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# Spin correlations: Tevatron vs. LHC

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**Summary.** — We compare theoretical expectations for the observation of spin correlations in top quark pair production and decay at the Fermilab Tevatron and the CERN Large Hadron Collider (LHC). In particular, we note that the differing top quark pair production mechanisms in the two environments test different aspects of the Standard Model and require different strategies to observe the correlations. At the Tevatron, production is dominated by  $q\bar{q} \rightarrow t\bar{t}$  and the strategy is to construct a double-decay angle distribution where one decay angle is measured in the t rest frame and the other in the  $\bar{t}$  rest frame. The dominant process at the LHC is  $gg \rightarrow t\bar{t}$ , with a rich spin structure that allows for a second option in observing spin correlations. Here the strategy is to select events where the  $t\bar{t}$  pair is produced at relatively low velocity in the zero momentum frame (ZMF). For these events, there are strong azimuthal correlations between t and  $\bar{t}$  decay products present. This measurement enjoys the advantage that it can be carried out in the laboratory frame.

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# 1. – Introduction

One of the major components of the physics program at the LHC will be a detailed study of the properties of the top quark. In this paper, we report on recent theoretical work on how to observe spin correlations in the production and decay of top quark pairs via gluon fusion [1]. The top quark is unique among the quarks in the Standard Model in that its decay lifetime is so short that it is predicted to decay before spin decorrelation takes place [2]. Thus, top quarks produced in a particular spin state will pass information about that state on to its decay products, and an investigation of the angular correlations among the decay products in select  $t\bar{t}$  events can shed light on the details of the production and decay mechanisms. The mere observation of  $t\bar{t}$  spin correlations would provide an upper limit on the top quark lifetime; conversely, the observation of a lack of significant  $t\bar{t}$  spin correlations would indicate the participation of the top quark in some sort of new strong interaction that causes spin decorrelation to occur more quickly than predicted by the Standard Model considerations of ref. [2]. Thus, in the near-term, establishing the presence or absence of these correlations is a worthwhile physics goal. In the longer

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Fig. 1. – The spin configurations for the process  $q_R \bar{q}_L \to t\bar{t}$  are best described by the off-diagonal basis. The angle between the top quark momentum and the spin vector in the zero momentum frame is given by  $\tan \Omega = (1 - \beta^2) \tan \theta$  where  $\beta$  is the speed of the top quark in the ZMF. The ratio of  $t_{\uparrow}\bar{t}_{\downarrow}$  to  $t_{\downarrow}\bar{t}_{\uparrow}$  production is  $(1 + \sqrt{1 - \beta^2 \sin^2 \theta})^2 : (1 - \sqrt{1 - \beta^2 \sin^2 \theta})^2$ . The results for the  $q_L\bar{q}_R$  initial state may be obtained by reversing all of the spins in the diagram. The same spin structure also applies to  $gg \to t\bar{t}$  with opposite helicity gluons.

term, a detailed study of as many aspects of the correlations as possible will provide a series of additional wide-ranging tests of the Standard Model.

The outline of this paper is as follows. We will begin with a detailed examination of the spin structure of the main top pair production mechanisms at the Tevatron  $(q\bar{q} \rightarrow t\bar{t})$ and LHC  $(gg \rightarrow t\bar{t})$ , with a particular emphasis on elucidating the spin state(s) of the final  $t\bar{t}$  pairs. In sect. **3** we will examine the angular distributions associated with the decay of polarized top quarks. Combining the production properties with the decays will lead us to the strategies for observing the spin correlations. These strategies will be different for the Tevatron (sect. **4**) and LHC (sect. **5**). In sect. **6** we will briefly consider some of the physics that could be accomplished with  $\mathcal{O}(10^6) t\bar{t}$  pairs at our disposal. Finally, sect. **7** contains a summary and our conclusions. Additional details may be found in ref. [1].

#### 2. – Detailed spin structure in top quark pair production

The main two top quark pair production mechanisms at hadron colliders are  $q\bar{q} \rightarrow t\bar{t}$ , which dominates at the Fermilab Tevatron, and  $gg \rightarrow t\bar{t}$ , which dominates at the LHC. In this section, we examine the spin structure of each of these processes in some detail.

The process  $q\bar{q} \to t\bar{t}$  proceeds through an *s*-channel gluon; in order to couple to this gluon, the incoming  $q\bar{q}$  pair must possess opposite helicities. The initial state is thus either  $q_R\bar{q}_L$  or  $q_L\bar{q}_R$ . We will focus our discussion on  $q_R\bar{q}_L$ : results for  $q_L\bar{q}_R$  may be generated by flipping all of the spins in the initial and final states.

The top quark pairs produced from the  $q_R \bar{q}_L$  are well described by the off-diagonal spin basis [3] (see fig. 1): at leading order the top quarks have opposite spins  $(t_{\uparrow}\bar{t}_{\downarrow} \text{ or } t_{\downarrow}\bar{t}_{\uparrow})$ 100% of the time using this spin quantization axis. At threshold, the off-diagonal basis coincides with the beamline basis [4], the appropriate choice of quantization axis in the  $\beta \rightarrow 0$  limit [5]. At the other extreme ( $\beta \rightarrow 1$ ), the off-diagonal basis coincides with the helicity basis. Between these two extremes, the off-diagonal basis smoothly interpolates from the beamline basis at small  $\beta$  to the helicity basis in the ultra-relativistic limit while maintaining a spin state consisting of 100% opposite spin  $t\bar{t}$  pairs.

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TABLE I. – Analyzing powers  $\alpha$  for both semi-leptonic and hadronic top quark decays at nextto-leading order. The coefficients for u, d, s, c and b are for partons: the values for jets differ slightly at NLO (see ref. [7]).

| Decay product | $\alpha$ |
|---------------|----------|
| $\ell^+$      | 0.998    |
| $ar{d},ar{s}$ | 0.966    |
| ν             | -0.314   |
| u, c          | -0.317   |
| b             | -0.393   |

The gluon fusion top pair production mechanism possesses a very rich spin structure: the spin state of the  $t\bar{t}$  is different for different gluon helicities. When the initial gluons have opposite helicity, the  $t\bar{t}$  spin state is the same as for  $q\bar{q}$  production (fig. 1). Thus, the off-diagonal basis provides the description with the maximum  $t\bar{t}$  spin correlation in this case. Opposite helicity gluon pairs form the dominant contribution to  $gg \to t\bar{t}$  when  $\beta\gamma \sin \theta > 1$ , or, equivalently, when  $\beta^2 > 1/(2 - \cos^2 \theta)$ .

On the other hand, when  $\beta\gamma\sin\theta < 1$ , like-helicity gluon pairs form the dominant contribution. In this case, it turns out that the helicity basis provides the best description of the  $t\bar{t}$  spin correlations, no matter what the value of  $\beta$ . Like-helicity gluons produce like-helicity  $t\bar{t}$  pairs. The ratio of  $t_R\bar{t}_R$  to  $t_L\bar{t}_L$  production from a  $g_Rg_R$  initial state is given by  $(1 + \beta)^2 : (1 - \beta)^2$ .

#### 3. – Decay of polarized top quarks

We now consider the angular distributions associated with the decay of spin-up top quarks. We define the decay angles in the top quark rest frame; we will denote the angle between the *i*-th particle and the top quark spin direction by  $\theta_i$ . Then, the decay angular distribution takes the simple form [6]

(1) 
$$\frac{1}{\Gamma} \frac{\mathrm{d}\Gamma}{\mathrm{d}(\cos\theta_i)} = \frac{1}{2} (1 + \alpha_i \cos\theta_i).$$

The analyzing power  $\alpha_i$  depends on which decay product we consider; the values of  $\alpha_i$  at next-to-leading order [7] are collected in table I.

The normalized distribution for the complete  $2 \rightarrow 6$  production and decay process is

(2) 
$$\frac{1}{\sigma} \frac{\mathrm{d}^2 \sigma}{\mathrm{d}(\cos \theta_i) \mathrm{d}(\cos \bar{\theta}_{\bar{\imath}})} = \frac{1}{4} \left[ 1 + \frac{N_{\parallel} - N_{\times}}{N_{\parallel} + N_{\times}} \alpha_i \bar{\alpha}_{\bar{\imath}} \cos \theta_i \cos \bar{\theta}_{\bar{\imath}} \right].$$

Equation (2) clearly displays the dependence on production  $(N_{\parallel})$  is the number of events with like-spin  $t\bar{t}$  pairs;  $N_{\times}$  is the number of events with opposite-spin  $t\bar{t}$  pairs) and decay  $(\theta_i, \alpha_i)$  refer to the t side of the event;  $\bar{\theta}_{\bar{i}}, \bar{\alpha}_{\bar{i}}$  refer to the  $\bar{t}$  side of the event; the angles are measured in the respective rest frames of the t and  $\bar{t}$ ). The next-to-leading-order corrections to this distribution have been presented in refs. [8,9]; these corrections will play an important role in future precision measurements of the correlations.

#### 4. – Tevatron strategy for observing spin correlations

At the Fermilab Tevatron, the strategy for observing the spin correlations in  $t\bar{t}$  production and decay consists of reconstructing the double decay angular distribution eq. (2) from the data. An examination of eq. (2) indicates how to maximize the size of the correlations. First, one should choose a spin quantization axis that maximizes the asymmetry  $(N_{\parallel} - N_{\times})/(N_{\parallel} + N_{\times})$ . Since the dominant process at the Tevatron is  $q\bar{q} \rightarrow t\bar{t}$ , this means using the off-diagonal basis, although in the long term it will be worthwhile to do the measurement using the helicity and beamline bases as well. Measurements with multiple spin bases probe different aspects of the  $q\bar{q} \rightarrow t\bar{t}$  matrix element and provide a cross-check on the analysis. Furthermore, the sources of systematic uncertainty are likely to vary greatly for different spin bases.

Second, one should choose t and  $\bar{t}$  decay products with large analyzing powers (*i.e.* the charged lepton or d-type quark). This consideration suggests looking in the dilepton or lepton+jets modes. The dilepton mode has the advantage of the maximum possible analyzing power, but suffers from a small branching ratio and the complications associated with the two (unobserved) neutrinos in the final state. The lepton+jets mode couples higher statistics with a much better ability to reconstruct the t and  $\bar{t}$  rest frames, but has a lower analyzing power since the jet associated with the d-type quark must be selected probabilistically. Eventually, given sufficient statistics, it will be useful to measure the correlations for as many different combinations of decay products as possible.

Although this is a very difficult analysis, both CDF and D0 have reported initial attempts to observe these correlations despite the limited data set currently available. At present, with only about half of the approximately  $7 \text{ fb}^{-1}$  delivered by the Tevatron analyzed so far, no unambiguous (>  $3\sigma$ ) sign of the correlations has been seen; however, both CDF and D0 plan on updates as well as a combined result in the near future [10].

## 5. – LHC strategy for observing spin correlations

The dominant top quark pair production mechanism at the LHC is gluon fusion. The detailed study of this process using an arbitrary spin axis performed in ref. [1] leads us to the following two conclusions: When  $\beta\gamma\sin\theta < 1$ , the best spin basis is the one which maximizes the like-spin fraction (like-helicity gluons dominate in this region). On the other hand, when  $\beta\gamma\sin\theta > 1$ , the best spin basis is the one which maximizes the opposite-helicity gluons dominate in this region).

Because the LHC will be a copious source of  $t\bar{t}$  pairs (about 10<sup>6</sup>  $t\bar{t}$  pairs per fb<sup>-1</sup> at full beam energy; even at reduced energy there will be 10<sup>4</sup> to 10<sup>5</sup> pairs per fb<sup>-1</sup>), there will be room to implement significant cuts on the data before statistical uncertainties become comparable to the systematic uncertainties. Thus, we will focus on the  $\beta\gamma\sin\theta < 1$ region. In this region, like-helicity gluons to like-helicity  $t\bar{t}$  pairs dominate. Since  $\beta$  is restricted to fairly moderate values in most of this region, the spin correlations will not be masked by large boosts, and one could, in principle, employ the same analysis as has been used at the Tevatron. Another option exists, however.

In order to motivate this alternative, we will examine the ratio

(3) 
$$S \equiv \frac{(|\mathcal{A}|_{RR}^2 + |\mathcal{A}|_{LL}^2)_{\text{corr}}}{(|\mathcal{A}|_{RR}^2 + |\mathcal{A}|_{LL}^2)_{\text{uncorr}}}$$

which compares the sum of the squares of the like-helicity amplitudes for the fully

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Fig. 2. – The differential distribution in  $\Delta\phi$ ,  $(1/\sigma_T)d\sigma/d(\Delta\phi)$ . The solid curve is for the fully correlated case whereas the dashed curve assumes that the top quarks decay spherically in their respective rest frames. In the plot on the left, a cut restricting the (true) invariant mass of the  $t\bar{t}$  pairs to a maximum of 400 GeV has been applied to the distributions; in the plot on the right, the cut restricts the (naive) unweighted average of all of the reconstructed values of  $m_{t\bar{t}}$  for each event to a maximum of 400 GeV.

correlated  $gg \rightarrow t\bar{t}$  matrix elements to the same sum for matrix elements calculated with a toy model employing top quark decays which are spherical in their rest frames (but with all other decays—*i.e.* the W's—fully correlated). In terms of the cosines of the angles in the ZMF between various pairs of particles and the ZMF speed of the top quarks,

(4) 
$$S = \left[\frac{1-\beta^2}{1+\beta^2}\right] \left[\frac{(1+\beta^2) + (1-\beta^2)c_{\bar{e}\mu} - 2\beta^2 c_{t\bar{e}}c_{\bar{t}\mu}}{(1-\beta c_{t\bar{e}})(1-\beta c_{\bar{t}\mu})}\right],$$

which reduces to  $1 + c_{\bar{e}\mu}$  for  $\beta \to 0$ . So, at smallish values of  $\beta$ , the difference between the correlated and uncorrelated versions of the matrix elements is sensitive to the angle between the two charged leptons, suggesting that we examine  $\Delta\phi$ ,  $\Delta\eta$ , and  $\Delta R$  for these two particles. Of these three distributions,  $\Delta\phi$  turns out to be the most sensitive to the presence or absence of  $t\bar{t}$  spin correlations. This distribution also enjoys the advantage of being invariant under longitudinal boosts; once the event sample has been selected, this measurement can be made in the laboratory frame without the complications associated with the reconstruction of and boost to some special frame of reference(<sup>1</sup>).

In fig. 2, we present a comparison of the correlated and uncorrelated distributions in  $\Delta\phi$  for the two leptons in a sample of dilepton events. In the plot on the left, the events were required to have a  $t\bar{t}$  invariant mass of less than 400 GeV to ensure that they sample the like-helicity-gluon-dominated  $\beta\gamma\sin\theta < 1$  region of phase space. Clearly this distribution has significant sensitivity to the presence or absence of spin correlations. Unfortunately, the pair of neutrinos in these events makes a full reconstruction of the event impossible<sup>(2)</sup>, and we must look for an alternate means of selecting the  $\beta\gamma\sin\theta < 1$ 

<sup>(&</sup>lt;sup>1</sup>) Unfortunately, the  $\Delta \phi$  distribution in  $q\bar{q} \rightarrow t\bar{t}$  is insensitive to the presence or absence of spin correlations. Thus, this observable is not particularly interesting at the Tevatron.

<sup>(&</sup>lt;sup>2</sup>) Although there are a total of 8 constraint equations for the 8 unknown components of the  $\nu$  and  $\bar{\nu}$  4-momenta, two of these constraints are quadratic, leading to up to 4 distinct solutions



Fig. 3. – The differential distribution in  $\cos \theta$ ,  $(1/\sigma_T) d\sigma/d(\cos \theta)$ , where  $\theta$  is the ZMF angle between the charged lepton and the *d*-quark jet (defined to be the jet which is spatially closest to the *b*-tagged jet in the *W* rest frame; this is also the jet with the lowest energy in the top quark rest frame). The solid curve is for the fully correlated case whereas the dashed curve assumes that the top quarks decay spherically in their respective rest frames. A cut restricting the invariant mass of the  $t\bar{t}$  pairs to a maximum of 400 GeV has been applied to these distributions.

region. In the plot on the right, the event selection is based on the value of the (naive) unweighted average of the various  $m_{t\bar{t}}$  values produced by the neutrino reconstruction routine. Cutting on this quantity has the unfortunate side-effect of producing a systematic depletion of events near  $\Delta \phi = \pi$ ; however, this depletion affects both models in a similar fashion and significant discriminating power remains. What we have here is a proof-of-concept: to do the actual measurement it will be necessary to understand the detector systematics and NLO corrections to the  $\Delta \phi$  distribution very well. Left open are the questions of whether a better substitute for the true value of  $m_{t\bar{t}}$  exists as well as the optimal maximum value of  $m_{t\bar{t}}$  to use in selecting the data.

We now turn to the lepton+jets mode. This channel has the advantage of larger statistics than dilepton mode, and only a single, over-constrained, neutrino to be reconstructed. Thus, it is possible to do a good job of locating the ZMF and calculating  $m_{t\bar{t}}$ . This mode has one disadvantage, however: it is not possible to know with 100% certainty which of the two W-decay jets corresponds to the d-type quark. Thus, it is necessary to use our best informed guess to select this jet on a probabilistic basis. The authors of ref. [4] suggest using the jet which has the smallest spatial separation from the b jet in the W rest frame; this is equivalent to selecting the jet with the lowest energy in the t rest frame [11]. Although this is the correct choice only about 60% of the time, it is good enough to provide an analyzing power of 0.4734 to 0.4736 at NLO [7], depending on what jet reconstruction algorithm is used.

Because the ZMF is well determined in lepton+jets mode, it is possible to examine the actual opening angle between the lepton and the *d*-jet candidate (fig. 3). Taking half of the area between the correlated and uncorrelated curves to be a measure of the potential sensitivity of this observable, we note that this half-area is equal to 0.07 for the  $\cos \theta_{\bar{e}d}$  distribution, as compared to a value of 0.11 for the  $\Delta \phi_{\bar{e}\mu}$  distribution. However, this somewhat reduced sensitivity could easily be offset by the higher statistics of the lepton+jets mode. Furthermore, the systematics of this measurement are certainly very

for each of the two possible pairings of W-bosons (leptons) with b jets. Thus, there are up to 8 distinct values of  $m_{t\bar{t}}$  generated by the reconstruction program.



Fig. 4. – Effects of varying the machine center-of-mass energy  $\sqrt{s}$ . (a) Total leading order cross section for  $pp \to t\bar{t}$ . These values should be multiplied by the branching fraction to dileptons (4.6%) or lepton-plus-jets (29%), as appropriate. We include only the e and  $\mu$  channels. (b) Fraction of dilepton and lepton plus jets events with  $m_{t\bar{t}} < 400 \,\text{GeV}$ . For dilepton events (crosses) we employ the unweighted average of the up to 8 solutions for the  $t\bar{t}$  invariant mass. For lepton+jets events (diamonds) the true value of  $m_{t\bar{t}}$  may be reconstructed and used in event selection. (c) Half of the area between the appropriate unit-normalized angular distributions for the fully correlated and spherical cases. For lepton+jets events (crosses), we use the distribution in  $\cos \theta$ , where  $\theta$  is the angle between the charged lepton and the d-jet candidate in the zero momentum frame of the event. For dilepton events (diamonds), we use the azimuthal opening angle  $\Delta \phi$  between the two charged leptons.

different than those associated with the  $\Delta \phi$  distribution; thus, further investigation of this variable by the experimental collaborations is warranted.

Before closing this section on the LHC, we present fig. 4, which summarizes the effect of running the LHC at reduced energy on the ability to observe  $t\bar{t}$  spin correlations. The conclusion to be drawn from the three plots in fig. 4 is that the biggest penalty of running at reduced energy comes from the much smaller  $t\bar{t}$  production cross section: the size of the spin correlations is essentially unchanged as one reduces  $\sqrt{s}$  from 14 TeV to 7 TeV. Thus, the prospects of observing these correlations with the data collected during the early stages of running are good.

### 6. – The future

Before concluding, let us take a moment or two to comment on the sorts of precision tests of the Standard Model that could be done with the few million  $t\bar{t}$  pairs that will ultimately be available at the LHC, beginning with the couplings between gluons and the top quark. All 3 of the diagrams for  $gg \to t\bar{t}$  influence the spin correlations; these diagrams are related in a very specific manner to satisfy SU(3) gauge-invariance. Thus, a test of the spin correlations can be viewed as a test of QCD gauge-invariance.

The time scale for the top quark is expected to be much shorter than the spin decorrelation time scale. This fact offers us the opportunity to perform a direct test of the predicted V - A structure of the Wtb vertex, the only quark for which such a test is possible. By measuring the size of the correlations for different combinations of decay products, it will be possible, with sufficient data, to perform measurements of the analyzing powers of all of the top quark decay products listed in table  $I(^3)$ .

 $<sup>(^3)</sup>$  Since the neutrino is over-constrained in lepton+jets mode, a measurement of  $\alpha_{\nu}$  may be possible, and a detailed feasibility study should be performed to investigate this possibility.

In addition, one ought to look for non-Standard Model couplings and/or couplings to new particles, such as Kaluza-Klein gluons, "extra" Higgs bosons, or "extra" Z-bosons. Should a new particle that couples strongly to the top quark be observed as a resonance in the  $t\bar{t}$  channel, then the spin correlations of the  $t\bar{t}$  pairs produced in the resonance region of phase space would bear the imprint of the spin and parity of the new particle and serve as a useful diagnostic in identifying the nature of the new physics.

## 7. – Conclusions

In summary, we have seen that the Tevatron and LHC probe two different aspects of spin correlations in top quark pair production and decay. At the Tevatron, the production cross section is dominated by  $q\bar{q} \rightarrow t\bar{t}$ . For this process, the off-diagonal spin basis provides the largest signal of spin correlations. The strategy for observing the correlations at the Tevatron involves extracting a joint decay distribution utilizing decay angles in the t and  $\bar{t}$  rest frames. At present, this challenging measurement is limited by low statistics.

At the LHC, the production cross section is dominated by  $gg \to t\bar{t}$ . At high values of the  $t\bar{t}$  invariant mass, opposite helicity gluons to opposite spin  $t\bar{t}$  pairs dominate the cross section; the correlations involved are the same as for  $q\bar{q} \to t\bar{t}$ . At low values of the  $t\bar{t}$  invariant mass, like-helicity gluons dominate the cross section; these gluons primarily produce like-helicity  $t\bar{t}$  pairs. By cutting on the value of  $m_{t\bar{t}}$ , it is possible to enhance the contributions from either like or opposite helicity gluon pairs. In addition to a repeat of a Tevatron-style analysis, promising observables for observing spin correlations include the azimuthal opening angle between the two leptons in dilepton events and the cosine of the angle between the lepton and d-jet candidate in the ZMF in lepton+jets mode. With millions of  $t\bar{t}$  pairs on the horizon, precision (%-level) measurements of the various correlation parameters should be possible during the next decade.

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