IL NUOVO CIMENTO Vol. 33 C, N. 4 Luglio-Agosto 2010
DOI 10.1393/ncc/i2010-10650-9

Colloquia: TOP2010

Measuring the running top-quark mass

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(ricevuto il 14 Luglio 2010; approvato il 15 Luglio 2010; pubblicato online il 24 Settembre 2010)

Summary. — In this contribution we discuss conceptual issues of current mass measurements performed at the Tevatron. In addition we propose an alternative method which is theoretically much cleaner and to a large extent free from the problems encountered in current measurements. In detail we discuss the direct determination of the top-quark's running mass from the cross section measurements performed at the Tevatron.

PACS 14.65.Ha – Top quarks.

1. – Introduction

The top-quark is the heaviest known elementary particle discovered so far. It plays a prominent role in the physics program of the Tevatron accelerator at Fermilab and the Large Hadron Collider (LHC) at CERN (for recent reviews see, *e.g.* [1,2]). The interest in top-quark physics stems from the fact that owing to its large mass the top quark is a sensitive probe of the mechanism of electroweak symmetry breaking. This is also the reason why the top quark plays a special role in many extensions of the Standard Model (SM) aiming to give an alternative description of the mass generation. From the Standard Model viewpoint top-quark physics involves only the mass and the matrix elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix as free parameters in addition to the strong coupling constant which we assume to be precisely measured by other means. Assuming that V_{tb} is close to one—which is supported by indirect measurements based on the assumption that only three flavour families exist—top-quark properties are thus precisely calculable in the SM provided the top-quark mass is known with good accuracy. We also note that the large top-quark decay width $\Gamma_t \approx 1.5\,\text{GeV}$ (a further consequence of the large mass) effectively cuts off non-perturbative effects. As a consequence top-quark physics provides an ideal laboratory for precise tests of the SM and its extension at the scale of electroweak symmetry breaking. The top-quark mass—a very fundamental property of the top quark—is not only important for top-quark physics. It enters as

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a very important parameter in electroweak fits constraining the Standard Model, *i.e.* giving rise to indirect limits on the mass of the Higgs boson (see, *e.g.* [3]). Currently, a value of $m_t = 173.1^{+1.3}_{-1.3}\,\text{GeV}$ is quoted for the mass of the top-quark [4] (For an updated value presented during the Moriond EW session see [5,6].) This amounts to an experimental uncertainty of less than 1%. Since the top-quark's width is so large that the top quark typically decays before it can hadronise [7] the mass measurements proceed via kinematic reconstruction from the decay products and comparison to Monte Carlo simulations. However the reconstruction of the four momentum of the coloured top quark from its uncoloured decay products introduces an intrinsic uncertainty due to the non-perturbative mechanism of hadronisation in which the coloured partons are transformed to colourless hadrons. There is a further conceptual problem with the determination of the top-quark mass from the kinematic reconstruction. Strictly speaking a higher-order theoretical prediction of the observable under investigation is required to extract a parameter of a model in a meaningful way. Only beyond the Born approximation the renormalisation scheme can be fixed. Thus, there is no immediate interpretation of the quantity currently measured at the Tevatron in terms of a parameter of the SM Lagrangian in a specific renormalization scheme. A more detailed discussion will be given in sect. **3**. In order to address this issue, we have chosen the following approach. We start from the total cross section for hadronic top-quark pair production, *i.e.* a quantity with well-defined renormalisation scheme dependence which is known to sufficient accuracy in perturbative Quantum Chromodynamics (QCD). Its dependence on the top-quark mass is commonly given in the on-shell scheme, although it is well known that the concept of the pole mass has an intrinsic theoretical limitation leading, for instance, to a poorly behaved perturbative series. This typically implies a strong dependence of the extracted value for the top-quark mass on the order of perturbation theory. Similar effects have been observed in e^+e^- annihilation [8]. So-called short-distance masses offer a solution to this problem. As we compute the total cross section as a function of the top-quark mass in the modified minimal subtraction ($\overline{\text{MS}}$) scheme [9-11] we demonstrate stability of the perturbative expansion and good properties of apparent convergence [12]. In particular, this allows for the direct determination of the top-quark's running mass from Tevatron measurements for the total cross section [13], which is of importance for global analyses of electroweak precision data. The direct extraction of the running mass also provides an important cross check of the current measurements. The outline of this contribution is as follows. In sect. **2** we briefly comment on the theoretical status of the predictions for top-quark pair production. In sect. **3** we discuss in some details conceptual issues of current measurements and how they can be avoided measuring the top-quark mass in the $\overline{\text{MS}}$ scheme often called the running mass for its dependence on the renormalisation scale. The application is shown in sect. **4**. A short summary is given in sect. **5**.

2. – The total cross section for top-quark pair production

We start by recalling the relevant formulae for the total cross section $\sigma_{pp\to t\bar{t}X}$ of top-quark hadro-production within perturbative QCD,

$$\sigma_{pp\to t\bar{t}X}(S, {m_t}^2) = \sum_{i,j=q,\bar{q},g} \int_{4{m_t}^2}^{S} \mathrm{d}s\, L_{ij}(s, S, {\mu_f}^2)\; \hat{\sigma}_{ij}(s, {m_t}^2, {\mu_f}^2), \tag{1}$$

$$L_{ij}(s, S, {\mu_f}^2) = \frac{1}{S}\int_{s}^{S} \frac{\mathrm{d}\hat{s}}{\hat{s}} F_{i/p}\left(\frac{\hat{s}}{S}, {\mu_f}^2\right) F_{j/p}\left(\frac{s}{\hat{s}}, {\mu_f}^2\right), \tag{2}$$

where S denotes the hadronic center-of-mass energy squared and m_t the top-quark mass (taken to be the pole mass here). The standard definition for the parton luminosity L_{ij} convolutes the two parton distributions (PDFs) $F_{i/p}$ at the factorization scale μ_f. Note that due to the additional factor $1/S$ the fluxes at the Tevatron and the LHC can be directly compared. The partonic cross sections $\hat{\sigma}_{ij}$ parameterize the hard partonic scattering process after factorzation of initial state singularities. Factoring out a common mass scale squared $1/{m_t}^2$ the remaining part of the cross section (often called scaling functions) depends only on dimensionless ratios of m_t, μ_f and the partonic center-of-mass energy squared s.

The QCD radiative corrections for the total cross section in eq. (1) as an expansion in the strong coupling constant α_s are currently known completely at next-to-leading order (NLO) [14-17] and, as approximation, at next-to-next-to-leading order (NNLO) [18, 19]. The latter result is based on the known threshold corrections to the partonic cross section $\hat{\sigma}_{ij}$, *i.e.* the complete tower of Sudakov logarithms in $\beta = \sqrt{1-4{m_t}^2/s}$ and the two-loop Coulomb corrections, *i.e.* powers $1/\beta^k$ (see also [20] for some recent improvements). It also includes the complete dependence on μ_f and the renormalization scale μ_r, both being known from a renormalization group analysis. The presently available perturbative corrections through NNLO lead to accurate predictions for the total hadronic cross section of top-quark pairs with a small associated theoretical uncertainty [12, 18, 19] (see also, *e.g.* [21] for related theory improvements through threshold resummation). For further refinements studied recently we refer to [20, 22-24]. We stress that aiming for a precision of the theoretical predictions at the per cent level also electroweak contributions need to be taken into account. At the LHC these corrections can amount up to 1–2%, for details we refer to [25-27]. Very close to the threshold the attractive part of the QCD potential may lead to remnants of a would-be bound state [28, 29]. These corrections affect significantly differential distributions in the threshold region. A prominent example is the m_{tt}-distribution, the invariant mass distribution of the top-quark pair. Due to bound-state effects the differential cross section obtains also a contribution from kinematic regions below the nominal production threshold. If one could resolve this region experimentally, it would provide a sensitive method to measure the top-quark mass similar to what is proposed for a future e^+e^- linear collider. The correction of the total cross section due to this effect is of the order of 10 pb at the LHC. At the Tevatron where colour octet production dominates this effect is less important.

3. – The top-quark mass

We may start the discussion with a few general remarks. When talking about the mass of an elementary particle one should always keep in mind what is actually meant by this parameter. This is in particular important for states which—due to confinement—do not appear as asymptotic states in the full field theoretical description. Since no free quarks exist we have to treat the quark mass similar to any other parameter/coupling appearing in the underlying model. In principle there is no difference between the treatment of the coupling constant of the strong interaction α_s and the self-coupling of the quarks denoted by m_t. Note that we restrict ourselves to pure QCD and ignore the fact that the masses are generated by the Higgs mechanism. To measure a parameter of the Lagrangian we have to compare the measurements with the theoretical predictions depending on the unknown parameters of the theory. The theoretical prediction should be as precise as possible so that a good agreement between data and theory can be assumed provided the parameters are chosen ("fitted") appropriate. In particular one should use at least a next-to-leading order prediction. There is a second even more important argument why at least a next-to-leading order prescription is required: In leading order no precise definition of a parameter can be given. The difference between different definitions implemented by a specific renormalisation schemes is formally of higher order in perturbation theory and thus only shows up when we go beyond the Born approximation. To illustrate the point let us come back to the quark mass. Two common schemes are frequently used in perturbation theory. One is the on-shell or pole-mass scheme. The mass parameter in the pole-mass scheme is defined as the location of the pole of the propagator. Since self-energy corrections can shift the location the pole-mass definition has to be enforced order by order in perturbation theory through the renormalisation procedure. That is the renormalisation constants are fixed order by order such that no shift in the renormalised pole mass occurs. Another scheme is the so-called modified minimal subtraction scheme ($\overline{\mathrm{MS}}$). This scheme is defined by subtracting the ultraviolet singularities appearing in the unrenormalised theory order by order in a minimal way. That is just the divergence itself (together with some irrelevant constants in case of the modified MS) is absorbed into the redefinition of the bare quantities. Since different renormalisation schemes should be equivalent it must also be possible to convert from one scheme to another. This is indeed the case. The relation between the pole mass m_t and the $\overline{\mathrm{MS}}$ mass $\overline{m}(\mu_r)$ reads, for example,

$$m_t = \overline{m}(\mu_r)\left(1 + \frac{\alpha_s(\mu_r)}{\pi} d_1 + \left(\frac{\alpha_s(\mu_r)}{\pi}\right)^2 d_2 + \dots\right). \tag{3}$$

Treating $(n_f - 1)$ flavours massless and expressing the QCD coupling constant in the n_f-flavour theory through the coupling constant in the $(n_f - 1)$-flavour theory—that is using a scheme in which the running of the coupling constant is solely determined by the massless quarks—the constants d_1, d_2 read

$$d_1 = \frac{4}{3} + \ell, \tag{4}$$

$$d_2 = \frac{307}{32} + 2\zeta(2) + \frac{2}{3}\zeta(2)\ln 2 - \frac{1}{6}\zeta(3) + \frac{509}{72}\ell + \frac{47}{24}\ell^2 - \left(\frac{71}{144} + \frac{1}{3}\zeta(2) + \frac{13}{36}\ell + \frac{1}{12}\ell^2\right) n_f + \frac{4}{3}\sum_l \Delta(m_l/m_t), \tag{5}$$

with $\ell = \ln(\frac{\mu_r}{\overline{m}(\mu_r)})$. As mentioned before we observe in eq. (3) that the difference between the pole mass and the running mass is formally proportional to α_s. We note that like α_s the $\overline{\mathrm{MS}}$ mass depends on the renormalisation scale. Since the top-quark mass is essentially measured at the Tevatron from a kinematical fit the renormalisation scheme is not unambiguously fixed. It is believed that the measured value should be interpreted as the pole mass. However one should keep in mind that the reconstruction of the top-quark momenta from the observed hadron momenta introduces a further uncertainty due to colour reconnection which is expected to be of the order of Λ_{QCD}. This is supported by a recent study by Skands and Wicke where the influence of different models for non-perturbative physics has been investigated [30]. There is a further reason why the use of the pole mass should be avoided when we are aiming for high accuracy. Qualitatively it is clear that the full $\mathcal{S}$-matrix cannot have a pole at the location of the quark mass since this would mean that the quark appears as asymptotic state which is not the case due to confinement. A more formal approach relates this uncertainty to a certain class of higher-order corrections spoiling the convergence of the perturbative series [31, 32]. Technically the problem becomes manifest when one uses a Borel summation of the perturbative series. The back transformation of the Borel transform is ill defined due to the existence of a pole on the real axis. Taking the residue of the pole as an estimate of the theoretical uncertainty it is found that an ambiguity of the order of Λ_{QCD} is introduced. That is, the pole mass scheme has an intrinsic uncertainty of the order of Λ_{QCD} [32]: It is thus conceptually impossible to measure the pole mass with an accuracy better than Λ_{QCD}.

Taken the last statements into account, a theoretical clean approach to measure the top-quark mass is to choose a specific observable, calculate the higher-order corrections choosing a well-defined renormalisation scheme like for example the running mass and then to compare with the measurements. This idea has been pursued in [12]. As observable the inclusive cross section has been used. In the next section we will comment on the details of this approach.

4. – The cross section using the $\overline{\mathrm{MS}}$ mass

As outlined in the previous section, the main idea to circumvent the aforementioned problems of the current experimental determination of the top quark mass is to choose a sensitive observable translated to the $\overline{\mathrm{MS}}$ scheme as far as the mass parameter is concerned. The mass value is then obtained from a direct comparison with experimental data. In [12] the results for the total cross section [18] were translated to the $\overline{\mathrm{MS}}$ scheme using eq. (3) and eq. (4). The translation is first done at a fixed renormalisation scale for three different factorisation scales. The full renormalisation scale dependence is recovered from a renormalisation group analysis. In fig. 1 the cross section is shown for three different choices of the factorisation scales $\mu_f = 0.5m, m, 2m$ as a function of the renormalisation scale μ_r. The left plot shows the cross section using a pole mass of 173 GeV. The right plot employs the running mass definition with a mass value $m(m) = 163\,\mathrm{GeV}$. The bands at the left side of the two plots show an estimate of what one may call a theoretical uncertainty. They are obtained by varying the relative scales μ_r/m and μ_f/m between 0.5 and 2. We note that there is typically a crossing of the different curves for a given order. In particular the central scale is not necessarily between the two extreme scales. This behaviour appears when the central scale corresponds to a plateau. If one studies the uncertainty bands two important features can be observed. Compared to the pole mass scheme the cross section prediction using the $\overline{\mathrm{MS}}$ mass is much more stable. The NLO band overlaps with the NNLO band, in fact the NNLO band is fully included in the NLO band. Furthermore the size of the bands is reduced compared to the predictions using the pole mass. The perturbative prediction becomes thus much more stable with respect to radiative corrections. Using the cross section to determine the mass parameter this leads to a much more stable determination in the running mass scheme compared to a determination in the pole mass scheme. In fig.2 the cross section is shown as a function of the $\overline{\mathrm{MS}}$ mass evaluated at $\mu_r = m$. The wide band is the NLO prediction while the narrow band is an approximation to the full NNLO result. The uncertainty bands are again due to a variation of the scales. The data point shown on the left is the recent Tevatron measurement [13] for the cross section:

$$\sigma = 8.18^{+0.98}_{-0.87}\,\mathrm{pb}. \tag{6}$$

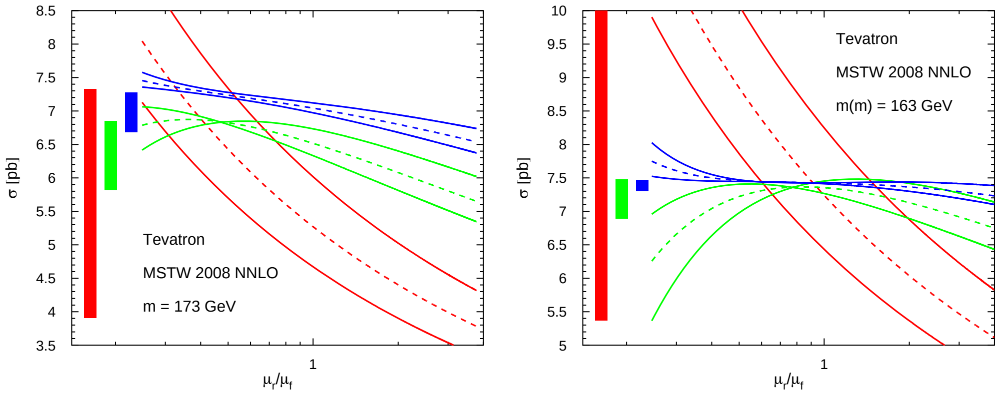

Fig. 1. – Cross section predictions using the pole mass (left) and the $\overline{\text{MS}}$ mass right as a function of the renormalisation scale for three different factorisations scales $\mu_f = 0.5m, m, 2m$.

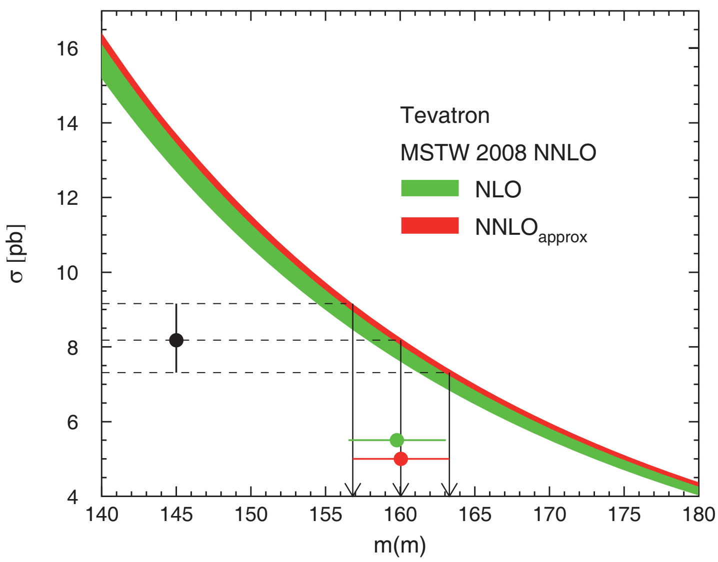

Fig. 2. – Cross section predictions using the $\overline{\text{MS}}$ mass as a function of the top quark mass.

Table I. – *The LO, NLO and approximate NNLO results for the top-quark mass in the* $\overline{\text{MS}}$ *scheme* $(m(m))$ *and the pole mass scheme* (m_t) *for the cross section measured at Tevatron.*

	$m(m)$ (GeV/c^2)	m_t (GeV/c^2)
LO	$159.2^{+3.5}_{-3.4}$	$159.2^{+3.5}_{-3.4}$
NLO	$159.8^{+3.3}_{-3.3}$	$165.8^{+3.5}_{-3.5}$
NNLO	$160.0^{+3.3}_{-3.2}$	$168.2^{+3.6}_{-3.5}$

We note that this measurement effectively depends on an assumed top-quark mass since detector efficiencies and other systematics are estimated from Monte Carlo simulations using a specific mass. In principle this dependence is known and can be taken into account. The dependence is however rather mild and thus does not give a significant shift in the cross section. In the current analysis it is not taken into account. The extraction of the top-quark mass in the $\overline{\mathrm{MS}}$ mass is now straightforward. Projecting the measured value on the curves we can immediately read off the corresponding mass value. An illustration of this procedure is visualized in fig. 2. The outcome of this procedure is presented in table I. For comparison we also show the results for the case that the pole mass is used. We observe that the extraction in the $\overline{\mathrm{MS}}$ scheme leads—as anticipated already—to very stable results with respect to different orders of the perturbative prediction. The determination using the pole mass scheme however shows large differences when going from LO to NLO and finally to NNLO. As final result the value corresponding to the NNLO approximation is quoted:

$$m(m) = 160^{+3.3}_{-3.2}\,\mathrm{GeV}/c^2. \qquad (7)$$

Converting the running mass to the on-shell mass yields a result which is consistent with the direct measurements at Tevatron. Due to the weak sensitivity of the cross section with respect to the mass the method is not competitive with the direct measurements as far as the uncertainty is concerned, however the method provides an independent cross check and is theoretically rather clean.

5. – Conclusions

The current top-quark mass measurements at the Tevatron claiming an accuracy at the per cent level suffer from various uncertainties (for a similar discussion see also [33]):

1) The renormalisation scheme is not uniquely defined since the measurement is based on a kinematic reconstruction without relying on higher-order predictions required to define unambiguously a specific renormalisation scheme.

2) The kinematic reconstruction of the top-quark momentum from the momenta of the decay products introduces an additional uncertainty due to the non-perturbative aspects of colour reconnection. The naive estimate that the uncertainty is of the order of Λ_{QCD} is supported by phenomenological studies [30] where the uncertainty was estimated to be of the order of 500 MeV.

3) The pole mass itself has an intrinsic uncertainty of the order of Λ_{QCD} which is usually attributed to IR renormalons.

One should note that each of the problems itself is hard to improve if not impossible. The intrinsic uncertainty of the pole mass for example cannot be improved. As a consequence we advocate an alternative method to determine the top-quark mass which is to a large extent free from the aforementioned problems. The basic idea is to extract the mass—as it is done in general for any parameter in a theoretical model—from a detailed comparison of the value of an experimentally measured observable with the theoretical predictions therefore. This leads to a clean definition of the renormalisation scheme adopted for the mass parameter. Using in addition a short-distance mass like the $\overline{\mathrm{MS}}$ mass the intrinsic uncertainties of the pole mass are circumvented. Along these lines we have used the total cross section written in terms of the $\overline{\text{MS}}$ mass to extract the top-quark mass from the cross section measurements at Tevatron. Our final result for the top-quark mass $m(m)$ in the $\overline{\text{MS}}$ scheme derived from the cross section measurements at the Tevatron is presented (eq. (7)). We find a remarkable stability with respect to the perturbative order of the theoretical predictions. Converted to the pole mass scheme the value is consistent with direct measurements. However we stress that despite the large uncertainty due the poor sensitivity of the total cross section with respect to the mass the result is theoretically rather clean and in particular free of uncertainties which are not quantified in the direct measurements.

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