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New theoretical results for tW and tH production

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Summary. — We summarise recent work on the production of a single top quark in association with either a W or a charged Higgs boson. Interference issues between Wt production and top pair production are discussed in detail, and previous approaches to this problem are reviewed. We then present an implementation of Wt production in the MC@NLO framework, and use this to argue that Wt is a well-defined production process at the LHC. We then discuss Ht production in MC@NLO, and show example results regarding the properties of b and light jets.

PACS `14.65.Ha` – Top quarks.
PACS `12.38.Bx` – Perturbative calculations.

1. – Introduction

Single top physics is of great interest at the LHC, both as a potential window for new physics, and as a background. In this contribution, we first focus on the production of a single top quark in association with a W boson [1-5]. This is sensitive to corrections to the Wtb vertex, but not to possible effective four-fermion interaction terms. Thus, Wt production gives complementary information to the other single top modes.

Single top production in the s- and t- channel modes is well understood theoretically. The Wt mode, despite having been intensively studied, is less well understood. At LO, the Wt cross-section is well defined, and much smaller than the cross-section for production of top quark pairs. At NLO, possible real emission corrections to the Wt mode include the diagrams shown in fig. 1. These diagrams give a large contribution when the invariant mass m_{bW} of the $W\bar{b}$ pair in the final state approaches the top mass m_t, *i.e.* when the propagator for the intermediate anti-top goes on-shell. Indeed, in this limit one may view the diagrams of fig. 1 in a different way, namely resonant top pair production (at LO) with decay of the anti-top. In practice, this means that the total cross-section for Wt production receives a huge correction at NLO, due to resonant pair production.

There are two main viewpoints which exist in the literature on how to proceed. Firstly, there is what one may call the "combined approach", as exemplified by [6]. Here one

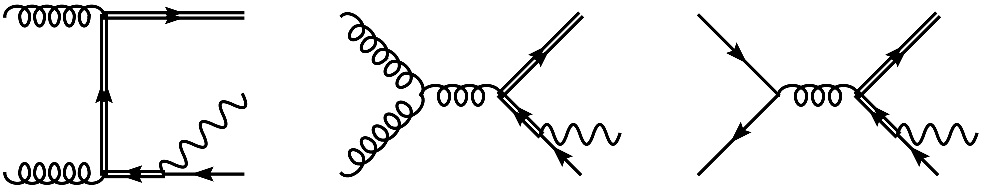

Fig. 1. – A subset of diagrams contributing to Wt production at next-to-leading order.

accepts that Wt production does not really exist, but is an accident of LO perturbation theory. Only WWb and $WWb\bar{b}$ final states are calculated, which is valid throughout all of phase space. However, this approach suffers from a significant theoretical deficiency at present in that NLO corrections to top pair production cannot be included (the relevant matrix elements have not yet been calculated). We know, however, that NLO corrections to resonant top pair production are large.

This has led to an alternative option, which we may call the "separated approach", *e.g.* [1-5, 7]. Here one considers Wt and $t\bar{t}$ as separate processes, subject to suitable analysis cuts. Namely, one may construct an efficient Monte Carlo event generator for Wt production, provided that one is away from the doubly resonant region. Such calculations include all diagrams in their domain of applicability, but are not valid in the doubly resonant region. Nevertheless, this approach allows one to add top pair and Wt production separately, thus including NLO corrections in both.

The right approach to use remains a subtle, delicate and somewhat controversial issue. Proponents of the separation approach essentially believe that NLO corrections to top pair production cannot be neglected, and in any case that one is entitled to construct a Monte Carlo generator whose validity is restricted to certain regions of phase space. Opponents, however, believe that only a description valid throughout all of phase space makes sense. We review previous separation approaches in the following section.

2. – Interference problem—previous approaches

The interference problem in Wt physics is not new, and has been dealt with by many previous calculations which go beyond LO, beginning with [2, 3]. There the total cross-section for Wt production was constructed by restricting the invariant mass of the $W\bar{b}$ pair via the requirement

$$|m_{bW} - m_t| > \eta\Gamma_t. \tag{1}$$

This indeed cuts out the contribution from the doubly resonant region, although it is difficult to straightforwardly implement this in an experimental setting, where it is not always possible to reconstruct the relevant decay products of the $\bar{t}$.

A different approach was taken in [4]. In that paper, the total Wt cross-section (including doubly resonant diagrams) was modified by a subtraction term of the form

$$\sigma_{\mathrm{subt}} = \sigma_{t\bar{t}}\, BR(\bar{t} \to Wb), \tag{2}$$

where $\sigma_{t\bar{t}}$ is the total cross-section for top pair production, which is multiplied by a branching ratio for the anti-top decay. This explicitly removes the contribution from resonant top pair production. A comparison was made with the approach of [2, 3], and it was found that the total cross-sections agreed if $\eta \simeq 15$ in eq. (1). One may argue whether or not this number is reasonable, however these results suggest that it is indeed possible to construct a definition of Wt production that is free of ambiguities outside the doubly resonant region.

A fully differential definition of Wt production was given in the NLO calculation of [5], and is implemented in the MCFM event generator. The calculation relies on a number of different ideas, including a veto on the transverse momentum of the second b quark (or $\bar{b}$) in an event, if such a quark is present. This is based on the fact that doubly resonant diagrams tend to have two b-flavoured quarks on average, where both are reasonably hard due to coming from a top decay. Also, $q\bar{q}$ initial states are removed, which is allowable outside the doubly resonant region as such subprocesses are numerically very small. Once these and other criteria are applied, a well-defined Wt cross-section is obtained, which works fully exclusively at the purely NLO level.

Having set the context for Wt calculations using a separation approach, we consider the extension of these ideas to a parton shower context in the next section.

3. – Wt production in MC@NLO

The state-of-the-art description for many scattering processes at hadron colliders is the combination of higher-order matrix elements with a parton shower, which estimates the effect of further QCD radiation from the incoming and outgoing partons, as well as modelling hadronisation effects. Ideally, one wishes to interface NLO matrix elements with a shower, and algorithms for doing this are by now well known [8,9]. The motivation for using a NLO matrix element is particularly strong for Wt production, as any LO Monte Carlo generator takes no account of complications due to interference with top pair production. In the previous section we saw a number of different approaches for recovering a well-defined meaning of Wt production at NLO. Whilst these work well within the context of a fixed-order calculation, the ideas are not immediately applicable in a parton shower context, and we discuss such a generalisation in this section.

Let us begin by noting the requirements that a MC@NLO generator for Wt production must satisfy:

1) It must be applicable when both initial and final state radiation are present.

2) It must be gauge invariant.

3) The definition of the Wt mode must be free of ambiguities away from the doubly resonant region.

The third requirement amounts to the statement that in phase space regions where one is allowed to use a separation approach, all diagrams contributing to Wt production (both singly and doubly resonant) should be present, in an unmodified form. Related to this, it is also helpful to have a means of checking the separation approximation, *i.e.* estimating the size of interference effects.

A definition of Wt production satisfying the above requirements was given in [7], and proceeds by modifying the Wt cross-section by a gauge-invariant subtraction term. Schematically, one writes

$$\mathrm{d}\sigma_{Wt} = \mathrm{d}\sigma - \mathrm{d}\sigma^{\mathrm{subt}}, \tag{3}$$

where the first term on the right-hand side is the sum of all singly and doubly resonant diagrams contributing to Wt production, and the second term is designed to remove the resonant top pair contribution. The above equation applies fully exclusively/locally in phase space, and a suitable choice must be made for the subtraction term consistent with the following requirements:

1) It must be gauge invariant.

2) It must be equal to the resonant top pair contribution when the invariant mass of the $W\bar{b}$ pair satisfies $m_{\bar{b}W} = m_t$.

3) It must fall off quickly in phase-space when the invariant mass of the $W\bar{b}$ pair moves away from the top mass, *i.e.* $|m_{\bar{b}W} - m_t| > m_t$.

The second requirement is just the definition of subtracting the top pair resonant contribution, and the third ensures that the resulting Wt calculation is free of ambiguities in its domain of applicability, *i.e.* that all diagrams are correctly included without modification in this region.

Naïvely, one may write a subtraction term based on the narrow width approximation for the anti-top that has the form

$$\mathrm{d}\sigma_{ab}^{\mathrm{subt}} = |\mathcal{A}(ab \to t\bar{t})|^2 f_{\mathrm{BW}}(m_{\bar{b}W})|\mathcal{A}(\bar{t} \to W\bar{b})|^2, \tag{4}$$

where a and b label initial state partons. The right-hand side consists of the amplitude for resonant top pair production, multiplied by a Breit-Wigner function and the amplitude for the decay of the anti-top. However, one wants the kinematics on the left-hand side to be applicable at any point in phase space, whereas the anti-top is restricted to be on-shell on the right-hand side. Furthermore, spin correlations are not included in the decay products of the anti-top, which are needed for local matching of the full amplitude and the subtraction term in the doubly resonant region.

Instead, one may write a subtraction term with the schematic form

$$\mathrm{d}\sigma_{ab}^{\mathrm{subt}} = |\tilde{\mathcal{A}}(ab \to tW\bar{b})_{t\bar{t}}|^2 \, \frac{f_{\mathrm{BW}}(m_{\bar{b}W})}{f_{\mathrm{BW}}(m_t)} \,. \tag{5}$$

Here the right-hand side contains the contribution from doubly resonant diagrams, but where the kinematics are reshuffled so as to place the anti-top on shell. Note that this is gauge invariant in the limit $m_{\bar{b}W} \to m_t$, as required. One then damps this as $m_{\bar{b}W}$ moves away from m_t with a ratio of Breit-Wigner factors. Equation (5) is only one of a number of possible solutions for the subtraction term, but it indeed satisfies all of the requirements outlined above. One may see this more directly by studying fig. 2, which shows the behaviour of the subtraction term as a function of the invariant mass of the $W\bar{b}$ pair. The amplitude at $m_{\bar{b}W} = m_t$ is equal to the resonant top pair contribution, and the subtraction term indeed dies away very quickly outside the double resonance region, as required.

The above prescription for the Wt mode is implemented in the latest public release of MC@NLO [10], and is called “Diagram Subtraction” (DS) in the code. Also provided is a calculation in which doubly resonant diagrams are removed at the amplitude level, called “Diagram Removal” (DR). The difference between the two codes then gives a measure of the size of interference effects between Wt and top pair production. If this

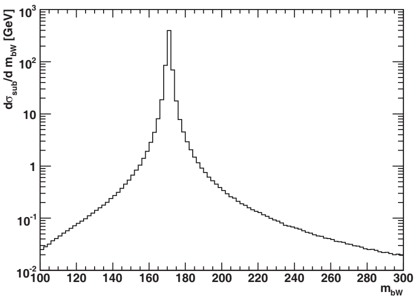

Fig. 2. – The integrated subtraction term as a function of the invariant mass of the $W\bar{b}$ pair.

difference is small compared with other uncertainties in any given analysis (*e.g.*, scale variation, PDF uncertainty, statistical uncertainty), then one indeed trusts the MC@NLO calculation. A word of caution is in order regarding the DR code. Given that diagrams are removed at the amplitude level, this breaks gauge invariance. A full discussion of this point is given in [7], including an exploration of the numerical size of any gauge invariance violating effects. It is also the case that no direct measure of the size of interference effects can be made gauge invariant. Ultimately, one must decide whether the estimate of the size of interference obtained by comparing DR and DS is a useful number, and it is argued in [7] that this is indeed the case. In any case, the DS code, as explained above, is gauge invariant.

4. – Results for Wt production using MC@NLO

A detailed study was carried out in [11] addressing the issue of interference effects in Wt. Specifically, the authors investigated whether the separation approach is justified for various choices of analysis cuts. There are two contexts in which one must evaluate the sum of Wt and top pair production. Firstly, when Wt production is the signal, and top pair production a (significant) background. Secondly, when Wt and top pair production are both backgrounds to a third process (the example of a Higgs boson decaying to two W bosons was considered in [11]).

In the former case, the authors considered the following cuts for isolating Wt production, based on observing semileptonic decays:

1) Exactly one b jet ($p_T > 50\,\text{GeV}$, $|\eta| < 2.5$). No other b jets with $p_T > 25\,\text{GeV}$ and $|\eta| < 2.5$.
2) Exactly two light jets with $p_T > 25\,\text{GeV}$ and $|\eta| < 2.5$. Also, $55\,\text{GeV} < m_{j_1 j_2} < 85\,\text{GeV}$.
3) Exactly one isolated lepton ($\Delta R < 0.4$ w.r.t. jets) with $p_T > 25\,\text{GeV}$ and $|\eta| < 2.5$.
4) Missing transverse energy $E_T^{\text{miss}} > 25\,\text{GeV}$.

These cuts are fairly minimal. The idea was to show that interference between Wt and top pair production is small for fairly loose cuts, so that more realistic analyses will be even safer. Total cross-sections were evaluated using MC@NLO for the above cuts, for Wt production (in both the DR and DS codes) and top pair production. The results are

TABLE I. – *Total cross-sections obtained using MC@NLO, for Wt and top pair production. Uncertainties come from variation of the common renormalisation and factorisation scale ($\mu = m_t$) by a factor of two.*

e_b	r_{lj}	$\sigma^{\mathrm{DR}}_{Wt}$/pb	$\sigma^{\mathrm{DS}}_{Wt}$/pb	$\sigma_{t\bar{t}}$/pb
1.0	10^4	$1.206^{+0.039}_{-0.017}$	$1.189^{+0.021}_{-0.010}$	$5.61^{+0.74}_{-0.54}$
0.6	30	$0.717^{+0.020}_{-0.014}$	$0.696^{+0.020}_{-0.005}$	$4.29^{+0.45}_{-0.46}$
0.6	200	$0.748^{+0.014}_{-0.011}$	$0.726^{+0.014}_{-0.007}$	$4.36^{+0.56}_{-0.42}$
0.4	300	$0.505^{+0.026}_{-0.009}$	$0.494^{+0.008}_{-0.008}$	$3.31^{+0.40}_{-0.37}$
0.4	2000	$0.512^{+0.011}_{-0.010}$	$0.503^{+0.001}_{-0.007}$	$3.35^{+0.37}_{-0.38}$

shown in table I, for a variety of choices of b tagging efficiency e_b and light-jet rejection rate r_{lj}. One notes from the table that the DR and DS results for Wt production agree well within scale uncertainties, thus justifying the use of the separation approach. Furthermore, the Wt cross-section is larger than the scale variation uncertainty of the top pair results, indicating that Wt production is indeed a well-defined signal at the LHC. Kinematic distributions were also examined in [11], and the DR and DS results were seen to agree within statistical uncertainties, thus demonstrating that for analysis cuts which remove the doubly resonant region, systematic uncertainty due to interference effects is small locally in phase space.

The authors also considered $H \to WW$, for which both Wt production and top pair production are backgrounds. Again the DR and DS codes in MC@NLO gave very similar results, thus justifying the use of a separation approach. As we have seen, this then allows the inclusion of NLO effects in both Wt and top pair production, significantly enhancing the accuracy of each calculation. It was also noted in [11] that the K-factor for Wt production is different from that for top pair production, a fact which has also been observed in previous NLO calculations of the Wt mode [5, 12]. This is further evidence for the fact that Wt and top pair production should be considered as separate processes where possible.

In spite of the evidence presented both here and elsewhere in the literature, opposition to the separation approach remains. An uncontroversial solution for all concerned would be to calculate $WWb\bar{b}$ fully at NLO, and interface this with a parton shower. Until then, compromises are necessary and inevitable. Either NLO corrections in top pair production must be sacrificed, or calculations can be presented which do not work in all phase space regions, and which must be used with appropriate care.

5. – Ht production

Another process of interest at the LHC is that of single top quark production in association with a charged Higgs boson. The latter occur rather generically in extensions to the Standard Model, *e.g.*, the minimal supersymmetric standard model (MSSM), which itself is an example of a more general class of theories, two-Higgs doublet models. So-called type I and type II varieties exist, which differ according to how the Higgs doublets couple to the various up- and down-type fermions.

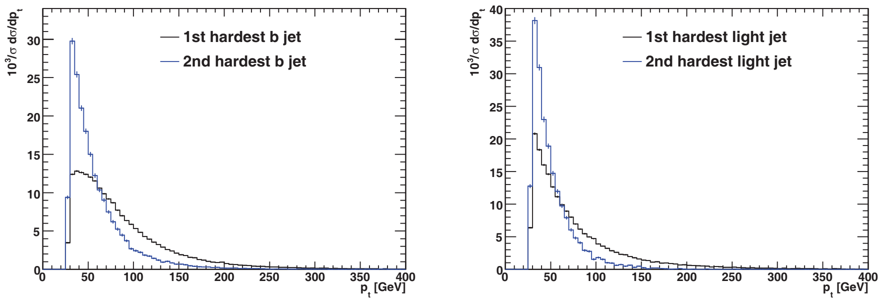

Fig. 3. – Transverse momentum spectra of the first and second hardest b and light jets in H^-t production, with $m_{H^-} = 300\,\mathrm{GeV}$, and leptonic decay of the top quark.

The H^-t process is very similar to the Wt process, although unlike the latter there are two distinct kinematic regimes depending on the mass of the charged Higgs boson:

1) $m_{H^-} > m_t$: The H^-t production mode is dominant in, *e.g.*, the MSSM.

2) $m_{H^-} < m_t$: H^-t interferes with $t\bar{t} \to tH^-b$.

The latter regime can be dealt with analogously to the Wt case, and we do not discuss this further here (for a full discussion, see [13]). From now on, we focus on results in the region of high m_{H^-}.

H^-t production has been implemented in MC@NLO, and will be in the next public release, including spin correlations of decay products using the method of [14]. It is also being implemented in POWHEG. The MC@NLO calculation is described in detail in [13]. It uses a five-flavour scheme, and is essentially the same as the NLO calculation of [15], which was previously implemented in the NLO event generator Prospino 2.1.

Some example physics results were also presented in [13], concerning the properties of b and light jets. Previous analyses have suggested that one may use additional b jets (*i.e.* those which do not come from a top decay) to design efficient event selection criteria for H^-t production. This relies on the assumption that additional b jets have sufficiently different properties (*e.g.*, p_t spectra) to radiated light jets. The advantages of testing this assumption in an MC@NLO (or POWHEG) framework are clear. The NLO matrix element gives a correct LO description of additional jet radiation, and the parton shower gives a realistic number of final state particles, jet substructure, etc.

In fig. 3 we show the transverse momentum spectra of the first and second hardest b and light jets, where the top quark decays leptonically. Jets are reconstructed using the k_T algorithm, in a detector volume $|\eta| < 2.5$ and $p_T > 25\,\mathrm{GeV}$. One sees from the figure that the additional b jet (*i.e.* the second hardest b jet, which does not come from the top decay most of the time) is not significantly harder than the hardest light jet. Similar results were observed for other Higgs masses.

More quantitatively, one may consider the question: Given that one hard b jet has been observed, what is the probability that one finds a second b jet by asking for the two hardest jets in the event? This was found to be rather low ($\simeq 35\%$), even for leptonic top decays. The reason for this is essentially combinatoric. There are typically more light jets than b jets, such that in forming the p_t spectrum of the hardest light jet (*vs.* the second hardest b jet) there are more jets to choose from, which hardens the resulting spectrum.

The above results suggest that it is very difficult to exploit any differences between radiated b and light jets. For a full discussion (and many more results) see [13], although this is admittedly a very preliminary study.

6. – Conclusion

We have described the implementation of Wt production in the MC@NLO event generator. This is potentially difficult due to interference issues with top pair production. However, we have seen that it is possible to implement Wt production in an NLO + parton shower framework in a gauge-invariant way. This allows higher-order effects to be included in both the Wt and top pair processes, subject to suitable analysis cuts. If there is any doubt about a given analysis, the two MC@NLO codes for Wt (DR and DS) should both be run. If the difference between them is small with respect to other uncertainties in the analysis, then the separation approximation is justified.

H^-t production has also been implemented, where the interference problem for $m_{H^-} < m_t$ can be dealt with using similar methods to in the Wt case. For the high Higgs mass region there is no interference problem, and we gave example results showing that there do not seem to be significant differences between additional b jets and radiated light jets.

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