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Neutrino physics and lepton flavour violation: A theoretical overview

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Summary. — We review the theoretical status of neutrino physics and its implications for physics beyond the Standard Model. We also discuss the prospects to observe flavour violation in the charged lepton sector, with special emphasis on the connection to neutrino parameters.

PACS `14.60.Pq` – Neutrino mass and mixing.
PACS `13.35.-r` – Decays of leptons.
PACS `12.60.Jv` – Supersymmetric models.

1. – Introduction

A series of experiments have firmly established the violation of lepton flavour in the neutrino sector [1], with dramatic implications for particle physics. In the Standard Model of particle physics the gauge interactions and the kinetic terms of the leptonic Lagrangian are invariant under the global symmetry $U(3)_{e_R} \times U(3)_L$. This symmetry is broken, however, by the Yukawa coupling of the charged leptons, which eventually lead to charged lepton masses. As a result, the full Standard Model Lagrangian has a smaller symmetry $U(1)_e \times U(1)_\mu \times U(1)_\tau$, which amounts to the conservation of all family lepton numbers. On the other hand, the disappearance of electron and muon neutrinos and antineutrinos observed in experiments constitute *evidences* that lepton flavour is not conserved in Nature, thus revealing the existence of new physics beyond the Standard Model.

The simplest, most elegant and probably correct explanation for the lepton flavour violation observed in experiments is three family neutrino oscillations(1). This statement, which seems obvious almost fifteen years after the discovery of neutrino oscillations, is nevertheless very non-trivial. It is important to remember that several mechanisms were

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(1) With the exception of the LSND experiment, which observed electron antineutrino appearance in a muon antineutrino beam from pion decay at short baselines, and which cannot be accommodated in this framework. This result, however, has not been confirmed by MiniBooNE.

TABLE I. – *Status of neutrino parameters (from* [2]*).*

$$\Delta m^2_{21} = 7.59 \pm 0.20 \ \left({}^{+0.61}_{-0.69}\right) \times 10^{-5}\,\mathrm{eV}^2$$

$$\Delta m^2_{31} = \begin{array}{l} -2.40 \pm 0.11 \ \left({}^{+0.37}_{-0.39}\right) \times 10^{-3}\,\mathrm{eV}^2 \ \text{(inverted)} \\ +2.51 \pm 0.12 \ \left({}^{+0.39}_{-0.36}\right) \times 10^{-3}\,\mathrm{eV}^2 \ \text{(normal)} \end{array}$$

$$\theta_{12} = 34.4 \pm 1.0 \ \left({}^{+3.2}_{-2.9}\right)$$

$$\theta_{23} = 42.3\,{}^{+5.3}_{-2.8} \ \left({}^{+11.4}_{-7.1}\right)$$

$$\theta_{13} = 6.8\,{}^{+2.6}_{-3.6} \ (\leq 13.2)$$

$$\left[\sin^2\theta_{13} = 0.014\,{}^{+0.013}_{-0.011} \ (\leq 0.052)\right]$$

$$\delta_{CP} \in [0,\ 360]$$

proposed in the past to explain, without invoking neutrino masses, the lepton flavour violation observed in experiments. All of them are nowadays excluded by experiments, whereas neutrino oscillations is still a viable possibility which moreover can explain *simultaneously* all the experiments. For example, the atmospheric neutrino deficit could be explained by neutrino decay or by quantum decoherence effects. However, these mechanisms could not explain the dip in the L/E dependence of the deficit which was observed by SuperKamiokande. On the other hand, the solar neutrino deficit could be explained by the resonant spin-flip flavour conversion of neutrinos in a postulated strong magnetic field in the interior of the Sun, which is again excluded now by the observation of electron antineutrino disappearance by KamLAND.

2. – Status of neutrino oscillations

If neutrinos are massive particles, the flavour eigenstates $|\nu_\alpha\rangle$ ($\alpha = e,\ \mu,\ \tau$) do not necessarily coincide with the mass eigenstates $|\nu_i\rangle$ ($i = 1, 2, 3$). Instead, they are related by the unitary transformation $|\nu_\alpha\rangle = (U_{\mathrm{lep}})_{\alpha i}|\nu_i\rangle$, where the leptonic mixing matrix U_{lep} is usually parametrized in terms of three angles θ_{12}, θ_{23} and θ_{13} and one phase δ for Dirac neutrinos or three phases δ, ϕ, ϕ' for Majorana neutrinos.

$$U_{\mathrm{lep}} = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \cdot \left(e^{-i\phi/2}, e^{-i\phi'/2}, 1\right), \tag{1}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$.

The neutrino mass eigenstates are labeled such that ν_3 is the eigenvalue which is most split in mass with respect to the other two, while ν_1 and ν_2 are ordered such that ν_1 is the lightest between them. Neutrino oscillation experiments are only sensitive to the mass splittings and not to the masses themselves. Therefore, present experiments allow two possible mass orderings: the "normal" hierarchy, $m_3 > m_2 > m_1$, and the "inverted" hierarchy, $m_2 > m_1 > m_3$. The present status of the determination of neutrino parameters from experiments is summarized in table I.

Even though the information about neutrino parameters is still rather limited one can already notice some features: i) neutrino masses are tiny, at most of the order of 1 eV, ii) there are two large mixing angles, one of them possibly maximal, while the third one is small, iii) the two heaviest neutrinos present a mild mass hierarchy, the ratio between their masses being smaller than six. In order to understand the origin of flavour, it is important to compare these parameters with the same ones in the quark sector or the charged lepton sector. In doing so, one notices very striking differences: i) quark masses and charged lepton masses are in the MeV or GeV range, while neutrino masses lie in the eV or sub-eV range, ii) in the quark sector the three mixing angles are small, while in the neutrino sector there are two large mixing angles, iii) the mass hierarchies between the quark masses are $m_t/m_c \simeq 140$, $m_c/m_u \simeq 550$, $m_b/m_s \simeq 44$, $m_s/m_d \simeq 19$, $m_\tau/m_\mu \simeq 17$, $m_\mu/m_e \simeq 208$, while the mass hierarchy between the two heaviest neutrino states is much smaller, at most a factor of six. Any model of flavour should therefore address the three following questions: why are the neutrino masses tiny?, why are there large mixing angles?, why is there at least one mild mass hierarchy?

3. – Neutrino parameters as a window to new physics

Many neutrino mass models have been proposed to answer these questions, which fall into two main categories depending on how the global symmetry $U(3)_{e_R}\times U(3)_L$ is broken. A few models incorporate Dirac neutrinos, where neutrino masses violate all the family lepton numbers while preserving the total lepton number ($U(3)_{e_R} \times U(3)_L \to U(1)_{\rm lep}$). On the other hand, most neutrino mass models proposed incorporate Majorana neutrinos, where all global quantum numbers in the leptonic sector are broken ($U(3)_{e_R} \times U(3)_L \to$ nothing). Whether neutrinos are Dirac or Majorana particles is still an open question which can only be resolved experimentally. Namely, the observation of neutrinoless double beta decay would constitute an evidence for the violation of total lepton number and thus an evidence for Majorana neutrinos. On the other hand, from the theoretical point of view these two possibilities provide different explanations to the puzzling differences between neutrino and quark parameters.

So far no Majorana fermion has been discovered, whereas we know of the existence of many fundamental Dirac fermions, therefore a very conservative assumption that one can make on the nature of neutrinos is that they are Dirac particles. If this is the case the Yukawa part of the leptonic Lagrangian reads

$$-\mathcal{L}_{\rm lep} = (h_e)_{ij}\bar{e}_{Ri}L_j\phi + (h_\nu)_{ij}\bar{\nu}_{Ri}L_j\tilde{\phi} + \text{h.c.} \tag{2}$$

Note that in this Lagrangian the conservation of lepton number has been imposed *by hand*: being the right-handed neutrinos singlets under the Standard Model gauge group, the gauge symmetry also allows the Majorana mass term $M_{ij}\bar{\nu}_{Ri}\nu^c_{Rj}$ which has been forbidden by invoking the total lepton number conservation. In this scenario, the tininess of neutrino masses can be explained by a Yukawa coupling $h_\nu \sim 10^{-12}$ for the heaviest generation. The mechanism that generates Yukawa couplings is currently unknown and such a small value cannot be precluded, however, in view of the values of the other Yukawa couplings for the third generation (one billion times larger than that), this explanation looks conspicuous. We can also get some insight on the possibility of Dirac neutrinos by comparing the mass ratio between the two heaviest neutrinos with the mass ratios in other sectors. Again, even though we ignore the concrete mechanism that generates Dirac couplings, measurements of the quark and charged lepton masses suggest that this mechanism tends to generate large mass hierarchies. Therefore, the existence of a mild mass hierarchy in the neutrino sector is another indication that neutrino masses have a different origin than the quark and charged lepton masses.

On the other hand, the possibility of Majorana masses is without any doubt the option preferred by most theorists, even though no fundamental Majorana fermion has been discovered so far. In this case the Yukawa part of the leptonic Lagrangian reads

$$-\mathcal{L}_{\rm lep} = (h_e)_{ij}\bar{e}_{Ri}L_j\phi + \frac{(\alpha_\nu)_{ij}}{\Lambda}L_i\tilde{\phi}L_j\tilde{\phi} + {\rm h.c.}\,, \tag{3}$$

which exhibits two remarkable facts. First, there are no new particles at low energies and secondly, this is the most general Lagrangian up to dimension five consistent with the Standard Model particle content and gauge symmetry (note that no global symmetry has been imposed). For Majorana neutrinos the tininess of the masses can be explained by invoking a small coupling α_ν and/or by invoking a large suppression of the dimension-five operator by a large Λ. Moreover, the coupling α_ν is not a "Dirac-like" Yukawa coupling, therefore the flavour structure can be completely different to the flavour structure of the known Yukawa couplings h_u, h_d, h_e, namely the hierarchy of the eigenvalues of α_ν does not have to be necessarily very large, as in the case of the known "Dirac-like" Yukawa couplings. The facts that the smallness of neutrino masses can be explained by a large Λ and that the coupling α_ν can have a flavour structure different to the quark and lepton Yukawa couplings opens new opportunities to understand the striking differences between neutrino parameters and quark parameters, making the possibility of Majorana masses very appealing from the theoretical point of view.

There are many proposals to explain the origin of Majorana neutrino masses. The most popular one (and perhaps the simplest and most elegant) consists on introducing new heavy degrees of freedom, possibility commonly known as see-saw mechanism. There are three types of see-saw mechanisms: the type I see-saw mechanism assumes the existence of new fermion singlets, type II, new scalar triples, and type III, new fermion triplets. Here we will just discuss the type I see-saw mechanism.

The type I see-saw mechanism consists on adding to the Standard Model particle content at least two right-handed neutrinos. With this particle content, the most general leptonic Lagrangian compatible with the Standard Model gauge symmetry reads

$$-\mathcal{L}_{\rm lep} = (h_e)_{ij}\bar{e}_{Ri}L_j\phi + (h_\nu)_{ij}\bar{\nu}_{Ri}L_j\tilde{\phi} - \frac{1}{2}M_{ij}\bar{\nu}_{Ri}\nu^c_{Rj} + {\rm h.c.}\,, \tag{4}$$

where in addition to the neutrino Yukawa coupling we have introduced a Majorana mass term for the right-handed neutrinos. Being the right-handed neutrinos singlets under the Standard Model gauge group, their mass scale is not related to the electroweak symmetry breaking scale: it can be of the same order, much larger or much smaller. The most interesting case arises when this mass scale is much larger than the electroweak scale. If this is the case, the right-handed neutrinos decouple at low energies and the effective theory can be described by the following Lagrangian:

$$-\mathcal{L}_{\rm lep} = \frac{1}{2}(L_i\tilde{\phi})[h_\nu^T M^{-1}h_\nu]_{ij}(L_j\tilde{\phi}) + {\rm h.c.}\,, \tag{5}$$

which gives, after the electroweak symmetry breaking, a neutrino mass matrix which reads $\mathcal{M}_\nu = h_\nu^T M^{-1}h_\nu\langle\phi^0\rangle^2$. Note that in the type I see-saw mechanism the neutrino masses are naturally small due to the large suppression by the large right-handed neutrino masses. Moreover, the neutrino Dirac Yukawa coupling enters in a complicated way in this formula. Therefore, it is plausible that even though the neutrino Dirac Yukawa coupling has very hierarchical eigenvalues (in accordance to our general expectation for "Dirac-like" couplings), the neutrino masses can have a mild mass hierarchy due to the complicated way it enters into this formula [3].

The type I see-saw mechanism has many attractive features: it is natural, simple and elegant, the particle content displays a suggestive left-right symmetry, it is nicely compatible with grand-unified theories, and could account for the observed baryon asymmetry of the Universe through the mechanism of leptogenesis [4]. For these reasons it is regarded as the "most standard extension of the Standard model". However, it has the disadvantage that since the new physics enters at very high energies it cannot be directly tested. Moreover, the best motivated see-saw scenario, where the right-handed neutrinos are much heavier than the electroweak symmetry breaking scale, suffers a serious fine-tuning problem. Namely, the Higgs mass acquires a quadratically divergent correction such that $\delta m_\phi^2 \sim 1/(16\pi^2) h_\nu^2 M^2$. There is a very appealing solution to this problem where the right-handed neutrinos can be arbitrarily heavy while the corrections to the Higgs mass being comparable to the Higgs mass itself. This is the supersymmetric see-saw model, where the large quadratic corrections to the Higgs mass from the right-handed neutrinos are compensated by large quadratic corrections to the Higgs mass from the right-handed *sneutrinos*, which are of the same size but of opposite sign. Therefore, supersymmetry is the natural arena to implement the high-scale see-saw mechanism and, as we will discuss later, offers new opportunities to (indirectly) test the see-saw mechanism.

4. – Flavour violation in the charged lepton sector

As discussed above, the lepton flavour violation observed in neutrino experiments have lead to a leptonic Lagrangian given either by eq. (2) for Dirac neutrinos or eq. (3) for Majorana neutrinos. Both can be regarded as effective Lagrangians containing terms up to dimension five. Clearly, in order to obtain additional information about which physics lies beyond the Standard Model it is desirable to find evidences for the higher-order terms in the effective Lagrangian, which can be inferred from the observation of flavour violating processes in the charged lepton sector. For instance, the dimension-six operators $\bar{e}_{Ri}\sigma_{\mu\nu}L_j\phi B_{\mu\nu}$ and $\bar{e}_{Ri}\sigma_{\mu\nu}\tau_I L_j\phi W^I_{\mu\nu}$, induce processes such as $\mu \to e\gamma$ or $\tau \to e\gamma$, whereas $(\bar{L}_i\gamma^\mu L_j)(\bar{L}_k\gamma_\mu L_l)$, $(\bar{e}_i\gamma^\mu e_j)(\bar{e}_k\gamma_\mu e_l)$ and $(\bar{L}_i\gamma^\mu e_j)(\bar{e}_k\gamma_\mu L_l)$ induce processes such as $\mu^+ \to e^+e^-e^+$ or $\tau^+ \to \mu^+\mu^-\mu^+$ [5]. There are presently very stringent constraints on these operators. For example, the lowest-dimension operator which induces the process $\mu \to e\gamma$ is $\mathcal{L} = -m_\mu\bar{\mu}(f^{\mu e}_{M1} + \gamma_5 f^{\mu e}_{E1})\sigma^{\mu\nu}eF_{\mu\nu} + \text{h.c.}$, where $f^{\mu e}$ are form factors. A reasonable parametrization of the form factors is $f^{\mu e} \sim \theta^2_{\mu e}\alpha/\Lambda^2$, which takes into account that these operators usually appear at the one-loop level. Then, the present experimental constraint on $\text{BR}(\mu \to e\gamma)$ implies $\Lambda \gtrsim 20\,\text{TeV}$ for generic mixing angles, $\theta_{\mu e} \sim 1/\sqrt{2}$. Conversely, if the new particles appear at the electroweak scale, $\Lambda \sim 300\,\text{GeV}$, the mixing angle has to be rather small, $\theta_{\mu e} \lesssim 0.01$. This fact has dramatic implications for new physics: many extensions of the Standard Model postulate new particles at the electroweak scale which couple to the leptons, therefore the non-observation of the process $\mu \to e\gamma$ imposes very severe constraints on these models. Conversely, from the optimistic point of view, the discovery of $\mu \to e\gamma$ might be around the corner.

This is in particular the case for the supersymmetric type I see-saw model. In this model, the neutrino Yukawa coupling introduces at tree level sources of lepton flavour violation in the interactions between the right-handed neutrino, the left-handed lepton and the up-type Higgs chiral superfields. Being the right-handed neutrinos and sneutrinos so heavy, this flavour violation decouples completely at low energies, since the dimension-six operator is suppressed by a large mass scale. Interestingly, this lepton flavour violation is transmitted to the soft supersymmetry breaking parameters via the quantum corrections, necessarily inducing at low energies flavour violating terms in the left-handed slepton mass matrices and in the leptonic trilinear term, with just a logarithmic dependence on the right-handed neutrino masses [6]:

$$(\mathbf{m}_L^2)_{ij} \simeq -\frac{1}{8\pi^2}(3m_0^2+|A_0|^2)(h_\nu^\dagger h_\nu)_{ij}\log\left(\frac{M_{\rm X}}{M}\right), \tag{6}$$

where m_0 and A_0 are the universal soft scalar mass and trilinear term and $M_{\rm X}$ is a cut-off, usually identified with the GUT scale. The lepton flavour violation in the scalar sector is suppressed by the loop factor, but can have a rather large impact in low energy phenomena, since the dimension-six operator generated is suppressed only by the scale of the scalar masses, which presumably lies between 100 GeV and 1 TeV.

The lepton flavour violating effects in the type I see-saw scenario are connected to the neutrino Yukawa couplings and the right-handed neutrino Majorana masses. These are the same parameters which generate the neutrino masses, therefore it is very important to analyze whether there is any connection between the rates for the lepton flavour violating processes and the measured neutrino parameters. Unfortunately, this is not the case. The complete see-saw Lagrangian contains twelve real parameters and six phases, whereas neutrino observations can fix at most six real parameters and three phases. Therefore, there are six real parameters and three phases which are completely unconstrained from neutrino observations and that prevent any model-independent prediction for the lepton flavour violating processes. Indeed, there are, compatible with the observed neutrino parameters, an infinite set of Yukawa couplings [7]: $h_\nu = \sqrt{D_M}R\sqrt{D_m}U_{\rm lep}^\dagger/\langle\phi^0\rangle$. Here, D_m is a diagonal matrix with the neutrino masses and $U_{\rm lep}$ is the leptonic mixing matrix, which can be in principle measured with experiments. On the other hand, D_M is a diagonal matrix with the right-handed neutrino masses and R is a complex orthogonal matrix, which cannot be determined with low energy experiments and are thus free parameters. Therefore, by changing R and the right-handed neutrino masses, any matrix $h_\nu^\dagger h_\nu$ can be obtained, and thus any value for the lepton flavour violating effects. Furthermore, it can be shown that there is a one-to-one correspondence between the high energy see-saw parameters $\{h_\nu, M\}$ and the low energy parameters that determine any possible low energy observable consequence of the see-saw mechanism in the fermionic sector and in the scalar sector, $\{\mathcal{M}_\nu, h_\nu^\dagger h_\nu\}$ [8]. As a consequence, from the mathematical point of view *any* low energy observation can be accommodated by a set of high energy see-saw parameters (at the price, perhaps, of tuning parameters).

Remarkably, under some well-motivated assumptions about the high energy parameters, it is possible to derive predictions for the lepton flavour violating processes. Namely, one can impose the absence of tunings among parameters and that the eigenvalues of the neutrino Yukawa coupling are hierarchical (as occurs in all Yukawa matrices known).

By assuming the absence of tunings it is possible to derive a lower bound on the rate for the process $\mu \to e\gamma$ as a function of the rates for the processes $\tau \to \mu\gamma$ and $\tau \to e\gamma$. Let us assume that the processes $\tau \to \mu\gamma$ and $\tau \to e\gamma$ are both observed. The

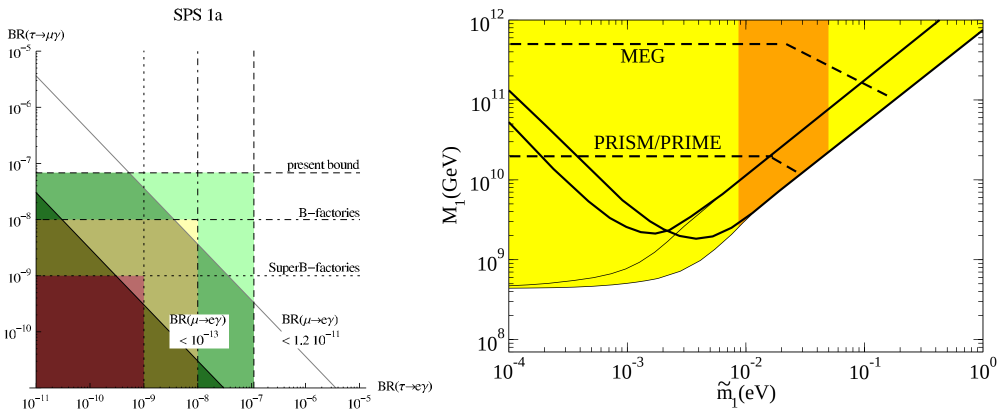

Fig. 1. – Left panel: constraints on the rare tau decays from present B-factories and from the non-observation of $\mu \to e\gamma$ in the type I see-saw mechanism for generic SUSY parameters. Right panel: constraints on the SUSY leptogenesis parameter space from the non-observation of $\mu \to e\gamma$. The "natural" region of this parameter space is shown in darker colour.

observation of the former implies new sources of tau and muon flavour violation, while the observation of the latter, new sources of tau and electron flavour violation. Therefore, the new physics that induces these two processes violate all flavour quantum numbers and hence this same physics necessarily generates at some level the process $\mu \to e\gamma$ [9,10]. Consequently, the following bound holds:

$$\mathrm{BR}(\mu \to e\gamma) \gtrsim C \times \mathrm{BR}(\tau \to \mu\gamma)\mathrm{BR}(\tau \to e\gamma), \tag{7}$$

where C is a model-dependent constant. This bound is saturated when the lepton flavour violation in the $\mu - e$ sector only appears at higher order, via the combination of $\mu - \tau$ and $\tau - e$ flavour violation, whereas much larger rates can arise if there is "direct" $\mu - e$ flavour violation. The impact of this constraint for the SUSY see-saw model is illustrated in fig. 1, left panel, for a typical choice of the SUSY parameters (the SPS1a benchmark point). It follows from eq. (7) that the present constraint on the rate of the process $\mu \to e\gamma$ rules out the possibility of observing *both* processes $\tau \to \mu\gamma$ and $\tau \to e\gamma$ in present B-factories. Moreover, if the MEG experiment reaches the sensitivity $\mathrm{BR}(\mu \to e\gamma) \sim 10^{-13}$ without finding a positive signal, the possibility of observing both rare tau decays at future super$B$-factories will also be ruled out. Conversely, if present B-factories observe both rare tau decays, the supersymmetric see-saw model will be disfavoured [10].

Moreover, assuming the absence of cancellations and that the neutrino Yukawa eigenvalues are hierarchical, it is possible to derive a lower bound on the process $\mu \to e\gamma$ as a function of the lightest right-handed neutrino mass [11]:

$$\mathrm{BR}(\mu \to e\gamma) \gtrsim 1.2 \times 10^{-11} \left(\frac{M_1}{5 \times 10^{12}\,\mathrm{GeV}}\right)^2 \left(\frac{m_S}{200\,\mathrm{GeV}}\right)^{-4} \left(\frac{\tan\beta}{10}\right)^2, \tag{8}$$

which allows to set an upper bound on the lightest right-handed neutrino mass from the constraint $\mathrm{BR}(\mu \to e\gamma) \leq 1.2 \times 10^{-11}$ [12], namely $M_1 \lesssim 5 \times 10^{12}$ GeV for typical SUSY parameters. This expression also allows to establish an interesting connection between baryogenesis through leptogenesis and the rate for $\mu \to e\gamma$. The leptogenesis mechanism to generate the observed matter-antimatter asymmetry in our Universe requires a rather large mass scale for the right-handed neutrinos, $M_1 \gtrsim 10^9\,\mathrm{GeV}$ [13]. Therefore, if leptogenesis is the correct mechanism to explain the matter-antimatter asymmetry in our Universe, it follows from eq. (8) that $\mathrm{BR}(\mu \to e\gamma) \gtrsim 5 \times 10^{-19}$ for typical SUSY parameters. Conversely, the non-observation of $\mu - e$ flavour violation in the charged lepton sector constrains the parameter space of leptogenesis, spanned by the lightest right-handed neutrino mass and by the washout parameter $\widetilde{m}_1 = (h_\nu h_\nu^\dagger)_{11}/M_1$. This is illustrated in fig. 1, right panel, where we show the region of the SUSY leptogenesis parameter space (adapted from [14]) that can be probed in present and future experiments searching for $\mu - e$ flavour violation. Furthermore, it can be shown that if there are no cancellations in the parameters that determine the washout of the baryon asymmetry, then $\sqrt{\Delta m^2_{\mathrm{sol}}} \lesssim \widetilde{m}_1 \lesssim \sqrt{\Delta m^2_{\mathrm{atm}}}$ (displayed as a darker region in the figure), which gives a more stringent lower bound on M_1. Therefore, in the absence of tunings, $\mathrm{BR}(\mu \to e\gamma) \gtrsim 5 \times 10^{-18}$. This sensitivity to $\mu - e$ lepton flavour violation is difficult to reach in experiments searching for $\mu \to e\gamma$, although it is not far from the projected sensitivity of future experiments searching for $\mu - e$ conversion in nuclei. Namely, the PRISM/PRIME experiment at J-PARC aims to achieve a single event sensitivity to the process $\mu\ \mathrm{Ti} \to e\ \mathrm{Ti}$ at the level of 10^{-18} [15], which is equivalent to a sensitivity to the process $\mu \to e\gamma$ at the level of $\sim 2 \times 10^{-16}$. One should also bear in mind that the lower bound $\mathrm{BR}(\mu \to e\gamma) \gtrsim 5\times 10^{-18}$ relies on extremely conservative assumptions, therefore if the SUSY leptogenesis mechanism is the actual origin of the observed matter-antimatter asymmetry in our Universe, there are good chances to observe $\mu - e$ flavour violation in future experiments; probing the whole parameter space of leptogenesis is unfortunately out of the reach of projected experiments.

5. – Conclusions

Many experiments have reported the observation of flavour violation in the neutrino sector, which can be described by adding to the Standard Model Lagrangian a new term of dimension four (Dirac neutrinos) or five (Majorana neutrinos). We have analyzed these two possibilities from the theoretical point of view and we have argued that the striking differences between neutrino and quark parameters are most naturally explained if neutrinos are Majorana particles. We have also discussed the dimension-six operators which presumably appear in the effective Lagrangian and which induce lepton flavour violation in the charged lepton sector. In supersymmetric scenarios these operators are only mildly suppressed, opening the possibility of observing charged lepton flavour violation in the next round of experiments. Lastly, we have analyzed in some detail some predictions for the lepton flavour violating processes in the supersymmetric type I see-saw mechanism and the connection to the observed neutrino parameters.

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