension of the order $R^{-1} \gtrsim 300\,\text{GeV}$. See, for example, ref. [13]. At the Fermilab Tevatron it will be possible to test compactification scales up to $R^{-1} \sim 500\,\text{GeV}$ at least within some particular scenario [14-16].

Lower bounds on the compactification radius arise also from analysis of electroweak precision measurements performed at the Z pole (LEP II). An important feature of this type of constraints is their dependence on the Higgs mass. A recent refined analysis [17] taking into account sub-leading contributions from the new physics as well as two-loop corrections to the standard model ρ parameter finds that $R^{-1} \gtrsim 600\,\text{GeV}$ for a light Higgs mass ($m_H = 115\,\text{GeV}$) and a top quark mass $m_t = 173\,\text{GeV}$ at 90% confidence level (CL). Only assuming a larger value of the Higgs mass the bound is considerably weakened down to $R^{-1} \gtrsim 300\,\text{GeV}$ for $m_H = 600\,\text{GeV}$, thus keeping the model within the reach of the Tevatron run II. The findings of this precision analysis are in qualitative agreement with previous results [18], but are at variance with the conclusions of a recent paper [19] where an analysis of LEP data including data from above the Z pole and two-loop electroweak corrections to the $\Delta\rho$ parameter pointed to $R^{-1} \gtrsim 800$ (at 95% CL).

It has been shown in ref. [20] that a refined analysis of $\bar{B} \to X_s\gamma$ including in addition to the leading-order contribution from the extra-dimensional KK states, the known next-to-next-to-leading order correction in the Standard model (SM) gives a lower bound on the compactification radius $R^{-1} \gtrsim 600\,\text{GeV}$ at 95% confidence level (CL) and independent of the Higgs mass.

We discuss here the formation, production and possible detection of bound states of Kaluza-Klein $n = 1$ excitations at e^+e^- collisions. To estimate the bound-state contribution to the threshold cross-section, an effect which can be as large as roughly a factor of three for strongly interacting particles, we use the method of the Green's function as opposed to previous works [21] which use a Breit-Wigner approximation. The interactions responsible for the formation of level-1 KK bound states is assumed to be an α_s-driven Coulomb potential. This allows the use of analytic expressions for the Green's function of the Coulomb problem. This method has also been recently used by the present authors in a study of *sleptonium* bound states within a slepton co-next to lightest supersymmetric particle (slepton co-NLSP) scenario of gauge mediated symmetry breaking (GMSB) [22].

2. – $u_1\overline{u}_1$ bound-state formation and production cross-section

In this section we shall review the possible creation of a bound state of the level-1 KK-excitation of the u-quark, *i.e.* a bound state $u_1\bar{u}_1$. The interaction among two Kaluza-Klein excitations are driven by the QCD interaction, thus bearing no differences with respect to the Standard Model; the strength of the interaction is given by α_s computed

TABLE I. – *Results of Coulombic model for the bound state of the level-1 iso-doublet U_1 quark. The strong coupling α_s is computed at the scale $Q = r_B^{-1}$, where $r_B = 3/(2m\alpha_s)$ is Bohr's radius. For each mass value m the scale $Q = r_B^{-1}$ depending itself on α_s must be solved numerically from the equation $Q = (2/3)\, m\, \alpha_s(Q)$.*

R^{-1} (GeV)	KK mass (GeV)	$\alpha_s(r_B^{-1})$	$M_{\mathcal{B}}$ (GeV)	E_{1S} (GeV)	$\Delta E(2P-1S)$ (GeV)
400	478.05	0.131	952.57	3.627	2.720
600	717.08	0.124	1429.30	4.903	3.677
800	956.11	0.120	1906.11	6.089	4.567
1000	1195.14	0.116	2382.98	7.214	5.411

at a suitable scale [23-25]. We shall adopt the same formation criterion stated there, namely that the formation occurs only if the level splitting depending upon the relevant interaction existing among constituent particles is larger than the natural width of the would-be bound state. This translates into the formation requirement

$$\Delta E_{2P-1S} \geq \Gamma, \tag{1}$$

where $\Delta E_{2P-1S} = E_{2P} - E_{1S}$ and Γ is the width of the would-be bound state. The latter is twice the width of the single KK quark, $\Gamma = 2\Gamma_{KK}$, as each KK quark could decay in a manner independent of the other.

In our model $V(\boldsymbol{x})$ is given by a Coulombic potential $V(r) = -4\alpha_s/(3r)$ with $r = |\boldsymbol{x}|$, and where α_s is the usual QCD coupling constant which has been taken at a suitable scale as described in [23, 24]. We are thus able to compute its energy levels given by the expression

$$\varepsilon_n = -\frac{4}{9}\frac{m{\alpha_s}^2}{n^2} \tag{2}$$

and the separation of the first two energy levels is given by $\Delta E_{2P-1S} = m\alpha_s^2/3$. The scale at which α_s is evaluated is given by the inverse of Bohr's radius $r_B = 3/(2m\alpha_s)$, the average distance of the constituents of the bound state. The mass of the n-th bound state is given by the expression $M_n = 2m + \varepsilon_n$ where m is the mass of the constituent u_1 quark and E_n is given by (2). The wave function at the origin, which will be needed in order to compute decay widths, for this particular model is given by the expression $|\psi(0)|^2 = (2m\alpha_s/3)^3/\pi$. The results are given in table I.

In order to determine whether the bound state will be formed we shall apply the criterion given in eq. (1). The KK-quark decay widths have been already computed in [21], where it has been shown that their values are at most of the order of 100 MeV, one order of magnitude less than the energy splittings. In this scenario eq. (1) requirement is always fulfilled, and the bound state is formed for KK-quark masses in this investigation range.

In order to describe the cross-section of a KK bound state in the threshold region we shall use the method of the Green's function. We briefly review here the essential features of the mechanism, and refer the reader to the literature for further details [26]. Let $\mathcal{G}_{1S}(\boldsymbol{x}, \boldsymbol{y}, E)$ be the Green's function of the Schrödinger equation which describes the bound state by means of a suitable potential $V(\boldsymbol{x})$.

The complete expression for the $1S$ Green's function of our problem as a function of energy from threshold is given with a slight change of notation by [27]

$$\mathcal{G}_{1S}(0,0,E+i\Gamma)=\frac{m}{4\pi}\left[-2\lambda\left(\frac{k}{2\lambda}+\log\left(\frac{k}{\mu}\right)+\psi(1-\nu)+2\gamma-1\right)\right], \tag{3}$$

where $k=\sqrt{-m(E+i\Gamma)}$, $\lambda=2\alpha_s m/3$ and the wave number is $\nu=\lambda/k$; $E=\sqrt{s}-2m$. With the position $E\to E+i\Gamma$ we take into account the finite width of the state. The ψ is the logarithmic derivative of Euler's Gamma-function $\Gamma(x)$, $\gamma\simeq 0.57721$ is Euler's constant and μ is an auxiliary parameter.

The final expression for the production cross-section of a KK bound state is thus given by

$$\sigma(m,E,\Gamma,\alpha_s)=\frac{18\pi}{m^2}\sigma_B\,\mathrm{Im}[\mathcal{G}_{1S}], \tag{4}$$

where σ_B is the Born expression of the cross-section [28]. The process $e^+e^-\to U_1\bar{U}_1$ proceeds through the annihilation into the standard model (level-0) gauge bosons γ and Z but in principle one should also consider the contribution of the level-2 gauge bosons $\gamma_{(2)}$ and $Z_{(2)}$. Especially so in our case of threshold production of the pair $u_1\bar{u}_1$. Indeed in this case $m\approx 1/R$, and $\sqrt{s}=2m+E\approx 2/R+E$ and since $m_{\gamma_2}\approx 2/R$ when producing at threshold the $u_1\bar{u}_1$ pair we would be close to the γ_2 and Z_2 resonances. However the mass spectrum is modified by the radiative corrections. We have verified that over the region of parameter space $300\,\mathrm{GeV}\le R^{-1}\le 1000\,\mathrm{GeV}$ and $2\le\Lambda R\le 70$ the pair production threshold $2m_{u_1}$ is always larger than m_{γ_2}, m_{Z_2} and thus these resonances should in principle be included in the calculation. We have also verified, cross checking our calculation with the output of a CalcHEP [29,30] session, that the numerical impact of these diagrams is completely negligible. Their contribution turns out to be five orders of magnitude smaller than that of the SM gauge bosons γ, Z. The analytic formula of the Born pair production cross-section $e^+e^-\to\gamma^*$, $Z^*\to U_1\bar{U}_1$ can be deduced for example from those of heavy quark ($t\bar{t}$) [31] taking into account the fact that the level-1 KK quarks are *vector-like*, *i.e.* their coupling to the Z is of the γ^μ type and has no axial component. Details of the calculation and the explicit expression of the cross-section are in [32].

In this work we shall concentrate on the continuum region of the cross-section, namely $E>0$. The region below threshold, $E<0$, has been already discussed in ref. [21], using a Breit-Wigner description of both the positions and the widths of the peaks. In this respect the Green's function approach is not expected to point to substantial differences relative to the Breit-Wigner one.

In fig. 1 we show the cross-section for selected values of the scale of the extra dimension, $R^{-1}=400$–$1000\,\mathrm{GeV}$. The results are less sensitive to the other parameter (ΛR) which only enters through the logarithmic factors in the radiative correction terms in the mass spectrum of the model [32]. In fig. 1 we have fixed $\Lambda R=20$ and varied R^{-1} computing the corresponding values of the level-1 KK quark mass, and assuming the energy of the collider being fixed at $\sqrt{s}=2m_{U_1}+E$, E being the energy offset from the threshold. We have used a value of $\Gamma=0.5\,\mathrm{GeV}$ for illustrative purpose, compatible with the formation of bound state. Different choices of Γ by even two orders of magnitude smaller will not make a visible difference on the figures.

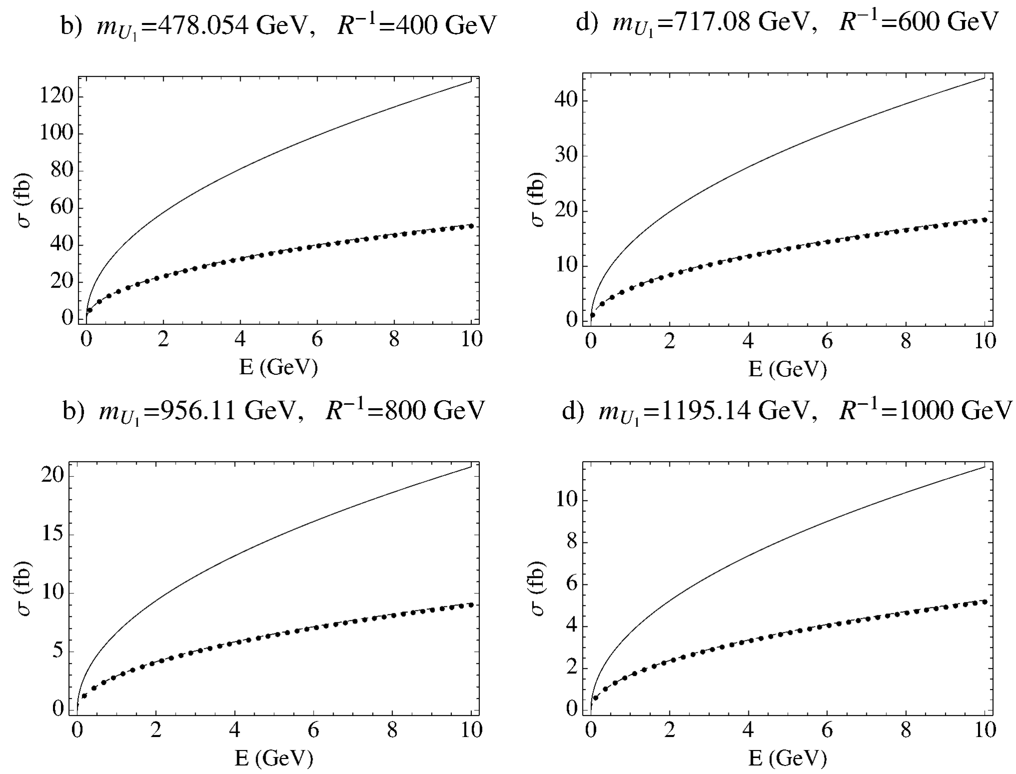

Fig. 1. – Production cross-sections of level-1 KK doublet quark bound states $U_1\bar{U}_1$ as a function of the energy offset from threshold ($\sqrt{s} = 2m_{U_1} + E$), for values of the scale of the extra-dimension R^{-1} in the range 400–1000 GeV and a total width of $\Gamma = 0.5$ GeV. The continuous line is the Green's function result, the dotted one is the Born approximation given by our analytical formula (see [32]). The full circles show the complete agreement with the Born cross-section from the CalcHEP [30] numerical session including also the annihilation diagrams of γ_2 and Z_2 whose contribution is however completely negligible. The cut-off scale Λ, at which perturbative expansions break down, has been fixed so that $\Lambda R = 20$.

3. – $u_1\overline{u}_1$ decay widths

The KK bound states we discuss here are the pseudoscalar ${}^1S_0$ and the vector one ${}^3S_1$. For the pseudoscalar state the decay channels are into two photons or two gluons for which the following Born level expressions hold (see for instance [24]):

$$(5)\qquad \Gamma_B({}^1S_0 \to \gamma\gamma) = q_i^4 \alpha^2 \frac{48\pi|\psi(0)|^2}{M^2} \qquad \text{and} \qquad \Gamma_B({}^1S_0 \to gg) = {\alpha_s}^2 \frac{32\pi|\psi(0)|^2}{3M^2}\,.$$

Here q_i is the charge of the constituent quark of the bound state, while M and $|\psi(0)|^2$ are given by the ones from Coulombic potential previously met.

The QCD radiative correction results [33], which are the same in the two decays, are shown in fig. 2 (left panel).

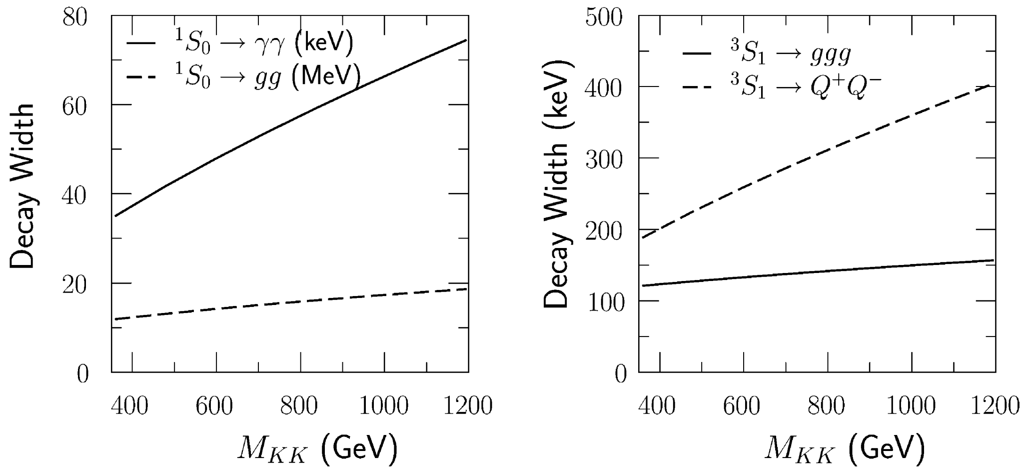

Fig. 2. – Left panel. Solid line: decay width (keV) of the pseudoscalar $u_1\bar{u}_1$ bound state to two photons as a function of KK excitation mass. Dashed line: decay width (MeV) of the pseudoscalar $u_1\bar{u}_1$ bound state to two gluons as a function of the KK mass. Right panel: decay widths of the $u_1\bar{u}_1$ vector bound state to two charged particles and into three gluons as a function of KK mass. Here we have considered all possible e.m. decay channels.

For the vector case $^3S_1$ the relevant decay channels are the one in charged pairs and the one into three gluons, for which one has

$$\text{(6)} \qquad \Gamma_B(^3S_1 \to q_f^+ q_f^-) = q_i^2 q_f^2 \alpha^2 \frac{16\pi|\psi(0)|^2}{M^2}$$
$$\text{and} \quad \Gamma_B(^3S_1 \to ggg) = \frac{(\pi^2-9)}{\pi}{\alpha_s}^3 \frac{160\pi|\psi(0)|^2}{81M^2}.$$

The charge of the final state charged particle is given by q_f. The QCD radiative corrections [33] depend as usual on α_s, that has to be computed at a scale of the order of $2m$. The two decays of the vector state are shown together in fig. 2 (right panel). We observe that only the pseudoscalar hadronic decay is in the MeV range and raises approximately linearly with KK mass. The $^1S_0$ photonic decay and $^3S_1$ decays are smaller by almost two orders of magnitude for the considered KK mass range. For the pseudoscalar case the hadronic is the dominant decay by far, while in the vector case the decay into charged particles, when taking into account all possible processes as seen in fig. 2 overtakes the hadronic decays.

Other electroweak decay channels are negligible. Those are proportional to α^2, thus their ratio to gluonic decays is suppressed by $(\alpha/\alpha_s)^2$, at least by two orders of magnitude.

For most scenarios depending upon the values of Λ and R [21] single quark decay becomes the dominant decay channel for the bound state.

Three-body decays are further suppressed with respect to previous formula by another power in coupling constant and phase-space reduction, resorting again in the keV range of energies.

From [21] one sees that in most cases single quark decays (SQD) are by far the most important decay channels of the bound state, to the order of hundreds of MeV, while as discussed above bound-state decays are essentially negligible. Moreover a comparison of those SQD widths with the results of table I through eq. (1) shows that for the considered mass range of KK there is formation of the bound state.

4. – Detection

As we have previously seen, for large R^{-1} values ($R^{-1} > 300\,\mathrm{GeV}$) SQD is the dominant decay channel for a KK bound state, thus leading to a dominant signature consisting of two monochromatic quarks plus missing energy. Following [26] we limit our analysis to the region above threshold, *i.e.* $E > 0$. The region below threshold, $E < 0$, is characterized by peaks in the cross-section for values of E equal to binding energies of the bound states. The widths of those peaks are given by the decay width of the bound state, which are at most of the order of the MeV for the SQD and much less, of the order of the keV, for other annihilation decay modes, as discussed in sect. **3**.

Because of ISR and beam energy spread, of the order of the GeV for a future linear collider, it is unclear whether it could be possible to resolve those peaks of keV magnitude with this machine. The only potentially detectable peaks should be the ones belonging to a SQD, provided one has a scenario with widths of the order of the MeV.

The situation above threshold changes drastically with respect to the "naive" Breit-Wigner estimate, as is clearly shown in fig. 1. A few GeV above threshold make for a factor of 3 of increase compared to the Born cross-section, allowing a clear distinction between the two cases. Assuming an annual integrated luminosity of $L_0 = 100\,\mathrm{fb}^{-1}$ and a scale of the extra-dimension $R^{-1} = 400\,\mathrm{GeV}$ one finds around 1.2×10^4 events per year of two quark decays for a center-of-mass energy of $10\,\mathrm{GeV}$ above threshold (we adopt here the scenario for which the branching ratio of SQD is essentially 1). The number of events per year loses an order of magnitude at $R^{-1} = 600\,\mathrm{GeV}$, that is about 4×10^3, as could be inferred from fig. 1.

For our $\overline{u}_1 u_1$ bound state there are two possible scenarios of decay pattern [34]. The first one concerns the iso-singlet u_{1_R} for which the decay channel into W_1 is forbidden while that into Z_1 is heavily suppressed $\mathcal{B}(u_{1_R} \to Z_1 u_{0_R}) \sim \sin^2\theta_1 \approx 10^{-2}$–$10^{-3}$ and the dominant channel is given by $u_{1_R} \to u_{0_R}\gamma_1$, with $\mathcal{B}(u_{1_R} \to u_{0_R}\gamma_1) \approx 0.98$ whose signature is a monochromatic quark and missing energy of the KK photon, the latter being the LKP [34].

For the iso-doublet u_{1_L} the situation is more interesting, as more channels are available [34], notably $u_{1_L} \to d_{0_L} W_1$ with $\mathcal{B}(u_{1_L} \to d_{0_L} W_1) \approx 0.65$ and $u_{1_L} \to u_{0_L} Z_1$, with $\mathcal{B}(u_{1_L} \to u_{0_L} Z_1) \approx 0.33$ while the branching ratio into γ_1 is negligible $\mathcal{B}(u_{1_L} \to u_{0_L}\gamma_1) \sim 0.02$. The decay chain into W_1 can follow the scheme: $u_{1_L} \to d_{0_L} W_1 \to d_{0_L}\ell_0\nu_{1_L} \to d_{0_L}\ell_0\nu_0\gamma_1$ with branching ratio given by

$$\mathcal{B}(u_{1_L} \to d_{0_L}\ell_0\nu_0\gamma_1) \approx \mathcal{B}(u_{1_L} \to d_{0_L} W_1)\mathcal{B}(W_1 \to l_0\nu_1)\,\mathcal{B}(\nu_1 \to \nu_0\gamma_1) \approx 0.65\,\frac{1}{6}\,1 \approx 10^{-1}$$

and alternatively, the same final state could be reached by the scheme: $u_{1_L} \to d_{0_L} W_1 \to d_{0_L}\ell_1\nu_{0_L} \to d_{0_L}\ell_0\nu_{0_L}\gamma_1$. As compared to the iso-singlet case, the result is a monochromatic quark, a *lepton* and missing energy in both cases.

The decay into the Z_1 channel is $u_{1_L} \to u_{0_L} Z_1 \to u_{0_L}\ell_0\ell_1 \to u_{0_L}\ell_0\ell_0\gamma_1$, resulting in a monochromatic quark, two leptons and missing energy. The branching ratio of the above chain is

$$\mathcal{B}(u_{1_L} \to u_{0_L}\ell_0\ell_0\gamma_1) \approx \mathcal{B}(u_{1_L} \to u_{0_L} Z_1)\,\mathcal{B}(Z_1 \to L_0 L_1)\,\mathcal{B}(L_1 \to \ell_0\gamma_1) \approx \frac{1}{3}\,\frac{1}{6}\,1 \approx 5 \times 10^{-2}.$$

These leptonic decays of u_1 have much cleaner signatures than the hadronic ones allowing, in principle, for a better detection of the signal.

In all cases we emphasize that the observable signal of the bound-state production at the linear collider would be similar to that of the Born pair production except for the absolute value of the cross-section.

The case of an iso-singlet bound state (or Born pair production of u_{1R}) would give rise to the signal $e^+e^- \to 2\text{jets} + \not{E}$ with cross-section

$$\sigma(e^+e^- \to 2\text{jets} + \not{E}) \approx \sigma_{\mathcal{B}_{KK}} \times [\mathcal{B}(u_{1R} \to u_0\gamma_1)]^2 . \tag{7}$$

We note that the $\sigma_{\mathcal{B}_{KK}}$ for the iso-singlet u_1 has to be computed *ex novo* and cannot be read from the values of fig. 1 since it refers to the iso-doublet U_1. The singlet and doublet have, when including radiative corrections, different masses and the corresponding pair production threshold is therefore different. See ref. [32] for details.

At an e^+e^- collider this signal has a standard model background from ZZ production with one Z decaying to neutrinos and the other decaying hadronically. Thus for the SM background we have

$$\sigma_{\text{SM}}(2\text{jets} + \not{E}) = \sigma_{ZZ}\,\text{fb} \times 0.7 \times 0.2. \tag{8}$$

In the case of an iso-doublet bound state (or Born pair production of U_1) the W_1 decay chain gives the signal: $e^+e^- \to 2\text{jets} + 2\ell + \not{E}$ with cross-section

$$\sigma(e^+e^- \to 2\text{j} + 2\ell + \not{E}) = \sigma_{\mathcal{B}_{KK}} [\mathcal{B}(u_{1L} \to d_{0_L}\ell_0\nu_0\gamma_1)]^2 , \tag{9}$$

while the Z_1 decay chain gives rise to the signature $e^+e^- \to 2\text{jets} + 4\ell + \not{E}$ with cross-sections

$$\sigma(e^+e^- \to 2\text{j} + 4\ell + \not{E}) = \sigma_{\mathcal{B}_{KK}} [\mathcal{B}(u_{1L} \to u_{0_L}\ell_0\ell_0\gamma_1)]^2 . \tag{10}$$

Triple gauge boson production, WWZ, ZZZ at a high energy linear collider has been studied in refs. [35, 36]. It has been found that these processes receive a substantial enhancement in the Higgs mass range $200\,\text{GeV} < m_H < 600\,\text{GeV}$ particularly the ZZZ channel. As these processes provide a source of standard model background for our signal we estimate them both at a value of $m_h = 120\,\text{GeV}$ and at a value of $m_h = 200\,\text{GeV}$ for which the cross-sections are enhanced. Production of WWZ can for instance give rise to the signature of $2\text{jets} + 2\ell + \not{E}$ via leptonic decay of the W gauge bosons and hadronic decay of the Z boson, while the ZZZ production can produce $2\text{jets} + 4\ell + \not{E}$ via hadronic decay of one Z while the others decay leptonically with one of them to a pair of τ which subsequently decay to $\ell\nu\bar{\nu}$ ($\ell = e, \mu$). Estimates of the resulting cross-sections are found using the CalcHEP [29] and CompHEP [37] software. We have verified agreement with previous results given in ref. [36]. We thus estimate within the standard model:

$$\begin{aligned} &\sigma_{\text{SM}}(2\text{j} + 2\ell + \not{E}) \approx \sigma_{WWZ} \times (0.1)^2 \times 0.7, \\ &\sigma_{\text{SM}}(2\text{j} + 4\ell + \not{E}) \approx \sigma_{ZZZ} \times (0.3)^2 \times 0.7 \times (0.17)^2. \end{aligned} \tag{11}$$

The $2\text{jets} + 2\ell + \not{E}$ channel could be potentially contaminated also from $t\bar{t}$ pair production cross-section which at such high energies is $\mathcal{O}(300)\,\text{fb}$ [38]. Assuming the top

TABLE II. – *The statistical significance SS as defined in the text corresponding to the annual integrated luminosity* $L_0 = 100\,\text{fb}^{-1}$ *for the three channels discussed in the text as a function of* R^{-1} *and* $\sqrt{s} = 2m_{U_1} + E$*, assuming an energy offset of* $E = 10\,\text{GeV}$ *from the threshold. In the second column we give the values of the* u_1 *iso-singlet level-1 KK quarks whose masses are different form those of the corresponding* U_1 *state from fig. 1. For the two multilepton channels SS has been computed for two values of the Higgs mass* $m_h = 120\,(200)\,\text{GeV}$ $(\Lambda R = 20)$.

R^{-1} (GeV)	m_{u_1} (GeV) (iso-singlet)	2 jets + $\not{E}$	2 jets + 2 ℓ + $\not{E}$ $m_h = 120$ (200) GeV	2 jets + 4 ℓ + $\not{E}$ $m_h = 120$ (200) GeV
400	469.0	81.1	8.2 (8.1)	5.6 (5.6)
600	703.5	44.4	4.1 (4.1)	3.3 (3.3)
800	938.0	29.0	2.4 (2.4)	2.3 (2.3)
1000	1172.5	20.8	1.6 (1.6)	1.7 (1.7)

quarks to decay with probability one to Wb and then the W gauge boson decay via the leptonic mode (with $\mathcal{B}(W \to \ell\nu_\ell) \approx 0.1$) would mimic the signal with a cross-section $\sigma_{\text{SM}}(2\text{jets} + 2\ell + \not{E}) \approx 3\,\text{fb}$. However in this case we expect b-tagging of the hadronic jets. Assuming an efficiency in b-tagging of 60% we would get a contribution of 1.2 fb to the 2jets + 2ℓ + $\not{E}$ cross-section which has to be added to that in eq. (11). This has been done in the calculation of the statistical significance of table II.

We conclude providing an estimate of the statistical significance $SS = N_\text{s}/\sqrt{N_\text{s} + N_\text{b}}$, of the three signals discussed above as related to an integrated luminosity of $L_0 = 100\,\text{fb}^{-1}$ (N_s is the number of signal events and N_b is the number of background events). These estimates are given in table II. Albeit quite encouraging (especially so the SS of the 2jets + $\not{E}$) we should bear in mind that the actual observation of these signals might be not be so easy from the experimental point of view. Indeed it is quite likely that in a framework of a quasi-degenerate KK mass spectrum the jets will be typically quite soft and therefore difficult to detect. It is therefore customary to concentrate on the much cleaner multilepton signatures [34, 39]. An analysis similar to the one given here, but with a perspective on signals arising at the Compact Linear Collider (CLIC), regarding the (Born) pair-production of level-1 KK-*leptons* and level-1 KK-*quarks* is given in [39].

5. – Conclusions

We have considered the formation and decay of a bound state of level-1 quark Kaluza-Klein excitation in UED and its consequent detection at a linear e^+e^- collider. Since m_{KK} should be larger than at least 300 GeV we have used a model with a Coulombic potential. Being a bound state we have used the Green's function technique for the evaluation of its formation cross-section in the threshold region, which is more appropriate than the standard Breit-Wigner picture as it takes into account the binding energy and the peaks of the higher level excitations that coalesce towards the threshold point. The net effect is a dramatic increase of the cross-section in the continuum region right of the threshold. This multiplicative factor is roughly 2.6 for $R^{-1} = 400\,\text{GeV}$ and drops down to 2.2 at $R^{-1} = 1000\,\text{GeV}$. The Green's function cross-section would allow more than $\approx 10^4$ events per year even at $R^{-1} = 400\,\text{GeV}$ ($m_{U_1} \approx 478\,\text{GeV}$) for a suitable integrated luminosity of the e^+e^- linear collider ($L_0 = 100\,\text{fb}^{-1}$). The number of events at $R^{-1} = 1000\,\text{GeV}$ ($m_{U_1} \approx 1200\,\text{GeV}$) would still be $\approx 10^3$ at the same integrated luminosity.

The large difference among the two descriptions of the cross-section should also possibly help in the determination of the correct model for such a heavy bound state outside the SM.

Our analysis of the backgrounds to the final states signals, though very simplified, indicates that the multi-lepton channels have a good statistical significance ($SS \gtrsim 2$) at least up to $R^{-1} = 600 \sim 700\,\mathrm{GeV}$, which certainly warrants further detailed and dedicated studies of these channels and their backgrounds. The potentially larger (by one order of magnitude) statistical significance of the $2j + \not{E}$ channel must be taken however with great caution because this signal may be difficult to observe as it is characterized by soft jets within the relatively degenerate mass spectrum of the extra-dimensional model. Further detailed studies are also needed for this channel.

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