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Top production at the LHC: The impact of PDF uncertainties and correlations

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Summary. — In this contribution we discuss the impact of PDF uncertainties on $t\bar{t}$ and t-channel single-top production at the LHC. We present predictions for total cross-sections computed at NLO accuracy with different PDF sets. For single-top production, we point out that the uncertainty arising from the choice of the bottom quark mass is one of the dominant theoretical uncertainties on the total cross-section. Finally, the possibility of using PDF-induced correlations between top quark and electroweak vector boson production cross-sections to improve the accuracy of LHC measurements is investigated.

PACS 12.38.-t – Quantum chromodynamics.
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1. – Introduction

A precise determination of Parton Distribution Functions (PDFs) with reliable estimate of their uncertainties is crucial for the success of the physics program at the LHC experiments. On the one hand, PDF uncertainties are often the dominant theoretical uncertainties for many relevant signal and background processes [1]. On the other hand, overestimated PDF errors might hinder the discovery of new physics effects, as shown for example in [2]. Top physics at the LHC is no exception and both top pair and single-top production present complementary and interesting properties as far as PDF determination/effects are concerned. This contribution aims to review some of the implications of PDFs for top quark physics at the LHC.

In the first part of the contribution we summarize the present status of the predictions for $t\bar{t}$, t- and s-channel single-top cross-sections, computed using different PDF sets. We show how differences in the predictions can directly be traced to both differences in the parton luminosities, and the values of physical parameters used in the PDF analyses, such as the strong coupling constant α_s or the the b-quark mass, m_b. In particular, we

TABLE I. – *Top pair cross-section at NLO with different PDF sets at LHC* 7 TeV.

CTEQ6.6 [8]	147.7 ± 6.4 pb
MSTW2008 [9]	159.0 ± 4.7 pb
NNPDF2.0 [6]	160.0 ± 5.9 pb
ABKM09 [10]	131.9 ± 4.8 pb
HERAPDF1.0 [11]	136.4 ± 4.7 pb

highlight the importance to account for the uncertainty on the b-quark mass for accurate predictions of single-top production at the LHC.

In the second part we study PDF-induced correlations between PDFs and $t\bar{t}$/single-top cross-sections and between top and $W^{\pm}/Z^0$ cross-sections. We briefly discuss how these correlations could be used in order to improve the accuracy of top cross-section measurements with early data at the LHC. These correlation studies are performed within the framework of the NNPDF parton analysis [3-6] which, by relying on Monte Carlo techniques for the estimation of uncertainties, provides an ideal tool for such statistical studies.

The baseline PDF set for the studies presented in this contribution is the recently released NNPDF2.0 [6], the first NLO global fit using the NNPDF methodology.

2. – Top-quark production at the LHC

2˙1. *$t\bar{t}$ production.* – Top pair production is the main channel for top quark production at Tevatron and LHC. In table I we collect the predictions for the top pair cross-section at LHC 7 TeV at NLO computed with the MCFM code [7] using different PDF sets.

We notice that the predictions from the three global fits, NNPDF2.0, CTEQ6.6 and MSTW08 agree at the 1-sigma level. The differences with PDF sets based on reduced datasets, ABKM09 and HERAPDF1.0, are larger. We note that top pair production depends strongly on the large-x gluon, and thus using sets which do not include Tevatron jet data might lead to rather different predictions for this observable. One should notice that differences between the predictions from different PDF sets also arise from the use of different values for the strong coupling constant α_s. It has been shown that using a common value of α_s brings predictions from different groups for various LHC observables, including top pair production, in better agreement [12, 13].

Once we subtract the difference introduced by different choices for α_s, the remaining differences can be directly traced to differences in the PDF luminosities at the typical scale of the process. This is illustrated in fig. 1, where the gluon-gluon luminosity for LHC at 7 TeV is plotted for the CTEQ6.6, MSTW2008 and NNPDF2.0 NLO sets. For example, the lower value for the cross-section obtained using the CTEQ6.6 set reflects the smaller gg luminosity as compared to the other sets at $Q^2 = m_t^2$.

2˙2. *Single-top production.* – Next we present predictions for single-top production at the LHC at 7 TeV. These predictions for various PDF sets are collected in table II, where we present results for both the t- and s-channel single-top cross-sections computed at NLO in QCD with the MCFM code. We have used the $N_f = 5$ (massless) calculation for the results presented here, recently the $N_f = 4$ calculation, which properly takes into account the effects due to finite b-quark mass, also became available [14].

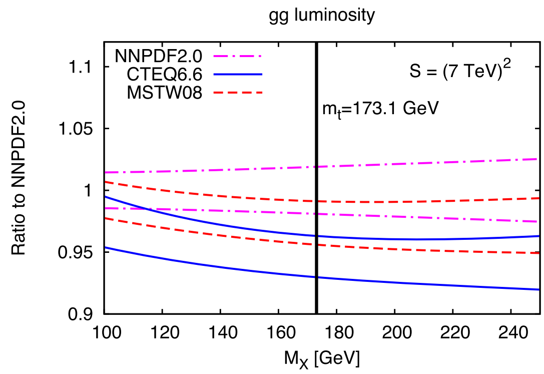

Fig. 1. – Gluon-gluon parton luminosities for CTEQ6.6, MSTW2008 and NNPDF2.0 including the associated PDF uncertainties. Results are showed as ratios to NNPDF2.0.

While at the Tevatron the contributions of t- and s-channel W exchange to single-top production are comparable in size, at the LHC t-channel production is by far the dominant production mechanism.

From the point of view of testing the predictions from different PDF sets t-channel single-top is also very interesting due to the fact that, in the so-called 5-flavour scheme (*i.e.* a scheme where the b is assumed to be a parton in the proton) the cross-section at LO probes directly the b-quark PDF, which in turn is closely related to the gluon distribution from which it is generated radiatively.

From table II one notices that the central predictions from the various PDF sets can differ by several times the quoted 1-sigma PDF uncertainty([1]). There are different contributions to this discrepancy. The first stems from the different values of the strong coupling constant α_s which are used by different parton sets. Since single-top production is mediated by electroweak gauge bosons, α_s enters only in radiative corrections, unlike the case of $t\bar{t}$ production discussed above, this effect is rather small.

In order to separate the differences in the single-top production cross-section which arise from the differences in the PDFs themselves and those from other physical parameters (like m_b or α_s) which also enter in the PDF analyses and in the computation of the partonic matrix element, we plot in fig. 2 the b-gluon parton luminosity, which determines the LO cross-section, for the CTEQ6.6, MSTW2008 and NNPDF2.0 NLO sets. It is clear that parton luminosities in the kinematic region relevant for single-top production differ by an amount much smaller than the cross-sections themselves, suggesting that the differences indeed come from variations of other physical parameters which enter the PDF analysis.

Indeed, it can be seen that the bulk of this difference is related to the different values of the b-quark mass used in the fits by the different collaborations. The NNPDF Collaboration sets $m_b = 4.3\,\text{GeV}$, CTEQ uses $m_b = 4.5\,\text{GeV}$ while the MSTW08 fit is performed setting $m_b = 4.75\,\text{GeV}$. In order to substantiate our claim that the different values used the b-quark mass explain the bulk of the difference for the t-channel single-top

([1]) Note that for the ABKM09 prediction the uncertainty includes the associated α_s, m_c and m_b uncertainties, which cannot be disentangled from the PDF uncertainties, and is thus much larger than the one obtained using other PDF sets.

TABLE II. – *Single-top cross-section at NLO with different PDF sets at LHC* 7 TeV.

	t-channel	s-channel
CTEQ6.6 [8]	40.85 ± 0.50 pb	2.33 ± 0.05 pb
MSTW2008 [9]	41.96 ± 0.26 pb	2.38 ± 0.04 pb
NNPDF2.0 [6]	44.33 ± 0.32 pb	2.38 ± 0.06 pb
ABKM09 [10]	43.17 ± 1.98 pb	2.40 ± 0.03 pb
HERAPDF1.0 [11]	40.04 ± 0.33 pb	2.38 ± 0.05 pb

cross-section, we produced two NNPDF2.0 sets with $m_b = 3.7$ GeV and $m_b = 5.0$ GeV, respectively. The results for the b-gluon parton luminosities for these modified sets are shown in the right plot in fig. 2 and the corresponding cross-sections for the t-channel single-top cross-section are collected in table III. It is clear that the value of m_b is anti-correlated with the bg luminosity and the t-channel single-top cross-section.

From table III one sees that variations of the b-quark mass of the order or $\delta m_b \sim 0.7$ GeV induce an uncertainty in the cross-section of $\delta\sigma \sim 3$ pb. If we take the PDG average as the best available determination of the b-quark mass and convert it from the $\overline{\text{MS}}$ scheme to the pole mass scheme we obtain an uncertainty of approximately $\delta m_b(\text{PDG}) \sim 0.2$ GeV. Rescaling errors, we are still left with an uncertainty in the t-channel single-top cross-section of $\delta\sigma \sim 0.8$ pb, still larger than the typical nominal PDF uncertainties quoted in table II. It is also clear from table III and fig. 2 that using a similar value of m_b would bring the predictions from different PDF sets into much better agreement. The uncertainty due to m_b should thus always be accounted for in the theoretical predictions for LHC single-top production.

3. – PDF-induced correlations

It is well known (see for example the discussion in ref. [8, 4]) that parton densities induce correlations among different observables measured at hadron colliders. These can be the PDFs themselves, one PDF and a physical observable or two physical observables.

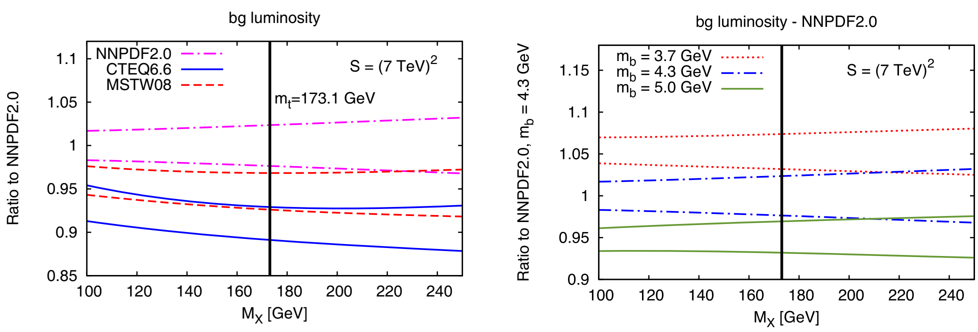

Fig. 2. – (Left) b-gluon parton luminosities for CTEQ6.6, MSTW2008 and NNPDF2.0, normalized to the NNPDF2.0 value. (Right) b-gluon parton luminosities for NNPDF2.0 fits with different values of the b-quark mass normalized to the standard NNPDF2.0.

TABLE III. – *t-channel single-top cross-section at NLO computed using NNPDF2.0 sets with different values of the b-quark mass.*

NNPDF2.0 ($m_b = 3.7\,\text{GeV}$)	$46.77 \pm 0.36\,\text{pb}$
NNPDF2.0 ($m_b = 4.3\,\text{GeV}$)	$44.33 \pm 0.32\,\text{pb}$
NNPDF2.0 ($m_b = 5.0\,\text{GeV}$)	$41.04 \pm 0.32\,\text{pb}$

The latter case is especially important from the experimental point of view, since it allows to define measurement strategies in which the PDF uncertainties between two observables cancel, for example in the case in which this correlation between the two observables is maximal.

In the case of a PDF set based on the Monte Carlo method, like NNPDF, the correlation coefficient $\rho[A, B]$ for two observables A and B which depend on PDFs is given by the standard expression for the correlation of two stochastic variables [4, 12]

$$\rho[A, B] = \frac{\langle AB\rangle_{\text{rep}} - \langle A\rangle_{\text{rep}}\langle B\rangle_{\text{rep}}}{\sigma_A \sigma_B}\,, \tag{1}$$

where the averages are taken over the ensemble of the N_{rep} values of the observables computed with the different replicas of the PDF set, and $\sigma_{A,B}$ are the standard deviations for the observables as computed from the MC ensemble. The value of ρ characterizes whether two observables are correlated ($\rho \approx 1$), anti-correlated ($\rho \approx -1$) or uncorrelated ($\rho \approx 0$). In the following we present results for the NNPDF2.0 set, the LHC cross-sections have been obtained as before using the MCFM code.

As a first example, we compute the correlation between the $t\bar{t}$ and t-channel single-top cross-section at the LHC (7 TeV) and different PDFs at the factorization scale $\mu_f = m_{\text{top}}$ as a function of x. The results are plotted in fig. 3. The most remarkable features are that the $t\bar{t}$ cross-section at the LHC at 7 TeV is mostly correlated to the gluon distribution at $x \sim 0.1$ and anti-correlated with it at small-x, with the same behaviour present for sea quark PDFs, generated radiatively from the gluon. We note also that the u- and d-quark distributions are anti-correlated with the cross-section at medium/large x.

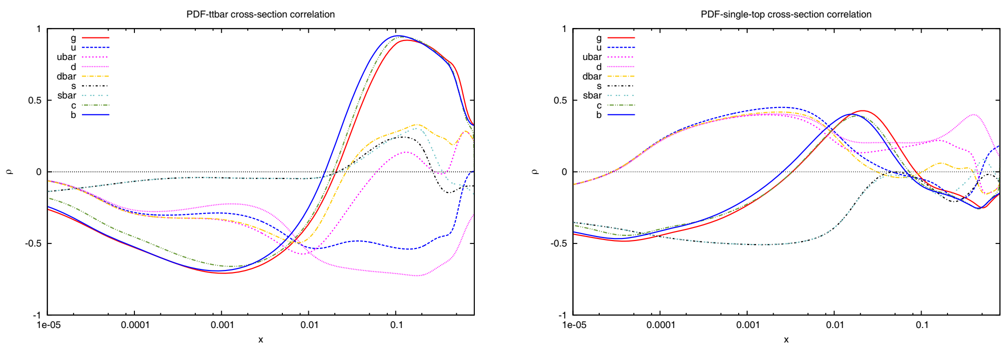

Fig. 3. – Correlation between parton densities and $t\bar{t}$ (left) and t-channel single-top (right) cross-sections at the LHC 7 TeV. The PDF set used is NNPDF2.0 and cross-sections have been computed with MCFM.

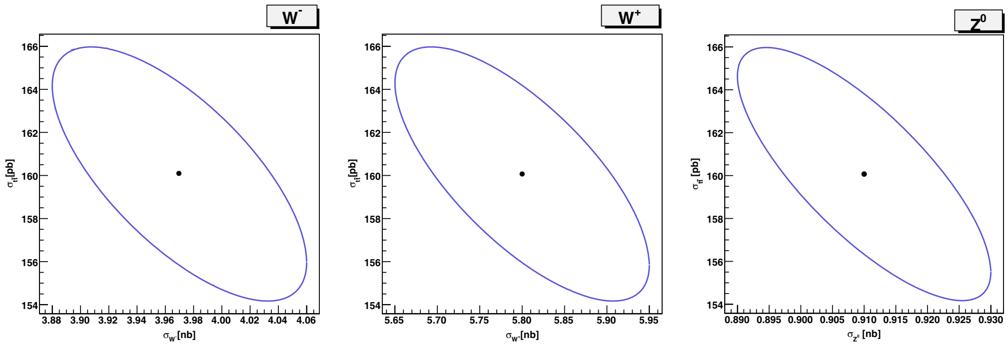

Fig. 4. – Correlation between $t\bar{t}$ and Electroweak Vector Boson cross sections at the LHC 7 TeV. The cross-sections have been computed with MCFM and the NNPDF2.0 parton set.

As for the case of the t-channel single-top cross-section, we point out that the strong correlation with the gluon (and therefore with the c- and b-quark distributions) present for $t\bar{t}$ is now milder and peaked at medium x, $x \sim 0.01$, and now we find a moderate correlation at medium/small x with the u and d PDFs, of the opposite sign as in the $t\bar{t}$ cross-section. We would like to stress as well the correlation of single-top cross-section and the s-,$\bar{s}$-quark PDFs at medium/small x, which is notably absent in the $t\bar{t}$ case.

As previously pointed out, the correlation coefficient eq. (1) can also be computed between two cross-sections, which is potentially relevant since in the case of a sizable correlation the measurement of one of these observables would provide useful information on the value of the other one. In this respect we have computed the correlation between the $t\bar{t}$ and t-channel single-top cross-sections on one side and $W^\pm$ or Z^0 cross-sections at the LHC on the other. The values for the correlation coefficients for the different pairs of observables are collected in table IV and the correlation ellipses are plotted in fig. 4 for the $t\bar{t}$ cross-section and in fig. 5 for t-channel single-top.

Both the values of ρ and the shape of the correlation ellipses show a significant anti-correlation between the $t\bar{t}$ cross-section and the $W^\pm$ and Z^0 ones. Given the fact that the vector boson cross-sections at the LHC are $\mathcal{O}(10)$ times larger than the $t\bar{t}$ cross-section and more accurately known from the theoretical point of view, it is foreseeable to use those in order to better calibrate the top pair cross-section measurement in early data.

On the other hand, the single-top cross-section shows a very mild correlation to the vector boson one thus rendering a similar approach based on the precision measurement of EW bosons difficult. However, if one is able to identify other observables which should be measured with a similar precision and that are correlated to the single-top cross-section, the discussion of the $t\bar{t}$ case would also apply here.

Table IV. – *Correlation coefficients between $t\bar{t}$ or t-channel single-top and $W^\pm$ or Z^0 cross-sections at the LHC* 7 TeV *in the NNPDF2.0 analysis.*

ρ	σ_{W^+}	σ_{W^-}	σ_{Z^0}
$\sigma_{t\bar{t}}$	−0.716	−0.694	−0.773
σ_t	0.330	0.140	0.240

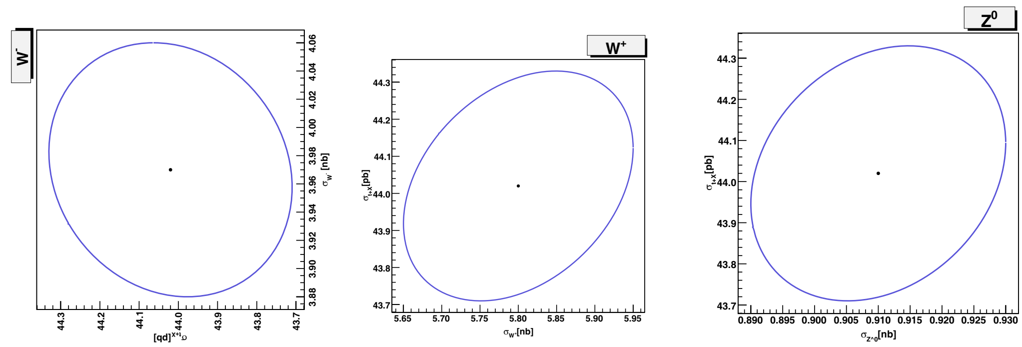

Fig. 5. – Correlation between t-channel single-top and Electroweak Vector Boson cross-sections at the LHC 7 TeV. The cross-sections have been computed with MCFM and the NNPDF2.0 parton set.

4. – Conclusions

The quality of top physics resulting from the LHC experiments will be affected, among other factors, by our knowledge of Parton Distribution Functions and their uncertainties. In this contribution we have reviewed the present status of predictions for $t\bar{t}$ and single-top cross-sections evaluated at NLO in QCD with different PDF sets and pointed out differences among them, trying to elucidate the reasons for these differences. For the case of single-top production, we have shown that one important source of difference among predictions obtained using different PDF sets is the value of the b-quark mass m_b used by the different collaborations.

In the second part we briefly discussed PDF-induced correlations between parton densities and top cross-sections and between the latter and electroweak vector boson production cross-sections at the LHC at 7 TeV. These correlations could be useful to define experimental strategies to measure the top quark cross-section in a way in which PDF uncertainties are reduced.

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