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QCD and hadronic interactions with initial-state radiation at B-factories

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Summary. — The efforts to improve on the precision of the measurement and theoretical prediction of the anomalous magnetic moment of the muon a_μ have turned into a test of our understanding of the hadronic contribution to vacuum polarisation. I describe how recent measurements of hadron production in e^+e^- interactions with initial-state radiation provide precision measurements of the hadron cross section, and have improved on the contribution to the prediction of the value of a_μ that dominates the global uncertainty.

PACS 13.66.Bc – Hadron production in e^-e^+ interactions.
PACS 12.38.Qk – Quantum chromodynamics: Experimental tests.
PACS 13.40.Em – Electric and magnetic moments.

1. – Introduction

Elementary particles have a magnetic moment $\vec{\mu}$ proportional to their spin $\vec{s}$, with $\vec{\mu} = (ge)/(2m)\vec{s}$. While pointlike Dirac particles would have $g = 2$, *i.e.* an "anomalous" relative deviation of $a \equiv (g-2)/2 = 0$, Nafe *et al.* observed the first hints of a significant deviation from $a_e = 0$ more than 60 years ago [1]. The following year, Schwinger computed [2] the first-order contribution to a, equal to $\alpha/(2\pi)$, the diagram for which is shown in fig. 1-left.

The development of quantum electro-dynamics (QED) followed, and later of gauge theories in general, making these early works the very basis of our present understanding of the elementary world. Tremendous efforts have been devoted to improving the precision of the theoretical prediction and of the direct measurement of a since then [3].

More than 60 years later the situation is pretty exciting, with the experimental and theoretical precision on the anomalous magnetic moment of the muon a_μ both of the order of $6. \times 10^{-10}$, and a discrepancy of $(29 \pm 9) \times 10^{-10}$ between them, *i.e.* amounting to 3.2 σ, should Gaussian statistics be assumed (table I).

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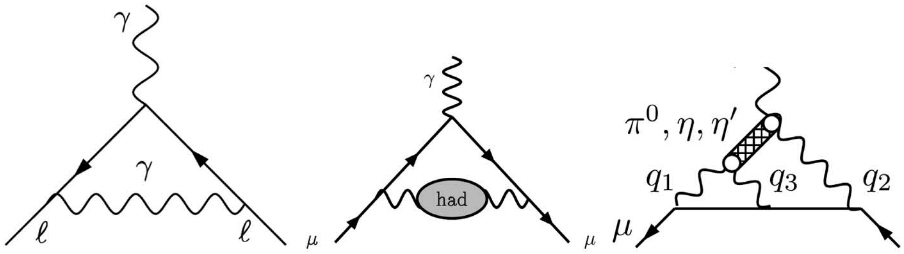

Fig. 1. – Left: 1st order contribution to a. Center: Lowest-order hadronic VP diagram. Right: A light-by-light diagram [3].

The largest contribution to a_μ by far is from QED but its contribution to the uncertainty is negligible. In terms of uncertainty, the main contribution is from the hadronic component of the one-loop vacuum polarisation (VP, fig. 1-center) and, to a lesser extent, from the hadronic component of the light-by-light processes (fig. 1-right).

The photon propagator with VP is obtained from the bare propagator by replacing the electric charge e by the energy-dependent quantity

$$e^2 \to e^2/\left[1 + (\Pi'(k^2) - \Pi'(0))\right],$$

where k is the photon 4-momentum. At low energy, hadronic processes are not computable with the desired precision. Instead the VP amplitude $\Pi'(k^2)$ is obtained from the dispersion relation

$$\Pi'(k^2) - \Pi'(0) = \frac{k^2}{\pi} \int_0^\infty \frac{\operatorname{Im}\Pi'(s)}{s(s - k^2 - i\epsilon)} \mathrm{d}s,$$

which in turn is related through the optical theorem

$$\operatorname{Im}\Pi'(s) = \alpha(s) R_{\text{had}}(s)/3$$

Table I. – *Summary of the contribution to the theory prediction of the value of a_μ, compared with the experimental measurement* [3].

QED	11 658 471.81	±0.02
Leading hadronic VP	690.30	±5.26
Sub-leading hadronic VP	−10.03	±0.11
Hadronic light-by-light	11.60	±3.90
Weak (incl. 2-loops)	15.32	±0.18
Theory	11 659 179.00	±6.46
Experiment [4]	11 659 208.00	±6.30
Exp – theory	29.00	±9.03

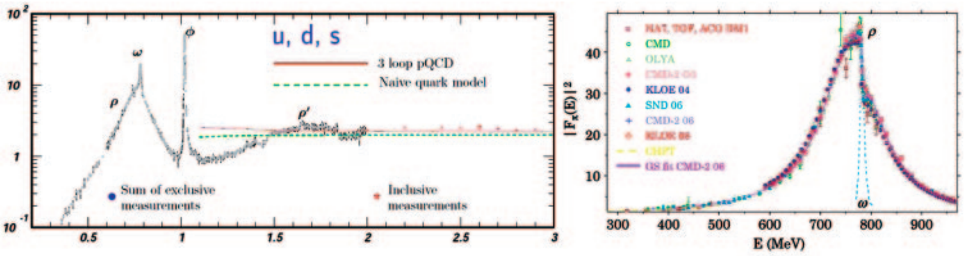

Fig. 2. – Left: $R_{\rm had}$ as a function of $\sqrt{s}$ (GeV) [5]. Right: variation of the pion form factor squared with energy [3] (KLOE 04 is superseded by KLOE 08).

to the ratio

$$R_{\rm had}(s) = \sigma_{\rm had}\frac{3s}{4\pi\alpha(s)} = \frac{\sigma_{e^+e^-\to{\rm hadrons}}}{\sigma_{e^+e^-\to\mu^+\mu^-}}.$$

Finally, the hadronic VP contribution is obtained from the "dispersion integral"

$$a_\mu^{\rm had} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int \frac{R_{\rm had}(s)\hat{K}(s)}{s^2}{\rm d}s,$$

where $\hat{K}(s)$ is an analytical function that takes values close to 1. We note, from the $1/s^2$ variation of the integrand, that the dominant contribution comes from the low energy part of the integral. A good experimental precision of the measurement of $R_{\rm had}(s)$ at low energy is therefore welcome. Figure 2-right shows a summary of the present measurements of $R_{\rm had}(s)$ [5], where the presence of $J^{PC} = 1^{--}$ mesons can be seen.

The $\pi^+\pi^-$ channel has both the largest contribution and dominates the uncertainty, with $a_\mu^{\pi^+\pi^-}[2m_\pi, 1.8\,{\rm GeV}/c^2] = (504.6 \pm 3.1({\rm exp}) \pm 0.9({\rm rad})) \times 10^{-10}$, compared to the full $a_\mu^{\rm had} = (690.9 \pm 5.3) \times 10^{-10}$ from table I. A summary of direct measurements in terms of the squared pion form factor is shown in fig. 2-left. The 3.2 σ discrepancy mentioned above is computed using this $\pi\pi$ input.

2. – τ decay spectral functions

The $I = 1$ part of the $e^+e^- \to \pi^+\pi^-$ cross-section can be estimated from the spectral function of τ decays to $\nu\pi^+\pi^0$, under the hypothesis of conservation of the weak vector current (CVC). The method was pioneered by the ALEPH Collaboration [6] and followed by OPAL [7] and CLEO [8]. Recently the Belle Collaboration has performed an analysis using much larger statistics [9], obtaining a value compatible with, and more precise than, the combination of all previous results [10].

The τ method provides a high experimental precision, but extracting the contribution to a_μ depends on making a number of isospin-breaking (IB) corrections. A recent update [11] of [10] lowers the correction by $\approx 7 \times 10^{-10}$, while the uncertainty on the correction is now 1.5×10^{-10}.

The branching fraction of the $\tau \to \nu\pi^+\pi^0$ decay also takes part in the calculation, with a 0.5% uncertainty.

3. – $e^+e^- \to \pi^+\pi^-$ using ISR method

Initial-state radiation (ISR) makes it possible to measure the cross-section of the production of a final state f in e^+e^- collisions at a squared energy s, over a wide range of energies, lower than $\sqrt{s}$, through the radiation of a high energy photon by one of the incoming electrons, after which the electrons collide at a squared energy s'.

The BaBar experiment has developed a systematic program of measurements of cross-sections of e^+e^- to hadrons at low energy using the ISR method [12]. The boost undergone by the final state f provides an excellent efficiency down to threshold. In all studies by BaBar, the ISR photon is observed (γ-tag) and its direction is compared to the direction predicted from the direction of f, providing a powerful rejection of background noise. Most of these measurements are more precise than the previously available results by about a factor of three.

3˙1. *KLOE's result on* $e^+e^- \to \pi^+\pi^-$. – The KLOE experiment, when running on the ϕ resonance, studied the e^+e^- annihilations to $\pi^+\pi^-$ with the ISR method [13]. Here the ISR photon is not reconstructed: the requirement that the photon direction be compatible with its having been emitted in the beam pipe allows mitigation of the background to some extent, but the systematical uncertainty on background subtraction is still a major component of the total uncertainty. The radiator function is provided from simulation, with systematics of 0.5%, and is the other major component.

The value of $a_\mu^{\pi^+\pi^-}$ obtained is compatible with the combination of previous results by CMD-2 & SND over the mass range that they have in common of (630–958 MeV/c^2).

3˙2. *BaBar's result on* $e^+e^- \to \pi^+\pi^-$. – BaBar uses a different approach: photon tagging with the ISR luminosity obtained from the muon channel, $e^+e^- \to \mu^+\mu^-\gamma$ [14]. The systematics related to additional radiation is minimized in this NLO measurement, *i.e.* radiation of one possible additional photon is allowed, so that the final states actually reconstructed are $\pi^+\pi^-\gamma(\gamma)$ and $\mu^+\mu^-\gamma(\gamma)$. The "bare" ratio $R_{\mathrm{had}}(s')$ mentioned above is obtained from the experimentally measured $R_{\mathrm{exp}}(s')$ after correction of final state radiation (FSR) in $e^+e^- \to \mu^+\mu^-$ and of additional FSR in ISR events $e^+e^- \to \mu^+\mu^-\gamma$.

A number of important systematics cancel when measuring the π/μ ratio, such as those associated with the collider luminosity, the efficiency of the reconstruction of the ISR photon, and the understanding of additional ISR radiation.

The limiting factor is then the understanding of the possible "double" $\pi-\mu$, MC-data efficiency discrepancies. These are studied in detail, with methods designed to disentangle correlations as much as possible. For example, inefficiency of the track-based trigger is studied using events selected with a calorimetry-based trigger—the small correlation between both triggers being studied separately. Likewise, μ and π particle identification (PID) efficiency is studied in good-quality, two-track ISR events, in which either one, or both, tracks meet the PID selection criteria. Concerning tracking, a sizable degradation of the efficiency for tracks overlapping in the detector was observed and studied in detail.

The systematics finally obtained are of the order of, or smaller than, 1% over the whole mass range studied; *i.e.*, from threshold to 3 GeV/c^2.

The $e^+e^- \to \pi^+\pi^-$ cross section measured by BaBar is shown in fig. 3. The sharp drop due to the interference between the ρ and the ω is clearly visible. The interference between the successive radial excitations of the ρ induces these dips in the cross-section.

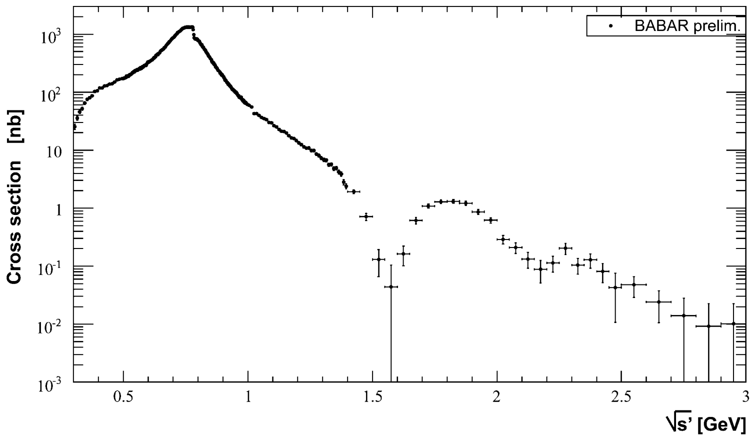

Fig. 3. – Bare, unfolded [15], $e^+e^- \to \pi^+\pi^-$ cross section measured by BaBar, using the ISR method [14].

The measured value of $a_\mu^{\pi^+\pi^-}[2m_\pi, 1.8\,\mathrm{GeV}/c^2] = (514.1 \pm 2.2 \pm 3.1) \times 10^{-10}$ has a precision similar to the combination of all previous e^+e^--based results, but is larger by about 2.0 σ.

In addition to the measurement of the π/μ ratio, and extraction of the $e^+e^- \to \pi^+\pi^-$ cross section, BaBar has compared its $\mu^+\mu^-$ spectrum to the Monte Carlo prediction, finding a good agreement within $0.4 \pm 1.1\%$, dominated by the collider luminosity uncertainty of $\pm 0.9\%$.

The distribution of the squared pion form-factor is fitted with a vector-dominance model including the resonances $\rho, \rho', \rho'', \omega$, with the ρ's being described by the Gounaris-Sakurai model. The fit (figure in ref. [16]) yields a good χ^2/n_{df} of 334/323, and parameters compatible with the world-average values. BaBar can then use the fitted model to compare their result with that of previous measurements (fig. 4). The BaBar result is a bit larger than that obtained by CMD2 [17] and SND [18], nicely compatible with the high-statistics τ-based result by Belle, but shows a clear disagreement with KLOE.

4. – a_μ: the present situation

The present situation in terms of a_μ is summarized in fig. 5:

– The four upper points show that there is a general agreement between the various recent combinations of direct e^+e^--based $\pi^+\pi^-$ measurement [19, 20, 11].

– The large discrepancy between computations of a_μ based on these and the experimental measurement by BNL-E821 [4] is clear.

– The combination of τ-based results, when corrected for isospin-breaking effects using the most recent calculation [11], is also significantly lower than the experimental measurement [4], by 1.8 σ.

– My computation of a_μ using the BaBar $\pi^+\pi^-$ measurement [14] only is larger than the combination of previous e^+e^--based measurements, and compatible with the τ-based result, but still 2.4 σ away from BNL-E821 [4].

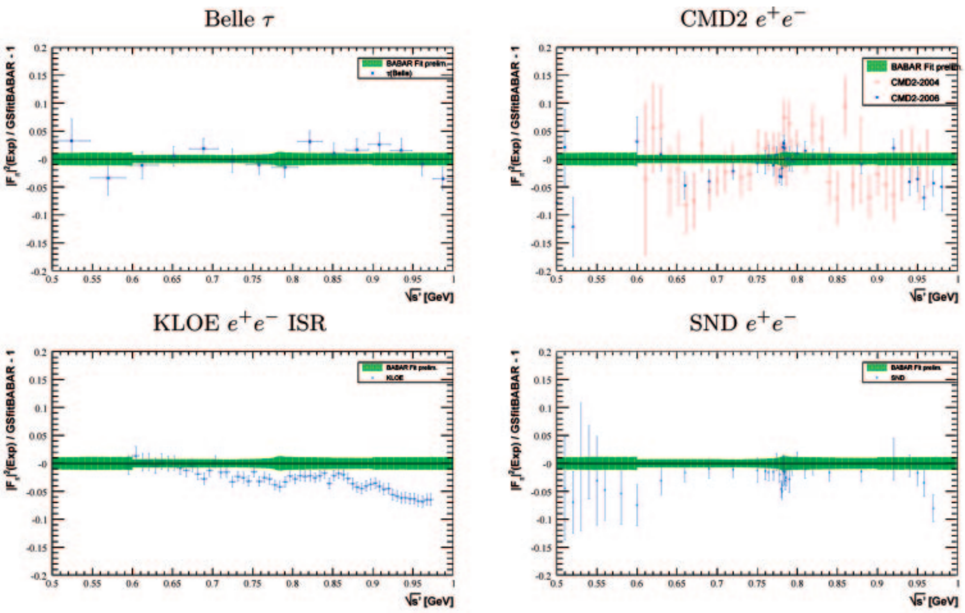

Fig. 4. – Relative difference between the BaBar result with that of previous experiments.

– The combination of all e^+e^--based measurements, including the recent one by BaBar, shows an uncertainty that has decreased significantly, and a central value that is larger. However, the significance of the difference with respect to BNL-E821 [4] is barely changed, of the order of 3.3 σ.

A more sophiticated combination of the available results, published recently [21], yields similar numbers.

5. – What might take place during this decade

5·1. a_μ *measurement*. – One single high-precision statistics-dominated experimental measurement of a_μ [4] is facing a prediction in which the contribution with largest

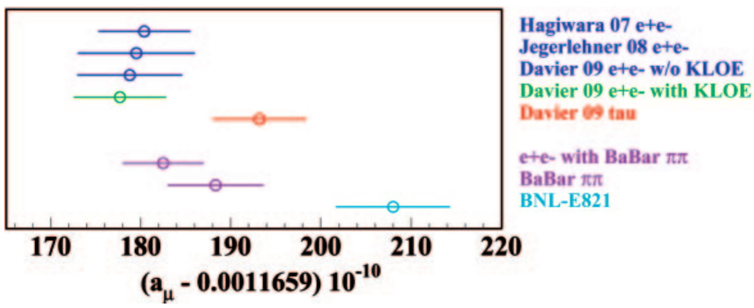

Fig. 5. – a_μ: the present situation.

uncertainty has been confirmed, within reasonable significance, by a number of measurements using various methods, each affected by its own systematics.

The obvious next step, before calling for new physics, is therefore to check the measurement:

- A new collaboration is planning to move the experimental apparatus from BNL to FNAL, and perform a new measurement with statistics increased by a factor of 50, and reduced systematics, bringing the experimental uncertainty down to 0.14 ppm, *i.e.* 1.6×10^{-10} [22].

- It would obviously be intensely desirable to cross-check such a measurement using a completely different set-up. An alternative scheme is explored at J-PARC, with a micro-emittance muon beam inside a high-precision magnetic field, mono-magnet storage "ring" [23].

5˙2. *Prediction.* – On the prediction side, the main effort is understandably devoted to the hadronic VP contribution.

- BaBar will complete its ISR program and provide measurements of all possible hadronic final states in the low energy range relevant to this discussion.

- Belle may check BaBar's $\pi^+\pi^-$ measurement and BaBar may check Belle's τ spectral functions. KLOE is working on an analysis with photon tagging too.

- BES-III will measure $R_{\text{had}}(s)$ in the range 2.0–4.6 GeV, something that will improve on a_μ only marginally, but will also measure the $\tau \to \nu\pi^+\pi^0$ branching fraction with improved precision [24], an important ingredient in the use of the τ-based spectral functions.

- The recent calculation of isospin-breaking corrections [11] will doubtlessly be cross-checked by other authors.

- The collider at Novosibirsk has been upgraded to VEPP-2000 [25], and the CMD [26] and SND experiments too.

Following the vacuum polarisation, the next target in line for improvement is the contribution of light-by-light scattering. Here too work is in progress and there is hope to improve the precision, both theoretically [27], and using results of the $\gamma\gamma$ programe at $DA\Phi NE$-2 [28].

In total, there is good hope to bring both the prediction and experimental uncertainties of a_μ at a very few 10^{-10}.

I regret I did not have the time to present the implications of the a_μ discrepancy, if assumed to be due to an underestimated hadronic cross-section, on the estimation of the Higgs mass [29]. Interpretation of the discrepancy as being due to contribution of yet-unknown heavy object(s) in loops is also an interesting possibility [30].

Finally, at higher energy, ISR can be used to understand QCD by exploring the new spectroscopy of $J^{PC} = 1^{--}$ charmonium-like states, opened by the discovery by BaBar of the $Y(4260)$ meson [31].

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